Intraday Liquidity and Money Market Dislocations*

Adrien d'Avernas[†] Baiyang Han[‡] Quentin Vandeweyer[§]

December 2024

Abstract

This paper proposes a new model of monetary policy implementation to account for two key developments: (i) the introduction of intraday liquidity requirements and (ii) the decreasing relevance of the federal funds market in favor of repurchase agreement (repo) markets with nonbank participants. Our paper studies how liquidity requirements prevent banks from arbitraging between the fed funds and repo markets and generate large repo spikes. We propose a simple measure of excess intraday reserves. Consistent with our theory, this metric is close to zero in 2019Q2, when US repo markets experienced a spike of 400 basis points.

Keywords: Repo, Money Markets, Reserves, Monetary Policy Implementation

JEL Classifications: E43, E44, E52, G12

*We dedicate this work to the honored memory of Livia Amato, to whom we owe a great debt of gratitude for her substantial contributions to this paper. A longer working paper version of this article was circulated under the same title with additional insights on Treasury markets. Many of those are now separately included in d'Avernas, Peterson and Vandeweyer (2023). We would like to thank Arvind Krishnamurthy, Simon Potter, Eric Fischer, Raghuram Rajan, Vincent Skiera, and Moritz Lenel for their valuable discussions as well as participants in seminars, workshops, and conferences at the University of Chicago: Booth, NYU: Stern, Duke Fuqua, UPenn: Wharton, the BI-SHoF Conference 2020, and the 2020 Macro-Finance Society Fall Meeting, the 2021 mini-symposium on Funding Markets, and 2022 Eastern Finance Association Meeting. We acknowledge gracious support from the Fama-Miller Center for Research in Finance. Wharton Research Data Services (WRDS) was used to prepare this article. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

[†]Stockholm School of Economics, adrien.davernas@hhs.se

[‡]University of Chicago Booth School of Business, baiyang.han@chicagobooth.edu

[§]University of Chicago Booth School of Business, quentin.vandeweyer@chicagobooth.edu

Over the last decade, the monetary policy frameworks of developed economies have transformed substantially, with a large increase in reserves following several rounds of quantitative easing, new banking regulations, and the decreasing relevance of unsecured interbank markets (such as the federal funds market) in favor of secured repurchase agreement markets (repo) featuring nonbank participants.[1] However, frequent disruptions in those repo markets—exemplified by the mid-September 2019 overnight surge in US dollar repo rates to 7%—are a source of concern about the sustainability of this new regime. These disruptions are particularly surprising as they take place after several rounds of quantitative easing (QE) policies, which have increased the quantity of reserves available to banks by several orders of magnitude beyond their reserves requirement.

Our goal in this work is to propose a consistent framework that accounts for these key innovations in monetary policy implementation and can be used to investigate the source of the observed disruptions. To do so, we update the seminal model of monetary policy implementation by Poole (1968)[2] with two key new elements: a nonbank sector that does not have access to the discount window at the central bank and an intraday liquidity constraint on banks as mandated by Basel III and implemented by Regulation YY in the US. We find that the latter element drastically reduces the elasticity of the reserves supply elasticity in daily payment flows. The main contributions of this paper are (i) to show that the combination of these two new elements can explain the recent volatility in US repo markets and require the Federal Reserve (Fed) to maintain a balance sheet larger than previously thought and (ii) to propose a new quantitative measure for excess *intraday* reserves, the relevant metric in this new regime.

Our model features two agents—banks and shadow banks—that trade in two money markets: a pure interbank market called *fed funds* and a bank-to-shadow-bank market called *repo*. Only banks can hold reserves at the Fed and have access to the discount window, to which they resort when their end-of-day reserves balance is short of the overnight requirement. Moreover, banks are connected through a real-time gross settlement (RTGS) system, which clears banks' reserves accounts in real-time. This RTGS system implies that reserves are instantaneously redistributed across banks following exogenous shocks driven by payment system activities. During the day, banks have the option to trade in both the fed funds and repo markets.

[1] As documented by Afonso, Entz and LeSueur (2013), the fed funds market has significantly reduced in size since 2007, from around $200 bn to $50 bn, while the remaining volumes are mostly driven by institutions not receiving interest on reserves.

[2] We note that Poole (1968) is referred to by most monetary economics textbooks as the seminal article on monetary policy implementation (see Bindseil, 2014; Walsh, 2017).

Shadow banks use the repo market to roll over a portfolio of illiquid securities. This feature captures the increasing involvement of shadow banking institutions, such as hedge funds, in arbitrage trades of asset mispricing financed with high leverage in repos.[3] Our main departure from Poole (1968) is to consider explicitly how requiring banks to hold a positive level of reserves *at each point during the day* as mandated by regulation YY in the US, affects the stability of repo markets and the liquidity provision to shadow banks.

We first show that, in an economy in which intraday liquidity is freely available through Fed overdrafts, banks always act as intermediaries between the repo and fed funds markets and prevent the repo rates from rising above the discount window rate. Although this result is standard in the setting of Poole (1968), our framework highlights its implicit reliance on the flexible intraday liquidity provision by the Fed, which is key to resolving the temporal mismatch of banks' disbursing reserves within the day when lending while only accessing the discount window at the end of the day. Consequently, intraday reserve flows are completely inconsequential as long as intraday overdrafts are accessible to banks. This result holds even in an economy with active shadow banks and featuring low reserve balances.

We then explore the consequences of introducing a regulatory intraday liquidity limit on traditional banks in an economy featuring shadow banks. Once banks have reached that limit, they are unable to further lend in fed funds or repo markets. As a consequence, the provision of repo supply is rationed, and its rate jumps up to the marginal cost of fire-sale portfolio liquidations for shadow banks, which cannot access the discount window. This mechanism formalizes the insight from Pozsar (2019) that an intraday liquidity constraint turns an elastic *credit system* into an inelastic *token system*, limiting the supply of reserves available for banks to settle overnight transactions such as repos to shadow banks. As those shadow banks don't have direct access to discount window liquidity, a scarcity of intraday reserves leads to a sharp increase in repo rates.

We conclude by proposing a simple formula to estimate the level of reserves in excess of intraday liquidity needs as the total quantity of excess reserves minus the portion of these

[3] Before 2008, securities dealers were not balance sheet constrained and, hence, largely active in multiple arbitrage trades, which enforced the law-of-one-price across markets. Since the financial crisis and subsequent regulations, large deviations from the law-of-one-price have emerged on multiple markets (Siriwardane, Sunderam and Wallen, 2022), and relative-value hedge funds have become increasingly active in those arbitrage trades. For instance, as documented by Barth and Kahn (2023), several hedge funds trade on mispricing between Treasuries and Treasuries' futures contracts while being financed in repos. Those hedge funds eventually suffered large losses in both September 2019 and March 2020. We further refer to d'Avernas, Peterson and Vandeweyer (2023) for a dynamic model in which those positions arise endogenously as a consequence of regulatory arbitrage created by leverage regulation such as the Supplementary Leverage Ratio.

reserves already mobilized in repo lending activities and in clearing flows occurring from ordinary payment system activities. We show that, unlike the overnight excess reserves, our intraday-adjusted metric is close to zero in 2019. With regulatory intraday, our model suggests that this shortage of intraday liquidity—brought about by the gradual reversal of QE policies from the Fed started in 2015—is likely responsible for the concomitant sharp increase in repo rates in 2019.[4]

Related Literature This paper contributes to the literature on monetary policy implementation. Adapting Poole (1968) to a modern over-the-counter setting, Afonso and Lagos (2015) and Bianchi and Bigio (2022) study disruptions in the over-the-counter interbank market and capture many elements of pre-2008 fed funds dynamics, including intraday liquidity movements.

In particular, this work contributes to a large literature studying the effect of regulation on the pass-through of monetary policy to various markets. Andersen, Duffie and Song (2019); Bech and Klee (2011); Duffie and Krishnamurthy (2016) show that leverage ratio regulation such as the Supplementary Leverage Ratio (SLR) creates a form of debt overhangs (Myers, 1977) and generates dispersion in many money market rates, with fed funds and repo rates typically trading below IOR. d'Avernas and Vandeweyer (2024) shows that leverage regulations also result in a segmentation of money markets in which the T-bill supply is the main driver of money market rates. Diamond, Jiang and Ma (2023) show that after a certain threshold, increasing the number of reserves through QE may crowd out bank lending. This paper contributes to this literature by modeling the driver of sporadic disruptions in repo rates above the discount window rate and by relating those spikes to the quantity of intraday liquidity available to banks.

In contemporaneous empirical work, Copeland, Duffie and Yang (2022) come to a similar conclusion that the supply of reserves is scarcer than previously believed due to a shortage of intraday liquidity by analyzing the timing of intraday Fedwire flows. We complement their work by providing a theoretical framework to understand the impact of intraday liquidity regulations. Afonso et al. (2022) estimate the slope of the reserves

[4] This intraday liquidity-centric explanation also anecdotally corresponds to declarations of market participants such as that of JPMorgan Chase CEO Jamie Dimon during a call report on Oct. 13, 2020: *"[W]e have $120 billion in our checking account at the Fed, and it goes down to $60 billion and then back to $120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there's extreme stress during the course of the day, it doesn't go below zero. If you go back to before the crisis, you'd go below zero all the time during the day. So the question is, how hard is that as a red line? That will be up to regulators to decide, but right now, we have to meet those rules, and we don't want to violate what we told them we are going to do."*

demand in the fed funds market; and Lopez-Salido and Vissing-Jorgensen (2023) stress the importance of bank deposits for its level. Building on earlier literature on intraday payment dynamics (Bech and Garratt, 2003; McAndrews and Rajan, 2000), Yang (2023) proposes a microeconomic model in which repo spikes appear as a consequence of strategic complementarity in intraday payment timing among banks. Afonso et al. (2020); Correa, Du and Liao (2021); Du, Hébert and Li (2023), and Avalos, Ehlers and Eren (2019) explore potential explanations for the September 2019 repo rate spike and point to the role of large global dealer-banks with balance sheet constraints. Kahn et al. (2023) finds that a confluence of factors caused the 2019 spike, exacerbated by a lack of price transparency along repo market segments. Building on this work, d'Avernas, Peterson and Vandeweyer (2023) study how repo spikes can spread to the Treasury market in a dynamic model with both leverage ratio and intraday liquidity constraints.

I Intraday Liquidity Regulation

This paper contributes to the literature by studying how the imposition of constraints on intraday liquidity flows in the payment system may disrupt short-term money markets. Such constraints were introduced by Basel III's emphasis on intraday liquidity through Principle 8[5] and implemented through several liquidity rules, tests, and supervision. In the US, Regulation YY includes rules covering intraday liquidity exposures through liquidity stress tests known as Resolution Liquidity Execution Need (RLEN) and Resolution Liquidity Adequacy and Positioning (RLAP) imposing failure planning requirements, articulated in the 2018 Guidance by the Federal Deposit Insurance Corporation and the Fed. Finally, the Fed's Large Institution Supervision Coordinating Committee is in charge of supervising the intraday liquidity risk of large banks based on the Comprehensive Liquidity Analysis and Review (CLAR) tests.[6] Overall, intraday liquidity regulations aim at avoiding the run-risks brought about when a large share of liquid assets are pledged

[5]Principles for Sound Liquidity Risk Management and Supervision (the Sound Principles) provide guidance for banks on their management of liquidity risk and collateral. Principle 8 of the Sound Principles focuses specifically on intraday liquidity risk and states that: "A bank should actively manage its intraday liquidity positions and risks to meet payment and settlement obligations on a timely basis under both normal." Specifically, amongst other objectives, a bank is required to "arrange to acquire sufficient intraday funding to meet its intraday objectives" and "have a robust capability to manage the timing of its liquidity outflows in line with its intraday objectives" (Basel Committee on Banking Supervision, 2013).

[6]Those Basel III principles are also implemented in other jurisdictions. For instance, the Financial Service Authority (FSA) handbook provides guidance from the United Kingdom's Financial Conduct Authority (FCA) and the regular Supervisory Review (SREP) in the European Union.

to obtain intraday liquidity, as had been the case for Lehman Brothers on the eve of its bankruptcy in 2008 (Ball et al., 2011).

Although no public account gives precise details of the implementation of these pieces of regulation by the regulators, converging evidence points to an emphasis on central bank reserves being necessary to meet these newly established requirements. Pozsar (2019) emphasize how RLAP supersedes the Liquidity Coverage Ratio (LCR) regulation[7] because it forces large banks to hold high-quality liquid assets over and above the quantum required by the LCR in order to ensure that banks are able to meet large "day one" outflows. Indeed, these regulations are all about intraday payment flows and intraday liquidity risks, and reserves are the only instruments that provide intraday liquidity for banks. Consistent with this interpretation, the Fed vice-chairman responsible for banking supervision stated the following in January 2020: "However, it may be difficult to liquidate a large stock of Treasury securities to meet large "day one" outflows. [...] The LCR does not capture these on-the-ground realities. But supervision does. Under Regulation YY's enhanced prudential standards, large firms are required to conduct internal liquidity stress tests. Supervisors expect firms to estimate day-one outflows and to ensure that their liquidity buffers can cover those outflows without reliance on the Federal Reserve. For firms with large day-one outflows, reserves can meet this need most clearly." Consistent with this notion, Nelson and Waxman (2021) reports that "[t]he size of the liquidity requirements imposed by RLAP and RLEN are treated as confidential supervisory information; however, many large banks have reported that resolution liquidity requirements are the most binding constraint."

In the model below, we study how the introduction of these regulations can generate spikes in the repo market and emphasize the difference between intraday and overnight liquidity constraints. We show that while these regulations were introduced to ensure banks maintain enough liquidity to weather a liquidity crisis, they also necessitate significantly larger reserve holdings and result in unintended consequences for money markets. The model below abstracts from the former and focuses on a positive analysis of the latter.

[7]The liquidity coverage ratio is a requirement under Basel III whereby banks are required to hold enough high-quality liquid assets to cover cash outflows for 30 days. Because of its focus on medium-term liquidity needs and in contrast to the intraday liquidity requirements, LCR regulatory accounting considers liquid long-term securities such as Treasury bonds without penalty.

II Environment

Our model's economic setup consists of two types of financial institutions: traditional and shadow banks. Traditional banks hold reserve accounts with the central bank, the Fed, and are granted access to the Fed's overnight discount window lending. They are also subject to specific liquidity regulations. All interbank transactions among them are instantaneously settled in reserves. In contrast, shadow banks operate without access to Fed reserves or the discount window, and their activities consist of financing a securities portfolio by borrowing from traditional banks via repos.

Timing Figure 1 summarizes the timing of the model. Traditional banks are exposed to two deposit shocks: one in the morning and one at the close of business. The two shocks feature both a common and an idiosyncratic component. After the first shock, the fed funds, repo, and securities markets are open for trading. Following the second shock, banks borrow at the discount window and receive interest payments on their reserves balance. The subscript $-$ denotes variables set before the morning shock, and the subscript $+$ denotes variables set after the close of business. The variable m accounts for traditional banks' reserves; d is their deposits; and p and f are the repo and fed funds they lend during the trading period, respectively. Let the unit mass of traditional banks be denoted by $j \in [0, 1]$ and the unit mass of shadow banks be denoted by $s \in [0, 1]$. For ease of notation, we write x instead of $x(j)$ or $x(s)$ when the indexation is unambiguous.

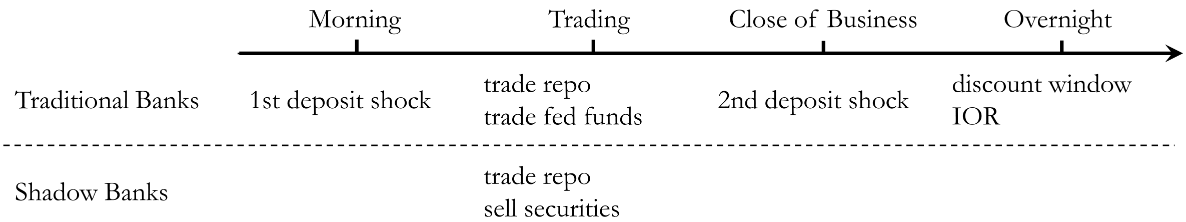

Figure 1: Model Timeline

Morning Shock At the beginning of the day, traditional banks are subject to a deposit common shock $d_-\varepsilon^c_-$ and a deposit idiosyncratic shock $d_-\varepsilon^i_-$ that shift deposits and corresponding reserves to other banks. These shocks are meant to capture the various intraday movements of reserves driven by firm and household payments. Although reserves do not leave the traditional banking sector, the common shock represents reserves encumbered by payment delays (the velocity of reserves is finite) or transferred to banks not active in the repo and fed funds lending markets. Thus, the quantity of reserves after the morning shock is given by $m_- + d_-(\varepsilon^c_- + \varepsilon^i_-)$. To derive our results analytically, we only need to define bounds on these shocks: $\varepsilon^c_- \in [-\sigma^c, \sigma^c]$ and $\varepsilon^i_- \in [-\sigma^i, \sigma^i]$. Furthermore, ε^i_- is such that $\int_0^1 d_-(j)\varepsilon^i_-(j)dj = 0$.

Close of Business Shock After the closing of the trading period, traditional banks are subject to a second deposit shock that results in reserve transfers. This shock also features two parts: the common component, $d_-\varepsilon^c_+ = -d_-\varepsilon^c_-$, which fully reverses the morning outflow, and the idiosyncratic component, $d_-\varepsilon^i_+$, which further reshuffles reserves across banks. Traditional banks that end the day with fewer reserves than stipulated by reserve requirements (RR) have to borrow the difference at the discount window facility on which they pay the discount window rate $r^m + r^w$. This feature generates a motive for traditional banks to hold reserves as a buffer against deposit shocks. Gross intraday deposit flows Δd are defined as

$$\Delta d = d_-(\varepsilon^c_- + \varepsilon^i_-) + d_-(\varepsilon^c_+ + \varepsilon^i_+) = d_-(\varepsilon^i_- + \varepsilon^i_+). \tag{1}$$

We denote by $\mathcal{F}(\varepsilon; j)$ the conditional distribution of the idiosyncratic shock $\varepsilon^i_+(j)$ for traditional bank j, also constrained on $[-\sigma^i, \sigma^i]$, such that $\int_0^1 d_-(j)\varepsilon^i_+(j)dj = 0$. For ease of notation, we define $\varepsilon_- \equiv \varepsilon^c_- + \varepsilon^i_-$ and $\varepsilon_+ \equiv \varepsilon^c_+ + \varepsilon^i_+$.

Intraday Flows In the trading period, traditional banks have the option to lend (reverse) repo to shadow banks and fed funds to other traditional banks. Lending triggers an instantaneous outflow of reserves. Hence, for traditional banks, lending in money markets amounts to swapping a quantity $p + f$ of reserves into repos and fed funds.

The intraday law of motion for deposits d_+ and reserves m_+ is given by

$$d_+ = d_- + \Delta d \quad \text{and} \quad m_+ = m_- + \Delta d - p - f. \tag{2}$$

Thus, lending in repo or fed funds encumbers reserves until the next day. Finally, daylight overdrafts at the Fed are created by banks carrying a negative balance on their reserves account during the day, with volume given by

$$o = \max\left\{0, p + f - m_- - d_-\varepsilon_-\right\}. \tag{3}$$

For simplicity, we assume that the interest rate on these intraday loans by the Fed is 0.

Regulation Traditional banks face two regulatory constraints. First, as in Poole (1968), they are subject to a traditional reserves requirement (RR)

$$m_+ \geq \chi^m d_+, \tag{RR}$$

where χ^m is the regulatory reserve ratio set by the regulator. If necessary to satisfy RR, a traditional bank must borrow reserves at the discount window rate $r^m + r^w$, a premium r^w over the interest paid on reserves r^m.[8]

Second, traditional banks are subject to an intraday liquidity constraint (IL)

$$p + f \leq m_- + d_- \varepsilon_-, \tag{IL}$$

where $m_- + d_- \varepsilon_-$ represents reserves after the morning shock before the trading period opens. Therefore, traditional bank lending is constrained by their buffer of reserves during the trading period.[9]

Traditional Banks The problem faced by traditional banks is given by

$$\max_{p,f} \left\{ \mathbb{E}\Big[m_+ r^m - (\chi^m d_+ - m_+) \mathbb{1}\{m_+ < \chi^m d_+\} r^w \Big] + p r^p + f r^f \right\} \tag{4}$$

$$\text{s.t.} \quad p + f \leq m_- + d_- \varepsilon_-. \tag{IL}$$

The variables r^p, r^f, and r^m are interest rates on repos, federal funds, and reserves, respectively. Banks maximize their expected profits subject to the intraday constraint IL, deciding on trading-period lending volumes in repos p and fed funds f. In scenarios in which the bank reserve stock does not meet the required reserve ratio RR at the end of the day, banks have to compensate for this shortfall by paying the penalty associated with borrowing at the discount window, r^w (including of potential stigma).

[8] Since March 2020, the reserves requirement has been lifted in the US. This case corresponds in our model to setting $\chi = 0$ or, equivalently, to impose a non-negativity constraint on m. Our qualitative results still hold in this case, given that close-of-business deposit shocks can still push banks to violate this non-negativity constraint and borrow at the discount window.

[9] The IL constraint prevents reserves to be negative during the day. We could impose a stricter constraint that reserves always need to be above a given threshold (enough to meet large "day one" outflows for example) without impacting our main results.

Shadow Banks We use the hat to denote shadow bank variables. Shadow banks borrow in repo to roll over a pre-existing portfolio of securities $\widehat{b}$:

$$\min_{0\leq\widehat{p}\leq\widehat{b}} \left\{ \widehat{p}r^p + (\widehat{b} - \widehat{p})\lambda \right\}. \tag{5}$$

This problem boils down to selecting the least expensive method to finance $\widehat{b}$. Without the ability to borrow repos from traditional banks, shadow banks' only option is to fire-sell securities at a loss rate of λ per unit.

Equilibrium All agents solve their respective problem, and the following market-clearing conditions are satisfied: $\int_0^1 p(j)dj = \int_0^1 \widehat{p}(s)ds$ for repos, and $\int_0^1 f(j)dj = 0$ for fed funds. We denote $B = \int_0^1 \widehat{b}(s)ds$ for shadow banks securities. The state of the economy in the trading period is given by the common shock ε_-^c, the joint distribution of $\{d_-(j), m_-(j), \varepsilon_-^i(j)\}$ for $j \in [0, 1]$, and the distribution of $\widehat{b}(s)$ for $s \in [0, 1]$.

III Analysis

In this section, we derive the theoretical implications of introducing an intraday constraint in a standard model of monetary policy with shadow banks. As a benchmark, we first derive the implications of the model for an economy in which the only piece of liquidity regulation is the reserve requirement, highlighting the banks' dependence on intraday overdrafts at the Fed. We then study the impact of the intraday constraint. We relegate all derivations and proofs to Appendix A. Below, we make three assumptions to simplify our analysis.

Assumption 1. *The bounds of the deposit shocks satisfy $\sigma^c + \sigma^i < 1$ and $\sigma^i < 0.5$ such that deposits are never negative at any time during the day.*

Assumption 2. *The fire-sale cost satisfies $\lambda > r^w + r^m$ such that shadow banks face higher liquidity risk than traditional banks.*[10]

Assumption 3. *Traditional banks never lend repo to other traditional banks, so they only trade with each other through the fed funds market.*[11]

[10] Note that if spikes are short-lived, even a relatively small transaction cost associated with trading Treasuries is large when compared to paying an annualized interest rate spread of around 7%, as with the September spike, for a couple of days.

[11] Although, in reality, banks may trade in repo markets because of their preference for collateralized lending, this assumption stems from our interpretation of the repo market as a bank-to-shadow-bank market.

A First Order Conditions

The first-order conditions of traditional banks with regard to p or f is given by

$$r^f = r^p = r^m + r^w \mathbb{P}\{m_+ < \chi^m d_+\} + \mu, \tag{6}$$

where μ is the Lagrangian multiplier for the intraday liquidity constraint IL and $\mathbb{P}\{m_+ < \chi^m d_+\}$ is the probability of having to borrow at the discount window, conditional on information available during the trading period. The first-order condition of shadow banks with respect to $\widehat{p}$ is given by

$$\widehat{p} = \begin{cases} \widehat{b} & \text{if} \quad r^p < \lambda, \\ (0, \widehat{b}) & \text{if} \quad r^p = \lambda, \\ 0 & \text{if} \quad r^p > \lambda. \end{cases} \tag{7}$$

When the repo rate r^p is below the fire-sale cost λ, shadow banks finance their entire portfolio with repos. When the repo rate is above λ, their repo demand falls to zero. In an in-between edge case in which the repo rate is precisely equal to λ, shadow banks are indifferent.

B Benchmark without the Intraday Liquidity Constraint

We first examine a benchmark case without the intraday liquidity constraint. This case corresponds to the setting studied in Poole (1968), extended to include a bank-to-shadow-bank repo market. We study how the repo market and the fed funds market interact with each other and find that traditional banks act as unconstrained arbitrageurs between these two market segments, ensuring that the two rates remain equal at all times. Proposition 1 characterizes the key property of an economy without an intraday liquidity constraint.

Proposition 1. *In an economy in which there is no intraday constraint, the repo rate is always equal to the fed funds rate, and both of these rates are bounded by the interest on reserves below and the discount window rate above:*

$$r^m \leq r^f = r^p \leq r^m + r^w. \tag{8}$$

As in Poole (1968), banks trade off the interest revenues in interbank markets (marginal benefits) and the probability of having to borrow at the discount window (marginal cost).

As this probability is bounded between 0 and 1, the two rates are contained within the boundaries of the interest rate on reserve r^m and the discount window rate $r^m + r^w$. The novelty of this setting resides in the additional option for banks to lend to shadow banks in repos. Because the marginal cost of lending in the repo market is the same as lending in the fed funds market, traditional banks' arbitrage repos and fed funds rates. In other words, by acting as arbitrageurs, traditional banks intermediate liquidity from the market for reserves into the broader repo market.

Our setting further allows us to derive the volume of intraday overdrafts as the integral of individual overdrafts over the unit mass of banks:

$$O = \int_0^1 \max\left\{\left[(1-\chi^m)\mathcal{F}^{-1}\left(\frac{r^p - r^m}{r^w}; j\right) - \chi^m - \chi^m \varepsilon^i_-(j) - \varepsilon^c_-\right] d_-(j), 0\right\} dj \tag{9}$$

when $r^m < r^f = r^p < r^m + r^w$. This equation highlights the key role played by Fed overdrafts in allowing banks to always arbitrage between the two markets. As indicated by Jamie Dimon (see footnote 4) and observed in Fed Board data for total intraday peak overdrafts and reserves in 2007, overdrafts would typically peak at around \$150 bn, whereas bank reserves would only be a third of this figure, around \$50 bn. In particular, a higher repo rate r^p or larger morning outflows, $-\varepsilon^i_-$ and $-\varepsilon^c_-$, lead to more overdrafts, which can then exceed the aggregate quantity of reserves.[12]

Finally, the volume of transactions in the fed funds market is given by

$$F = \int_0^1 \max\left\{m_-(j) + \left[(1-\chi^m)\left(\mathcal{F}^{-1}\left(\frac{r^p - r^m}{r^w}; j\right) + \varepsilon^i_-(j)\right) - \chi^m\right] d_-(j), 0\right\} dj \\ - \max\{B, 0\} \tag{10}$$

when $r^m < r^f = r^p < r^m + r^w$. As in Poole (1968), equation (10) implies that interbank lending depends on the intensity of the deposit shocks. The novelty of our setting is that a greater aggregate repo demand B decreases transactions in the fed funds market because reserves lent in repos cannot be simultaneously lent in fed funds.

[12]For example, we can derive the following sufficient condition: If $\varepsilon^c_- < (B - 2M)/D_-$, then $O > M$, where $M \equiv \int_0^1 m_-(j)dj$ is the aggregate quantity of reserves and $D_- \equiv \int_0^1 d_-(j)dj$ is the aggregate quantity of deposits in the morning.

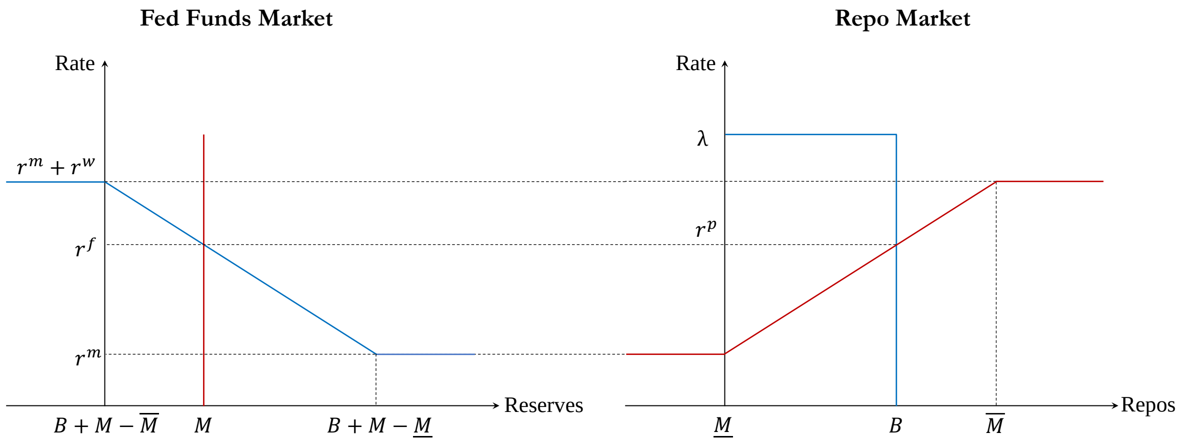

Figure 2: Benchmark without an Intraday Constraint. The figure shows the fed funds and repo markets in an economy without an intraday constraint, where $M \equiv \int_0^1 m_-(j)dj$ is the aggregate quantity of reserves, $D_- \equiv \int_0^1 d_-(j)dj$ is the aggregate quantity of deposits in the morning, $\underline{M}$ is defined as $M - \chi^m D_- - (1-\chi^m)\sigma^i D_-$, and $\overline{M}$ is defined as $M - \chi^m D_- + (1 - \chi^m)\sigma^i D_-$. For this illustration, we set $d_-(j)$ and $m_-(j)$ identical across all banks and ε_-^i, and ε_+^i are uniformly distributed on $[-\sigma^i, \sigma^i]$.

C Economy with an Intraday Liquidity Constraint

In this section, we characterize an economy with traditional banks subject to the intraday liquidity constraint. We find that this constraint prevents the use of overdrafts and limits traditional banks' ability to intermediate liquidity to shadow banks, resulting in large equilibrium repo spreads.

Define $\overline{P}$ as the maximum aggregate quantity of repos that could be supplied by traditional banks before hitting the intraday liquidity constraint, that is, $\overline{P} \equiv M + D_- \varepsilon_-^c$. When $B < \overline{P}$, at least some traditional banks do not hit their intraday constraint. If none of the traditional banks are constrained by (IL), the economy is equivalent to the benchmark case. If only a subset of traditional banks is constrained, equilibrium interest rates still remain within the monetary policy corridor as $r^f = r^p = r^m + r^w \mathbb{P}\{m_+ < \chi^m d_+\}$ must hold given the first-order conditions of the unconstrained banks. Nonetheless, because some banks are constrained, the supply curve becomes steeper, and repo and fed funds rates are higher when compared to the economy without an intraday liquidity constraint.

Proposition 2 mirrors Proposition 1, but for an economy in which all traditional banks reach the intraday liquidity constraint (IL) and the demand for funds by shadow banks cannot be fully intermediated by traditional banks, that is, $B > \overline{P}$.[13]

[13] See Appendix D for the case $B = \overline{P}$.

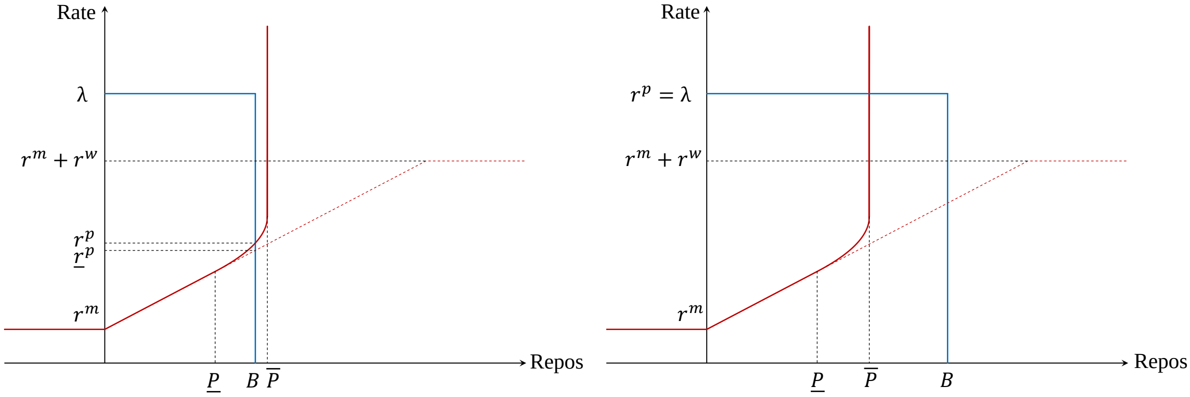

Figure 3: Economy with an Intraday Constraint. The figure shows repo markets in an economy with an intraday constraint: On the left panel, only a subset of banks are constrained by IL; On the right panel, all banks are constrained by IL. For this illustration, we set $d_{-}(j)$ and $m_{-}(j)$ identical across all banks and ε_{-}^{i}, and ε_{+}^{i} are uniformly distributed on $[-\sigma^i, \sigma^i]$.

Proposition 2. *In an economy in which the demand for repo funds cannot be fully fulfilled by traditional banks—that is, $B > \overline{P}$—the repo rate is above the discount window rate:*

$$r^p = \lambda > r^m + r^w. \tag{11}$$

When $B > \overline{P}$, shadow banks are able to finance only a subset of their portfolio in repos and have to fire-sell some of their securities. In that case, the cost for shadow banks of not accessing overnight liquidity is not bounded by the discount window rate $r^m + r^w$—which they cannot access—and jumps to the fire-sale cost λ.

Figure 3 illustrates the repo market for an economy with an intraday constraint and contrasts with the right panel of Figure 2. The left panel of Figure 3 shows an economy in which only a subset of banks are constrained by IL—that is, $\underline{P} < B < \overline{P}$, where $\underline{P}$ is defined as the threshold at which at least some banks are constrained. We denote by $\underline{r}^p$ the corresponding repo rate for the economy without an intraday liquidity constraint. The right panel shows an economy where all banks are constrained by (IL)—that is, $B > \overline{P}$—and the demand for repo by shadow banks cannot be supplied.

In contrast to the benchmark case in equation (9), there are no intraday overdrafts ($O = 0$) in an economy with an intraday liquidity constraint. Indeed, traditional banks cannot rely on intraday overdrafts at the Fed to satisfy the constraint.

Furthermore, transactions in the fed funds market are now given by

$$F = \int_0^1 \max\{-m_{\text{-}}(j) - d_{\text{-}}(j)\varepsilon_{\text{-}}(j), 0\}\, dj. \tag{12}$$

Thus, if all banks begin the trading period with a positive quantity of reserves ($m_{\text{-}}(j) + d_{\text{-}}(j)\varepsilon_{\text{-}}(j) \geq 0$ for all j), there are no transactions in the fed funds market. Since traditional banks have access to both repo and fed funds markets, the fed funds rate should be equal to the repo rate in equilibrium. However, no bank borrows fed funds at a rate higher than the discount window rate, unless they must in order to satisfy IL, because to borrow at the discount window rate is their worst-case scenario. This result corresponds to the empirical observation of sharp drops in fed funds volume on days with large repo spikes, such as September 16-17, 2019.[14] When no traditional banks violate the intraday liquidity constraint after the morning shock, transactions in the fed funds market drop to zero.

In this economy, we can also compute the probability of observing the repo rate spiking above the discount window rate as

$$\mathbb{P}\left[\overline{P} < B\right] = \mathbb{P}\left[M - B < -D_{\text{-}}\varepsilon^c_{\text{-}}\right]. \tag{13}$$

This equation shows that lower demand for repos by shadow banks B, or a larger quantity of reserves M decreases the probability of a repo spike. This result is intuitive as our model illustrates that conditions in repo markets depend on the demand for repo from shadow banks relative to a supply from banks, which is limited by the quantity of reserves through the intraday constraint.

In sum, we explore the implications of implementing an intraday liquidity constraint for money markets. The constraint alters money market rates by preventing banks from intermediating liquidity from reserves markets to repo markets, causing spikes in repo rates. When shadow banks are short of funds and the repo rate jumps above the discount window rate up to the cost of fire sales λ. Note that this model with intraday liquidity regulation cannot generate the empirical observation that repo and fed funds rates sometimes trade below the interest on reserves; we refer to articles by Bech and Klee (2011), d'Avernas and Vandeweyer (2024), and d'Avernas, Peterson and Vandeweyer (2023) for models in which balance sheet costs do generate such a phenomenon.

[14]See series EFFRVOL from the Federal Reserve Bank of New York, available at https://fred.stlouisfed.org/series/EFFRVOL.

IV Estimating Intraday Excess Reserves

In this section, we propose a simple formula to estimate the supply of reserves in excess of intraday liquidity needs. As shown in equation (13), we observe a repo spike whenever reserves M_t are smaller than repo lent by traditional banks B_t plus peak intraday flows $-D_{t\text{-}}\varepsilon^c_{t\text{-}}$. We thus define intraday excess reserves (IER) as

$$\text{IER}_t \equiv \text{Reserves}_t - \text{Bank Repos}_t - \text{Peak Intraday Flows}_t. \tag{14}$$

We compute this measure by making use of call reports and data from the Federal Reserve Bank System for bank repos and reserves.[15] Before 2008, we use peak intraday overdrafts[16] O_t to approximate[17] peak intraday flows $D_{t\text{-}}\varepsilon^c_{t\text{-}}$ according to the model:

$$-D_{t\text{-}}\varepsilon^c_{t\text{-}} = O_t + M_t - B_t. \tag{15}$$

After 2008, regulations prevents banks from using intraday overdrafts. Consequently, to proxy for peak intraday liquidity needs from the payment system after 2008, we estimate a simple linear model by regressing peak intraday flows $-D_{t\text{-}}\varepsilon^c_{t\text{-}}$, using pre-2008 data, on a constant, a linear trend, US gross domestic product, and velocity of money.[18] For reference, this figure is around \$560 bn in 2007, and estimated by the model at \$750 bn in 2019.

We plot the time series for IER in Figure 4. A first observation is that this series shifted from negative to positive around 2009 as a consequence of the rounds of quantitative easing following the global financial crisis. The IER then peaks around 2015 at \$1.5 tr and steadily decreases between 2015 and 2019 as the Fed reduced the size of its balance sheet. Importantly, the IER gets very close to zero in the summer of 2019 following reductions in central bank reserves through the reversal of QE policies. This time corresponds to the large repo spike that took place on September 17, 2019. After the Fed reacted by injecting reserves back into money markets through open market operations, the IER drifted back into positive territory and kept increasing after March 2020 as a consequence of the new pandemic programs. As of 2023, the IER remains largely in positive territory at around

[15]More precisely, we use TOTRESNS from FRED for Bank Reserves. For banks' repo, we use repo asset BHCKB989 minus repo liabilities BHCKB995 from FRY9C.

[16]We use the Total Intraday Peak Overdrafts from the Fed's Daylight Overdrafts and Fees data.

[17]This is an approximation because of the idiosyncratic dispersion of bank initial reserves and deposit shocks. That is, since $O = \int_0^1 \max\{0, p(j) + f(j) - m_{\text{-}}(j) - d_{\text{-}}(j)\varepsilon_{\text{-}}(j)dj\}$, then $-D_{\text{-}}\varepsilon^c_{\text{-}} \leq O + M - B$. If $m_{\text{-}}(j) = m_{\text{-}}(i)$ and $d_{\text{-}}(j)\varepsilon_{\text{-}}(j) = d_{\text{-}}(i)\varepsilon_{\text{-}}(i)\ \forall j \neq i$, then $-D_{\text{-}}\varepsilon^c_{\text{-}} = O + M - B$.

[18]Time series GDPC1 and M2V from FRED.

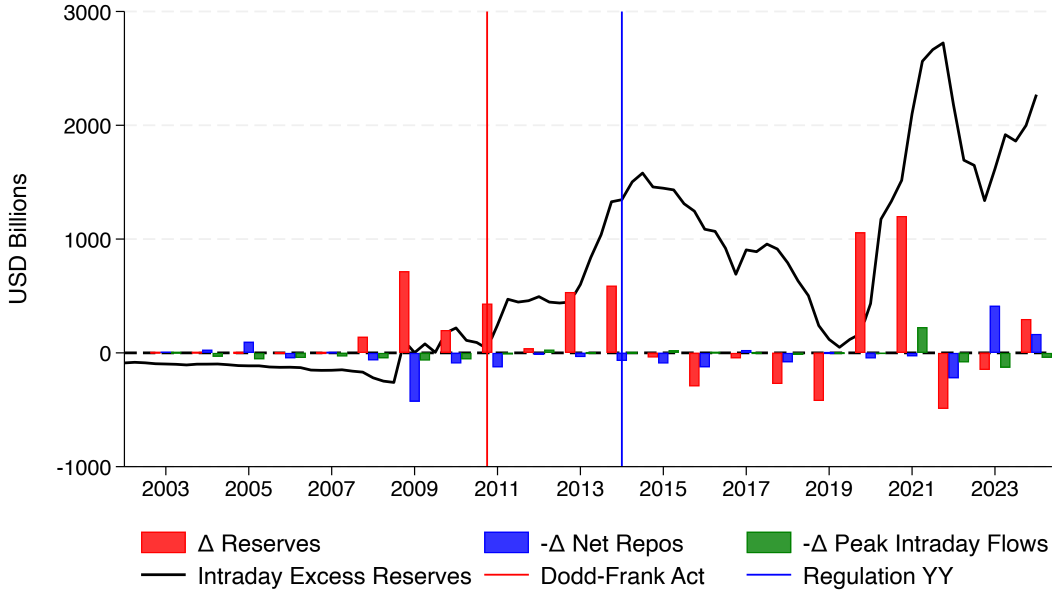

Figure 4: Intraday Excess Reserves. The figure shows the time series $\text{IER}_t \equiv \text{Excess Reserves}_t - \text{Bank Repos}_t - \text{Peak Intraday Flows}_t$ from 2002Q1 to 2024Q1 on a quarterly basis along with the decomposition into the components of its yearly changes.

\$2 tr. The growth decomposition exercise in Figure 4 highlights that an important driver of IER volatility post-GFC lies in changes in reserves, while peak intraday flows and bank repos mostly impact its level.

Our model is also informative as to whether the intraday constraint IL or the overnight constraint RR is more binding at a given point in time. Lemma 1 provides a sufficient condition under which intraday liquidity requirements are more restrictive than overnight regulations. Essentially, if the morning common shock, ε^c_-, which reverses at the close of business, is substantially larger than the idiosyncratic shocks, ε^i_- and ε^i_+, then intraday liquidity management becomes the primary concern.

Lemma 1. *If $-\varepsilon^c_- > \chi^m(1 + \varepsilon^i_- + \varepsilon^i_+) - \varepsilon^i_+$, then the intraday liquidity constraint IL is always binding before a bank needs to borrow at the discount window to satisfy the reserve requirement constraint RR.*

Assuming banks are identical with the same d_- and m_-, and a uniform probability distribution for ε^i_- and ε^i_+ over $[-\sigma^i, \sigma^i]$, we can calculate the probability that the condition in Lemma 1 is empirically satisfied.[19] Our results show that the probability of

[19] We estimate σ^i based on federal funds trading volumes prior to 2008. From equation (10), we get $\sigma^i_t = \left(2F_t - 2\min\{B_t, 0\} + B_t + 2\sqrt{(F_t - \min\{B_t, 0\})\,(F_t - \min\{B_t, 0\} + B_t)}\right) \Big/ \left((1 - \chi^m_t)D_{t\text{-}}\right)$

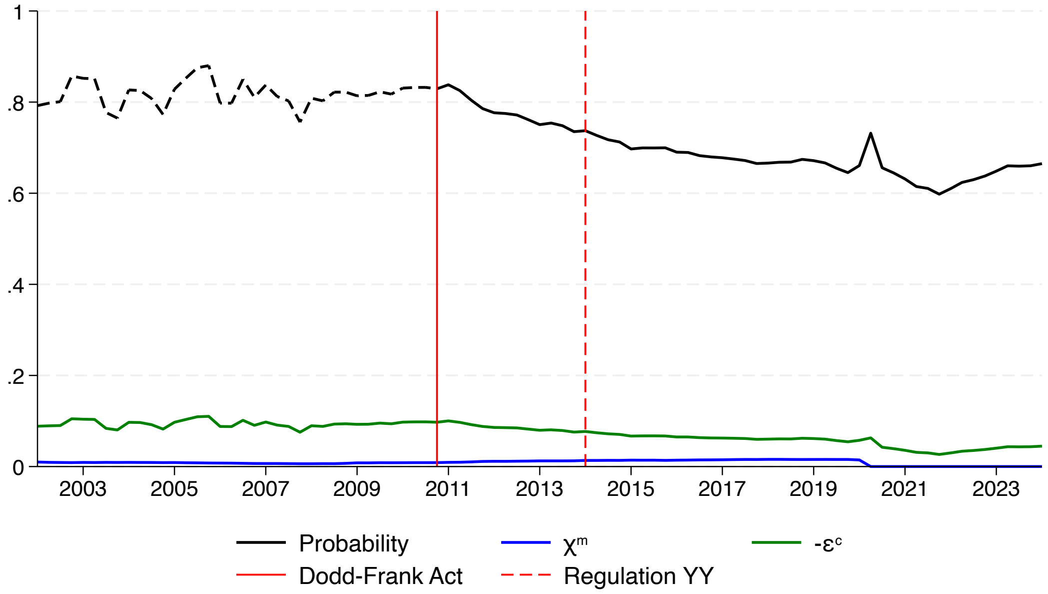

Figure 5: Intraday vs. Overnight Liquidity. The figure shows the time series of $\mathbb{P}[-\varepsilon^c_- > \chi^m(1+\varepsilon^i_- + \varepsilon^i_+) - \varepsilon^i_+]$ given $\varepsilon^i \sim \mathcal{U}(-\sigma^i, \sigma^i)$ and the time series of $-\varepsilon^c_-$.

this conservative condition being met ranges between 0.6 and 0.84, reflecting substantial intraday payment flows relative to day-to-day liquidity changes.

V Conclusion

This article proposes a parsimonious framework to study the interplay between intraday and overnight liquidity and how this interaction can lead to spikes in money market rates. The analysis reveals that regulatory requirements for banks to pre-fund intraday outflows with reserves limit the supply of repurchase agreements (repos) to shadow banks by limiting the supply of reserves available to settle transactions within the day. This finding implies that in a system with inelastic intraday reserves supply as per regulation YY, the relevant metric for the supply of reserves needed to operate an "ample reserves" operational framework can be of orders of magnitude larger than with fully elastic overdrafts.

Those results suggest a regulatory interpretation for the ratchet effect of QE docu-

assuming $d_-(j)$ and $m_-(j)$ identical across all banks, and ε^i_- and ε^i_+ uniformly distributed on $[-\sigma^i, \sigma^i]$. The detailed derivation is provided in Appendix F. We get the fed funds trading volumes F from the call reports by aggregating all banks with positive fed funds holding (fed funds asset BHDMB987 minus fed funds liabilities BHDMB993 from FRY9C). We use DPSACBM027NBOG from FRED for deposits D_-. And finally, we get the reserve ratio χ^m by dividing banks' required reserves (REQRESNS from FRED) by D_-. We set $\chi^m = 0$ after March 2020, when the reserve requirement is lifted. We average σ^i_t before 2008 to approximate the bounds of the idiosyncratic shock σ^i throughout the entire period.

mented by Acharya and Rajan (2022) and Acharya et al. (2023). By increasing the quantity of reserves available to banks, QE allowed regulators to require banks not to rely on daylight overdrafts for intraday liquidity needs. Consequently, the reversal of QE policies resulted in disruptions in money markets and now requires permanently larger balance sheets.

Overall, our findings have important implications for financial regulators, as shadow banks, which hold significant Treasury debt, lack access to the central bank's emergency lending facilities. Sudden spikes in repo rates generated by banks with too few reserves to lend may then trigger fire sales and cause an increase in Treasury yield, as observed in March 2020.

References

Acharya, Viral V, and Raghuram Rajan. 2022. "Liquidity, Liquidity Everywhere, Not a Drop to Use – Why Flooding Banks with Central Bank Reserves May Not Expand Liquidity." National Bureau of Economic Research Working Paper 29680.

Acharya, Viral V, Rahul S Chauhan, Raghuram Rajan, and Sascha Steffen. 2023. "Liquidity Dependence and the Waxing and Waning of Central Bank Balance Sheets." National Bureau of Economic Research Working Paper 31050.

Afonso, Gara, Alex Entz, and Eric LeSueur. 2013. "Who's Lending in the Fed Funds Market?" Federal Reserve Bank of New York Liberty Street Economics 20131202.

Afonso, Gara, and Ricardo Lagos. 2015. "Trade Dynamics in the Market for Federal Funds." *Econometrica*, 83(1): 263–313.

Afonso, Gara, Domenico Giannone, Gabriele La Spada, and John C. Williams. 2022. "Scarce, Abundant, or Ample? A Time-Varying Model of the Reserve Demand Curve." Federal Reserve Bank of New York Staff Report 1019.

Afonso, Gara, Marco Cipriani, Adam M. Copeland, Anna Kovner, Gabriele La Spada, and Antoine Martin. 2020. "The Market Events of Mid-September 2019." *Economic Policy Review*, 27(2).

Andersen, Leif, Darrell Duffie, and Yang Song. 2019. "Funding Value Adjustments." *Journal of Finance*, 74(1): 145–192.

Avalos, Fernando, Torsten Ehlers, and Egemen Eren. 2019. "September Stress in Dollar Repo Markets: Passing or Structural?" Bank for International Settlements BIS Quarterly Review.

Ball, Alan, Edward Denbee, Mark Manning, and Anne Wetherilt. 2011. "Intraday liquidity: risk and regulation." *Financial Stability Paper No. 11.*

Barth, Daniel, and Jay Kahn. 2023. "Hedge Funds and the Treasury Cash-Futures Disconnect." *OFR Working Paper Series*, 21-01.

Basel Committee on Banking Supervision. 2013. "Monitoring tools for intraday liquidity management."

Bech, Morten L., and Elizabeth Klee. 2011. "The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market." *Journal of Monetary Economics*, 58(5): 415–431.

Bech, Morten L., and Rod Garratt. 2003. "The Intraday Liquidity Management Game." *Journal of Economic Theory*, 109(2): 198–219.

Bianchi, Javier, and Saki Bigio. 2022. "Banks, liquidity management, and monetary policy." *Econometrica*, 90(1): 391–454.

Bindseil, Ulrich. 2014. *Monetary Policy Operations and the Financial System.* Oxford:Oxford University Press. online edn. https://doi.org/10.1093/acprof:oso/9780198716907.001.0001.

Copeland, Adam, Darrell Duffie, and Yilin Yang. 2022. "Reserves Were Not So Ample After All." Federal Reserve Bank of New York Staff Report No. 974.

Correa, Ricardo, Wenxin Du, and Gordon Liao. 2021. "U.S. Banks and Global Liquidity." NBER Working Paper 27491.

d'Avernas, Adrien, and Quentin Vandeweyer. 2024. "Treasury Bill Shortages and the Pricing of Short-Term Assets." *Journal of Finance*, 79(6): 4083–4141.

d'Avernas, Adrien, Damon Peterson, and Quentin Vandeweyer. 2023. "Central Bank Balance Sheet and Treasury Market Disruptions." Unpublished.

Diamond, William, Zhengyang Jiang, and Yiming Ma. 2023. "The Reserve Supply Channel of Unconventional Monetary Policy." National Bureau of Economic Research Working Paper 31693.

Duffie, Darrell, and Arvind Krishnamurthy. 2016. "Passthrough Efficiency in the Fed's New Monetary Policy Setting." In *Designing Resilient Monetary Policy Frameworks for the Future. Federal Reserve Bank of Kansas City, Jackson Hole Symposium.* 21–102.

Du, Wenxin, Benjamin Hébert, and Wenhao Li. 2023. "Intermediary Balance Sheets and the Treasury Yield Curve." *Journal of Financial Economics*, 150: 103722.

Kahn, R. Jay, Matthew McCormick, Vy Nguyen, Mark Paddrik, and H. Peyton Young. 2023. "Anatomy of the Repo Rate Spikes in September 2019." Office of Financial Research OFR Working Paper 23-04.

Lopez-Salido, David, and Annette Vissing-Jorgensen. 2023. “Reserve Demand, Interest Rate Control, and Quantitative Tightening.”

McAndrews, James, and Samira Rajan. 2000. “The Timing and Funding of Fedwire Funds Transfers.” *Economic Policy Review*, 6(2): 17–32.

Myers, Stewart C. 1977. “Determinants of corporate borrowing.” *Journal of Financial Economics*, 5(2): 147–175.

Nelson, Bill, and Brett Waxman. 2021. “Bank Treasurers’ Views on Liquidity Requirements and the Discount Window.” *Bank Policy Institute Blog.* https://bpi.com/bank-treasurers-views-on-liquidity-requirements-and-the-discount-window/.

Poole, William. 1968. “Commercial Bank Reserve Management in a Stochastic Model: Implications For Monetary Policy.” *Journal of Finance*, 23(5): 769–791.

Pozsar, Zoltan. 2019. “Collateral Supply and o/n Rates.” Credit Suisse Global Money Notes 22.

Siriwardane, Emil, Aditya Sunderam, and Jonathan Wallen. 2022. “Segmented Arbitrage.” NBER Working Paper 30561.

Walsh, Carl E. 2017. *Monetary Theory and Policy.* . 4th ed., Cambridge,MA:MIT Press.

Yang, Yilin. 2023. “What Quantity of Reserves Is Sufficient?” Working Paper.

Appendices

A First-order Conditions for Traditional Banks

As in the main text, for ease of notation, we write x instead of $x(j)$ and $x(s)$ when the indexation is not necessary. The traditional banks' problem is given by

$$\max_{p,f} \left\{ \mathbb{E}\Big[m_+ r^m - (\chi^m d_+ - m_+) \mathbb{1}\{m_+ < \chi^m d_+\} r^w \Big] + p r^p + f r^f \right\} \tag{16}$$

$$\text{s.t.} \quad p + f \le m_- + d_- \varepsilon_-, \tag{IL}$$

where $d_+ = d_- + \Delta d$, $m_+ = m_- + \Delta d - p - f$, and $\Delta d = d_-(\varepsilon_-^i + \varepsilon_+^i)$. The Lagrangian multiplier μ on (IL) is given by

$$\mathcal{L} = \mathbb{E}\Big[m_+ r^m - (\chi^m d_+ - m_+) \mathbb{1}\{m_+ < \chi^m d_+\} r^w \Big] + p r^p + f r^f - \mu(p + f - m_- + d_- \varepsilon_-). \tag{17}$$

Denote $\omega \equiv p + f$, then we have $m_+ = m_- + \Delta d - \omega$. Let us further define $\phi(\omega) = (\chi^m d_- - m_- + \omega) / [(1 - \chi^m) d_-] - \varepsilon_-^i$, which is the value of ε_+^i given d_- that satisfies $m_+ = \chi^m d_+$. Given the definition of $\mathcal{F}(\varepsilon; j)$, we have

$$\mathbb{P}\{m_+ < \chi^m d_+\} = \begin{cases} 0 & \text{if} \quad \omega \le \underline{m} \\ \mathcal{F}(\phi(\omega); j) & \text{if} \quad \omega \in (\underline{m}, \overline{m}) \\ 1 & \text{if} \quad \omega \ge \overline{m} \end{cases}, \tag{18}$$

where $\underline{m} \equiv m_- - \chi^m d_- + (1 - \chi^m) d_-(\varepsilon_-^i - \sigma^i)$ and $\overline{m} \equiv m_- - \chi^m d_- + (1 - \chi^m) d_-(\varepsilon_-^i + \sigma^i)$. The first-order conditions for p or f is given by $r^p = r^f = r^m + r^w \mathbb{P}\{m_+ < \chi^m d_+\} + \mu$.

B Proof of Proposition 1

First, we define the total quantity of reserves as $M \equiv \int_0^1 m_-(j) dj$, the total quantity of shadow bank securities as $B \equiv \int_0^1 \widehat{b}(s) ds$, and the total quantity of deposits in the morning as $D_- \equiv \int_0^1 d_-(j) dj$, $\underline{M} \equiv \int_0^1 \underline{m}(j) dj$, and $\overline{M} \equiv \int_0^1 \overline{m}(j) dj$.

When there is no intraday constraint or the constraint is not binding, $\mu = 0$, and $r^p - r^m = \mathbb{P}\{m_+ < \chi^m d_+\} r^w$. Thus, $r^m \le r^f = r^p \le r^m + r^w$ and $\widehat{p} = \widehat{b}$.

Whenever traditional banks lend more repo and fed funds than reserves they hold—

that is, $m_- + d_-\varepsilon_- < \omega$—they borrow with intraday Fed overdrafts. The total volume of overdrafts is given by $O = \int_0^1 \max\{\omega(j) - m_-(j) - d_-(j)\varepsilon_-(j), 0\}dj$. Given Assumption 3, the trading volume in the fed funds market is $F = \int_0^1 \max\{\omega(j), 0\}dj - B$.

- **Case 1:** $\exists j : \omega(j) \leq \underline{m}(j)$**.** Then $\mathbb{P}\{m_+ < \chi^m d_+\} = 0$, $r^f = r^p = r^m$,

$$B \leq \underline{M} \tag{19}$$

  and $\omega(j) \leq \underline{m}(j)$ for all j.

- **Case 2:** $\exists j : \omega(j) \geq \overline{m}(j)$**.** Then $\mathbb{P}\{m_+ < \chi^m d_+\} = 1$, $r^f = r^p = r^m + r^w$,

$$B \geq \overline{M} \tag{20}$$

  and $\omega(j) \geq \overline{m}(j)$ for all j.

- **Case 3:** $\underline{m}(j) < \omega(j) < \overline{m}(j)$ **for all** j**.** Then $\mathbb{P}\{m_+ < \chi^m d_+\} \in (0, 1)$ and $r^m < r^f = r^p < r^m + r^w$. Thus,

$$\omega(j) = \left(\mathcal{F}^{-1}\left(\frac{r^p - r^m}{r^w}; j\right) + \varepsilon_-^i(j)\right)(1 - \chi^m)d_-(j) + m_-(j) - \chi^m d_-(j). \tag{21}$$

  Integrating over (21), we get

$$B = M + \int_0^1 \left(\mathcal{F}^{-1}\left(\frac{r^p - r^m}{r^w}; j\right)(1 - \chi^m) - \chi^m\right) d_-(j)dj. \tag{22}$$

C Derivation of $\underline{P}$

We define $\underline{P}$ such that, if $B < \underline{P}$, then $\mu(j) = 0$ for all j, and if $B > \underline{P}$, then $\exists j$ such that $\mu(j) > 0$. Below, we derive the value of $\underline{P}$. We also define

$$\underline{\ell}(j) \equiv \min\{m_-(j) + d_-(j)\varepsilon_-(j), \underline{m}(j)\} \tag{23}$$

$$\overline{\ell}(j) \equiv \min\{m_-(j) + d_-(j)\varepsilon_-(j), \overline{m}(j)\}. \tag{24}$$

$\underline{L} \equiv \int_0^1 \underline{\ell}(j)dj$ and $\overline{L} \equiv \int_0^1 \overline{\ell}(j)dj$. It is direct to see that $\underline{L} \leq \underline{M} < \overline{M}$, and $\underline{L} \leq \overline{L} \leq \overline{M}$. Also note that $\int_0^1 \omega(j)dj = B$ when $B \leq \overline{P}$.

- **Case 1:** $\underline{L} < \underline{M}$**.**

Note that $\underline{L} < \underline{M}$ implies that $\exists j' : \underline{\ell}(j') < \underline{m}(j')$. Thus, $m_-(j') + d_-(j')\varepsilon_-(j') < \underline{m}(j') < \overline{m}(j')$. Furthermore, $\overline{\ell}(j') < \overline{m}(j')$. Therefore, $\overline{L} < \overline{M}$ and we do not need to investigate what happens if $\underline{L} < \underline{M}$ and $\overline{L} = \overline{M}$.

- If $B < \underline{L}$, $\exists j' : \omega(j') < \underline{\ell}(j')$ and $r^m = r^p = r^f$. Thus, $\mu(j) = 0$ for all j.
- If $B > \underline{L}$, $\exists j' : \omega(j') > \underline{\ell}(j')$. Since $\omega(j') \leq m_-(j') + d_-(j')\varepsilon_-(j')$, $\omega(j') > \underline{m}(j')$ and $r^p > r^m$.

  In addition, since $\underline{L} < \underline{M}$, $\exists j'' : \underline{\ell}(j'') < \underline{m}(j'')$. Thus, $\omega(j'') \leq m_-(j'') + d_-(j'')\varepsilon_-(j'') < \underline{m}(j'')$, which implies $\mathbb{P}\{m_+(j'') < \chi^m d_+(j'')\} = 0$ from equation (18). So, $\mu(j'') > 0$.

Therefore, $\underline{P} = \underline{L}$.

- **Case 2:** $\underline{L} = \underline{M} \leq \overline{L} < \overline{M}$**.**

  - If $B < \underline{L} = \underline{M}$, $\exists j' : \omega(j') < \underline{\ell}(j')$ and $r^m = r^p = r^f$. Thus, $\mu(j) = 0$ for all j.
  - If $B > \overline{M} > \overline{L}$, $\exists j' : \omega(j') > \overline{m}(j')$, and $r^p \geq r^m + r^w$. In addition, since $\overline{L} < \overline{M}$, $\exists j'' : \overline{\ell}(j'') < \overline{m}(j'')$. Thus, $\omega(j'') \leq m_-(j'') + d_-(j'')\varepsilon_-(j'') < \overline{m}(j'')$, which implies $\mathbb{P}\{m_+(j'') < \chi^m d_+(j'')\} < 1$ from equation (18). So, $\mu(j'') > 0$.

Thus, $\underline{M} \leq \underline{P} \leq \overline{M}$. Let us define $\underline{\mathcal{F}}$ as

$$\underline{\mathcal{F}} \equiv \min_{j\in[0,1]} \{\mathcal{F}(\varepsilon(j); j)\} \tag{25}$$

where $\varepsilon(j)$ is defined as

$$\varepsilon(j) \equiv \frac{m_-(j) + d_-(j)\varepsilon_-(j) - m_-(j) + \chi^m d_-(j)}{(1-\chi^m)d_-(j)} - \varepsilon^i_-(j). \tag{26}$$

We now show that $\underline{P}$ is given by

$$\underline{P} = M + \int_0^1 \left(\mathcal{F}^{-1}\left(\underline{\mathcal{F}}; j\right)(1-\chi^m) - \chi^m\right) d_-(j)dj. \tag{27}$$

- If $\underline{M} < B < \underline{P}$, then $\exists j' : \omega(j') < (\mathcal{F}^{-1}(\underline{\mathcal{F}}; j') + \varepsilon^i_-(j'))(1-\chi^m)d_-(j') + m_-(j') - \chi^m d_-(j') \leq m_-(j') + \varepsilon_-(j')$. Thus, $\mu(j') = 0$ and $(r^p - r^m)/r^w < \underline{\mathcal{F}}$.

Using equations (21) and (25), we get

$$\begin{aligned}&\left(\mathcal{F}^{-1}\left(\frac{r^p-r^m}{r^w};j\right)+\varepsilon_-^i(j)\right)(1-\chi^m)d_-(j)+m_-(j)-\chi^m d_-(j)\\&<m_-(j)+d_-(j)\varepsilon_-(j)\end{aligned}\tag{28}$$

for all j. Thus, $\mu(j)=0$ for all j.

– If $\underline{P}<B<\overline{M}$, it is impossible that $\mu(j)=0$ for all j. Otherwise, $(r^p-r^m)/r^w>\underline{\mathcal{F}}$, and there exists j'' such that

$$\begin{aligned}\omega(j'')&\le m_-(j'')+d_-(j'')\varepsilon_-(j'')\\&<\left(\mathcal{F}^{-1}\left(\frac{r^p-r^m}{r^w};j''\right)+\varepsilon_-^i(j'')\right)(1-\chi^m)d_-(j'')+m_-(j'')-\chi^m d_-(j'').\end{aligned}\tag{29}$$

Thus, $\mu(j'')>0$.

Therefore, $\underline{P}$ is given by equation (27).

- **Case 3.** If $\overline{L}=\overline{M}$:

This implies $\overline{m}(j)\le m_-(j)+d_-(j)\varepsilon_-(j)$ for all j. Thus, $\overline{L}=\overline{M}\le\overline{P}$.

– If $B<\overline{L}=\overline{M}$, then $\exists j':\omega(j')<\overline{m}(j')\le m_-(j')+d_-(j')\varepsilon_-(j')$. Thus, $\mu(j')=0$. Furthermore, $\mathbb{P}\{m_+(j')<\chi^m d_+(j')\}<1$ by the definition of $\overline{m}(j')$. Therefore, $r^p<r^m+r^w$. Thus, $\mathbb{P}\{m_+(j)<\chi^m d_+(j)\}<1$ and $\omega(j)<\overline{m}(j)$ for all j. Thus, $\mu(j)=0$ for all j.

– If $\overline{M}\le B<\overline{P}$, then $\exists j':\omega(j')\ge\overline{m}(j')$. Thus, $\mathbb{P}\{m_+(j')<\chi^m d_+(j')\}=1$ by the definition of $\overline{m}(j')$. Therefore, $r^p\ge r^m+r^w$.

In addition, $B<\overline{P}$ implies that $\exists j'':\omega(j'')<m_-(j'')+d_-(j'')\varepsilon_-(j'')$ and $\mu(j'')=0$. Therefore, $r^p\le r^m+r^w$.

Thus, $r^p=r^m+r^w$.

Finally, if $\exists j''':\mathbb{P}\{m_+(j''')<\chi^m d_+(j''')\}<1$, then $\omega(j''')<\overline{m}(j''')$ and $\mu(j''')>0$. This is not possible given that $\overline{m}(j''')\le m_-(j''')+d_-(j''')\varepsilon_-(j''')$. Thus, $\mathbb{P}\{m_+(j)<\chi^m d_+(j)\}=1$ for all j. Thus, $\mu(j)=0$ for all j.

– If $B>\overline{P}$, we show in the proof of Proposition 2 in the main text that $r^p=\lambda>r^m+r^w$. Thus, $\mu(j)>0$ for all j.

Therefore, $\underline{P} = \overline{P}$.

Overall, we have

$$\underline{P} = \begin{cases} \underline{L} & \text{if } \underline{L} < \underline{M}, \\ M + \int_0^1 \left(\mathcal{F}^{-1}\left(\underline{\mathcal{F}}; j\right)(1-\chi^m) - \chi^m\right) d_-(j) dj & \text{if } \underline{L} = \underline{M} \leq \overline{L} < \overline{M}, \\ \overline{P} & \text{if } \overline{L} = \overline{M}, \end{cases} \tag{30}$$

where $\underline{\mathcal{F}}$ is defined as in equation (25). Note that the proof of Proposition 2 in the main text implies that $r^p = \lambda > r^m + r^w$ when $B > \overline{P}$. Thus, $\mu(j) > 0$ for all j when $B > \overline{P}$. Therefore, $\underline{P} \leq \overline{P}$ by the definition of $\underline{P}$.

D Economy with an Intraday Constraint and $B = \overline{P}$

In this appendix, we characterize the case in which the demand for repo funds is just at the limit of the intraday constraint, $B = \overline{P}$.

First, we show that for *every* shadow bank, $\widehat{p}(s) = \widehat{b}(s)$. If $\exists s' : \widehat{p}(s') < \widehat{b}(s')$, then $r^p = \lambda$ given the maximization problem of shadow banks and $\int_0^1 \omega(j)dj = \int_0^1 \widehat{p}(s)ds < B = \overline{P} = \int_0^1 m_-(j) + d_-(j)\varepsilon_-(j)dj$. Thus, $\exists j' : \omega(j') < m_-(j') + d_-(j')\varepsilon_-(j')$ and $\mu(j') = 0$, which implies that $r^p \leq r^m + r^w$, a contradiction with $\lambda > r^m + r^w$. Therefore, $\widehat{p}(s) = \widehat{b}(s)$ for all s. In addition, $\int_0^1 \omega(j)dj = \int_0^1 \widehat{p}(s)ds = B = \overline{P}$ implies that $\omega(j) = m_-(j) + d_-(j)\varepsilon_-(j)$ for all j.

Given that $\widehat{p}(s) = \widehat{b}(s)$ for all s, $r^p \leq \lambda$ given the maximization problem of shadow banks. In this edge case, there are multiple equilibria, and the rates, r^p and r^f, can land in the monetary policy corridor between r^m and $r^m + r^w$ or jump above the discount window rate, $r^m + r^w$. Below, we characterize the two types of equilibria.

- **Type 1: $\mu(j) = 0$ for all j.**

  This implies that $r^p \leq r^m + r^w$. The equilibrium is equivalent to the benchmark case in Proposition 1.

- **Type 2: $\exists j' : \mu(j') > 0$.**

  The rates r^p and r^f are higher compared to those in the benchmark case.

Turning to the volume of transactions in the fed funds market, in both types of equilibria, we have $F = \int_0^1 \max\{-m_-(j) - d_-(j)\varepsilon_-(j), 0\}dj$, as in Proposition 2. Given $\underline{P} \leq \overline{P} = B$, we have two cases.

- **Case 1: $\underline{P} = \overline{P} = B$.**

  Both types of equilibria exist in this case.

- **Case 2: $\underline{P} < \overline{P} = B$.**

  Only equilibrium type 2 exists in this case. It is impossible to have $\mu(j) = 0$ for all j, because by the definition of $\underline{P}$, when $B > \underline{P}$, then $\exists j' : \mu(j') > 0$.

E Proof of Lemma 1

When the intraday liquidity constraint IL is binding, $p + f = m_- + d_-\varepsilon_-$. Hence, the bank has zero reserves before the close-of-business shock, which further implies that the reserves flowing back during the close-of-business shock are the only source to meet the reserve requirement. Hence, the reserve requirement constraint RR can be written as

$$d_-\varepsilon_+ \geq \chi^m d_-(1 + \varepsilon_- + \varepsilon_+); \tag{31}$$

$$-\varepsilon_-^c + \varepsilon_+^i \geq \chi^m(1 + \varepsilon_-^i + \varepsilon_+^i); \tag{32}$$

$$-\varepsilon_-^c \geq \chi^m(1 + \varepsilon_-^i + \varepsilon_+^i) - \varepsilon_+^i. \tag{33}$$

Hence, when condition (33) is satisfied, banks do not need to borrow at the discount window to meet RR given that the IL is binding in advance. This concludes the condition in Lemma 1.

In this appendix, we also provide the probability that condition (33) holds with the assumption that ε_-^i and ε_+^i are uniformly distributed within $[-\sigma^i, \sigma^i]$. Rearranging condition (33), we get

$$-\varepsilon_-^c - \chi^m \geq \chi^m \varepsilon_-^i - (1 - \chi^m)\varepsilon_+^i.$$

The RHS of the inequality is bounded within $[-\sigma^i, \sigma^i]$ as both ε_-^i and ε_+^i are also within $[-\sigma^i, \sigma^i]$. Hence, if $-\varepsilon_-^c - \chi^m \geq \sigma^i$, then the probability (33) holds is 1; if instead $-\varepsilon_-^c - \chi^m \leq -\sigma^i$, the probability is 0.

We then turn to the case where $-\sigma^i < \varepsilon_-^c - \chi^m < \sigma^i$[20]. If $\chi^m = 0$, we have

$$P(\chi^m \varepsilon_-^i - (1 - \chi^m)\varepsilon_+^i \leq -\varepsilon_-^c - \chi^m) = P(-\varepsilon_+^i \leq -\varepsilon_-^c) = \frac{-\varepsilon_-^c + \sigma^i}{2\sigma^i}. \tag{34}$$

[20] To limit the number of different cases, we only consider $\chi^m \leq 1/2$. This is empirically correct as shown in the series of χ^m in Figure 5.

If $\chi^m > 0$, given that the morning and close-of-business idiosyncratic shocks are independently and uniformly distributed over $[-\sigma^i, \sigma^i]$, we have

$$P(\chi^m \varepsilon_-^i - (1-\chi^m)\varepsilon_+^i \leq -\varepsilon_-^c - \chi^m) = \int_{-\chi^m\sigma^i}^{\chi^m\sigma^i} P\left(-(1-\chi^m)\varepsilon_+^i \leq -\varepsilon_-^c - \chi^m - x\right) f_{\chi^m \varepsilon_-^i}(x) dx$$

$$= \begin{cases} \frac{(-\varepsilon_-^c - \chi^m + \sigma^i)^2}{8(1-\chi^m)\chi^m(\sigma^i)^2} & \text{if } -\varepsilon_-^c - \chi^m \leq -(1-2\chi^m)\sigma^i \\ \frac{1}{2} + \frac{-\varepsilon_-^c - \chi^m}{2(1-\chi^m)\sigma^i} & \text{if } |-\varepsilon_-^c - \chi^m| < (1-2\chi^m)\sigma^i \\ 1 - \frac{\left(-\varepsilon_-^c - \chi^m - \sigma^i\right)^2}{8(1-\chi^m)\chi^m(\sigma^i)^2} & \text{if } -\varepsilon_-^c - \chi^m \geq (1-2\chi^m)\sigma^i \end{cases} . \tag{35}$$

F Derivation of σ^i from F

Under the assumption that ε_-^i and ε_+^i are uniformly distributed over $[-\sigma^i, \sigma^i]$, and m_- are d_- are identical across banks, we have that $\omega = (1-\chi^m)D_-\varepsilon_-^i + B$ from equations (21) and (22). Together with $F \equiv \int_0^1 \max\{\omega(j), 0\}dj - \max\{B, 0\} = -\int_0^1 \min\{\omega(j), 0\}dj + \min\{B, 0\}$, we obtain

$$F = -E[(1-\chi^m)D_-\varepsilon_-^i + B | (1-\chi^m)D_-\varepsilon_-^i + B < 0] + \min\{B, 0\}. \tag{36}$$

Given that we see positive trading volumes of fed funds, i.e., $F > 0$, from the data, $\varepsilon_-^{i*} \in (-\sigma^i, \sigma^i)$ where $(1-\chi^m)D_-\varepsilon_-^{i*} + B = 0$. Hence,

$$F - \min\{B, 0\} = \frac{(1-\chi^m)D_-}{4\sigma^i}\left(\frac{B}{(1-\chi^m)D_-} - \sigma^i\right)^2. \tag{37}$$

given ε_-^i is uniformly distributed on $[-\sigma^i, \sigma^i]$.

We solve the above equation and get[21]

$$\sigma^i = \frac{2F - 2\min\{B,0\} + B + 2\sqrt{(F - \min\{B,0\})(F - \min\{B,0\} + B)}}{(1-\chi^m)D_-} \tag{38}$$

[21] Since $(1-\chi^m)D_-\sigma^i + B > 0$, we take the larger root.