Disentangling Credit Spreads and Equity Volatility

Job Market Paper

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What Drives Predictors of the Business Cycle?

Financial indicators are powerful predictors of economic activity.

- equity volatility and corporate bond credit spreads (Bloom 09, GZ 12)

No generally accepted understanding of what shocks drive these indicators.

- source of the predictive content

I propose a structural framework to quantify the drivers of financial indicators.

- dynamic capital structure model with liquidity frictions
Potential Driving Forces

To explain credit spreads and equity volatility, the model features shocks to

- firms’ asset values
- firms’ aggregate asset volatility
- firms’ idiosyncratic asset volatility
- bankruptcy costs
- stochastic discount factor
- liquidity frictions

structurally estimated from 300,887 monthly firm-level observations of

- corporate bond credit spreads (Lehman/Warga and Merrill Lynch)
- equity prices and equity volatilities (CRSP)
- accounting statements (Compustat)
- bond recovery ratios (Moody)

in the U.S. from 1973 to 2014.
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in the U.S. from 1973 to 2014.
The model yields predictions for the levels and *joint* macrodynamics of:

- corporate bond credit spreads
- equity volatility
  - default risk
  - excess bond premium
  - bond bid-ask spreads
  - aggregate equity volatility
  - idiosyncratic equity volatility
  - leverage
Results

(1) Model-implied financial indicators match historical counterparts.

(2) Shocks to firms’ aggregate asset volatility are key for joint dynamics.

(3) Shocks to firms’ aggregate asset volatility strongly predict economic activity.
Predictors of Real Economic Activity
Gilchrist, Yankov, and Zakrajšek (2009); Stock and Watson (2012); Gilchrist and Zakrajšek (2012); Faust, Gilchrist, Wright, and Zakrajšek (2013); Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016); Gilchrist and Zakrajšek (2012); Kelly, Manzo, and Palhares (2016); López-Salido, Stein, and Zakrajšek (2016)

Credit Spreads
Collin-Dufresne, Goldstein, and Martin (2001); Longstaff, Mithal, and Neis (2005); Hackbarth, Miao, and Morellec (2006); Almeida and Philippon (2007); David (2008); Chen, Collin-Dufresne, and Goldstein (2009); Bhamra, Kuehn, and Strebulaev (2010)

Equity Volatility
Schwert (1989), Campbell and Taksler (2003); Bloom (2009), Arellano, Bae, Kehoe (2012), Christiano, Motto, and Rostagno (2014); Atkeson, Eisdeldt, and Weill (2014); Jurado, Ludvigson, and Ng (2015); Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)

Structural Models of Default and Liquidity
Merton (1974), Leland (1994); Chen (2010); He and Milbradt (2014); Chen, Cui, He, and Milbradt (2016)
Macrodynamics of Credit Spreads and Equity Volatility
Model
Model Overview

- firms have access to productive assets
- debt and equity as liabilities
- tax shield and bankruptcy costs
- stochastic discount factor
- liquidity as in Chen, Cui, He, and Milbradt (2016)
- structurally estimate shocks driving credit spreads and equity volatility
Firms’ Assets

Type-\(j\) firm’s asset cash flows follow a diffusion

\[
\frac{dy_{it}}{y_{it}} = \mu^{j}_{Y,F} dt + \sigma^{j}_{Y,A}(s_t) dZ^A_t + \sigma^{j}_{Y,I}(s_t) dZ^I_{it}
\]

Shocks on fundamental parameters follow a Markov chain \(s_t \in \{1, \ldots, S\}\)

- aggregate asset volatility \(\sigma^{j}_{Y,A}(s_t)\)
- idiosyncratic asset volatility \(\sigma^{j}_{Y,I}(s_t)\)
Equity and Debt

Equity holders earn

\[ y_{it} - (1 - \tau)c^j + m(D^j(y_{it}; s_t) - p) \]

where

- \( \tau \) tax shield; \( c^j \) coupon payment; \( p \) principal; \( 1/m \) debt maturity
- \( D^j(y_{it}; s_t) \) endogenous debt value
- equity holders optimally choose when to default

Upon bankruptcy, bondholders receive

\[ (1 - \alpha^j(s_t))V^j(y_{it}; s_t) \]

where

- \( \alpha^j(s_t) \) fraction lost to bankruptcy costs
- \( V^j(y_{it}; s_t) \) value of firm’s unlevered assets
Stochastic Discount Factor

Stochastic discount factor is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta(s_t)dZ^A_t + \sum_{s_t \neq s^-_t} \left( \kappa(s^-, s_t) - 1 \right) \left( dN(s^-, s_t) - \zeta(s^-, s_t) \right)$$

where

- $r(s)$ the risk-free rate; $\eta(s)$ market price of risk
- $N(s, s')$ Poisson jumps; $\zeta(s, s')$ transition intensity
- $\kappa(s, s')$ jump-risk premium

$2S + (S^2 - S)/2$ parameters to calibrate for $r(s)$, $\eta(s)$, and $\kappa(s, s')$
Parametrization

Three assumptions similar to Chen (2010):

- Epstein-Zin preferences
- Aggregate production:
  \[
  \frac{dY_t}{Y_t} = \mu_Y(s_t) dt + \sigma_Y(s_t) dZ_t^A
  \]
- Systemic volatility \( \sigma_Y(s_t) \) and firms' aggregate asset volatility \( \sigma_{Y,A}(s_t) \):
  \[
  \sigma_{Y,A}(s_t) = \bar{\sigma}_{Y,A} + \theta \left( \sigma_Y(s_t) - \bar{\sigma}_Y \right)
  \]

With these assumptions, \( 5 + S \) parameters to calibrate:

- Relative risk aversion \( \gamma \); intertemporal substitution \( \psi \); time discount rate \( \rho \)
- Long-run volatility \( \bar{\sigma}_Y \); sensitivity \( \theta \); growth rates \( \mu_Y \)
Liquidity

- over-the-counter search frictions as in Chen, Cui, He, and Milbradt (2016)
- it takes time to find an intermediary
- investors hit by a liquidity shock bear a holding cost
  \[ \chi(N - P^j(y; s)) \]
- Nash bargaining generates bid-ask spreads
- bid-ask spreads are endogenous
Solution

Finding firms’ asset, equity, and bond prices

\[
\{ V^j(y; s), E^j(y; s), D^{H,j}(y; s), D^{L,j}(y; s) \}_{s \in S; j \in J}
\]

requires to solve a system of second order ordinary differential equations with endogenous default boundary conditions.
Calibration
Asset Return Drift

\[ \frac{dy_{it}}{y_{it}} = \mu_{Y,F}^j dt + \sigma_{Y,A}^j (s_t) dZ_t^A + \sigma_{Y,I}^j (s_t) dZ_t^I \]

- drift \( \mu_{Y,F}^j \) affects overall match between credit spreads and leverage
- calibrated to target the 8-10 years cumulative default rate from Moody
### Parameters

#### Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
</table>
| relative risk aversion                        | $\gamma$ 7.5 | Bansal and Yaron (2004)
| intertemporal substitution                    | $\psi$ 1.5 | Bansal and Yaron (2004)
| time discount rate                            | $\rho$ 0.02 | Bansal and Yaron (2004)
| systemic volatility                           | $\bar{\sigma}_Y$ 0.0293 | Bansal and Yaron (2004)
| tax shield                                    | $\tau$ 0.15 | Graham (2003)
| bargaining power                              | $\beta$ 0.03 | Feldhütter (2012)
| meeting intensity                             | $\lambda$ 50 | Chen, Cui, He, and Milbradt (2015)
| liquidity shock intensity                     | $\xi$ 0.7 | Chen, Cui, He, and Milbradt (2015)

#### Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
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</tr>
</thead>
</table>
| $\Delta \sigma_{Y,A}(s)/\Delta \sigma_Y(s)$  | $\theta$ 6 | average equity premium
| holding cost slope                            | $\chi$ 0.06 | average bid-ask spreads
| holding cost intercept                        | $N$ 0.12 | average bid-ask spreads
| average debt maturity                          | $1/m$ 8 | average maturity
Estimation
Firm-level Model Variables $x^i(t|s)$

I invert the model to infer firm-level model variables from observations.

<table>
<thead>
<tr>
<th>observations</th>
<th>model variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ market equity/book debt</td>
<td>▶ firms’ asset value</td>
</tr>
<tr>
<td>▶ aggregate equity volatility</td>
<td>▶ aggregate asset volatility</td>
</tr>
<tr>
<td>▶ idiosyncratic equity volatility</td>
<td>▶ idiosyncratic asset volatility</td>
</tr>
<tr>
<td>▶ bond recovery ratio</td>
<td>▶ bankruptcy costs</td>
</tr>
</tbody>
</table>

- ▶ data
- ▶ more
Mapping between Aggregate Equity and Asset Volatility

The model maps observations to model variables.

\[ \sigma_{E,A} \ E = \frac{\partial E(y;s)}{\partial y} \sigma_{Y,A} \]
States Identification

States estimated with Baum-Welch algorithm for hidden Markov models

1. Initiate with values for the Markov chain \( \mathcal{M} = \left\{ \sigma_{Y,A}^j(s), \sigma_{Y,I}^j(s), \zeta(s,s') \right\} \)

2. Solve the structural model and estimate \( \mathcal{Y} = \left\{ \sigma_{Y,A}^j(t|s), \sigma_{Y,I}^j(t|s) \right\} \)

3. Maximize the likelihood of being in state \( s \) at time \( t \) given \( \mathcal{Y} \)

4. Get new estimates of firms’ aggregate and idiosyncratic asset volatilities

\[
\sigma_{Y,A}^j(s) = \frac{\sum_{t=1}^{T} \sigma_{Y,A}^j(t|s) \mathbf{1}\{s_t = s\}}{\sum_{t=1}^{T} \mathbf{1}\{s_t = s\}}
\]

\[
\sigma_{Y,I}^j(s) = \frac{\sum_{t=1}^{T} \sigma_{Y,I}^j(t|s) \mathbf{1}\{s_t = s\}}{\sum_{t=1}^{T} \mathbf{1}\{s_t = s\}}
\]

5. Update transition intensities \( \zeta(s,s') \) with historical transitions

6. Iterate on 2-5 until convergence
I solve the model for two types $j$ of firms: investment- and speculative grade.

Thus, I structurally estimate

- firms’ asset value $y_{it}$
- aggregate asset volatility $\sigma_{Y,A}(s_t)$
- idiosyncratic asset volatility $\sigma_{Y,I}(s_t)$
- bankruptcy costs $\alpha^j(s_t)$

for month $t$, firms $i$, firms’ types $j$, and states $s_t$. 
Investment-Grade Aggregate Asset Volatility

\[ \sigma_{Y,A}(t | s) \]

Speculative-Grade Aggregate Asset Volatility

\[ \sigma_{Y,A}(s) \]
Investment-Grade Idiosyncratic Asset Volatility

Speculative-Grade Idiosyncratic Asset Volatility

\[ \sigma_{Y,I}(t|s) \]

\[ \sigma_{Y,I}(s) \]
Investment-Grade Idiosyncratic Equity Volatility

Speculative-Grade Idiosyncratic Equity Volatility
## Equity and Bond Premia

<table>
<thead>
<tr>
<th></th>
<th>Investment-grade</th>
<th>Speculative-grade</th>
<th>Equity Premium</th>
<th>Bond Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td></td>
</tr>
<tr>
<td>1973–2014</td>
<td>520 bps</td>
<td>611 bps</td>
<td>622 bps</td>
<td>40 bps</td>
</tr>
<tr>
<td>1987–2014</td>
<td>564 bps</td>
<td>656 bps</td>
<td></td>
<td>86 bps</td>
</tr>
</tbody>
</table>

### Market Price of Aggregate Shocks

The graph shows the market price of aggregate shocks from 1973 to 2014. The y-axis represents the market price, ranging from 0 to 0.5, and the x-axis represents the years from 1973 to 2014. The data points are connected by a line, indicating the trend over time. The peak in 2001 indicates a significant event affecting market prices.
External Validation
Goodness of Fit

<table>
<thead>
<tr>
<th></th>
<th>$R^2(cs_t, \hat{cs}_t)$</th>
<th>$R^2(\Delta cs_t, \Delta \hat{cs}_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>investment-grade firms</td>
<td>0.61</td>
<td>0.70</td>
</tr>
<tr>
<td>speculative-grade firms</td>
<td>0.71</td>
<td>0.74</td>
</tr>
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</table>

**Goodness of Fit for Credit Spreads in Levels and First Differences** The variable $cs_t$ is the average of credit spreads within a rating class, while $\hat{cs}_t$ corresponds to the model prediction. When taking first differences, the series are first averaged over each year.
Investment-Grade Average Bid-Ask Spreads

- **Bao et al. (2011)**
- **Edwards et al. (2007)**

Speculative-Grade Average Bid-Ask Spreads
Decomposition
Investment Grade Credit Spreads Decomposition

Speculative Grade Credit Spreads Decomposition

- Liquidty
- Risk Aversion
- Default Risk


%
NO LIQUIDITY FRICTIONS

Investment-Grade Credit Spreads

Speculative-Grade Credit Spreads
NO SHOCKS TO IDIOSYNCRATIC VOLATILITY

Investment-Grade Credit Spreads

Speculative-Grade Credit Spreads
NO SHOCKS TO AGGREGATE VOLATILITY

Investment-Grade Credit Spreads

Speculative-Grade Credit Spreads
NO SHOCKS TO MARKET PRICE OF RISK

Investment-Grade Credit Spreads

Speculative-Grade Credit Spreads
NO SHOCKS TO MARKET PRICE OF RISK AND AGGREGATE VOLATILITY
NO MARKOV STATES

Investment-Grade Credit Spreads

Speculative-Grade Credit Spreads
Histograms of Estimated Aggregate Shocks to Firms’ Asset Values

With Macroeconomic Shocks

Without Macroeconomic Shocks
NO LIQUIDITY FRICTIONS

Investment-Grade Equity Volatility

Speculative-Grade Equity Volatility
NO SHOCKS TO DEFAULT LOSSES

Investment-Grade Equity Volatility

Speculative-Grade Equity Volatility
NO SHOCKS TO MARKET PRICE OF RISK
NO SHOCKS TO IDIOSYNCRATIC VOLATILITY

Investment-Grade Equity Volatility

Speculative-Grade Equity Volatility
NO SHOCKS TO AGGREGATE VOLATILITY

Investment-Grade Equity Volatility

Speculative-Grade Equity Volatility
NO MARKOV STATES

Investment-Grade Equity Volatility

Speculative-Grade Equity Volatility
Forecasting
An excellent approximation to firms’ aggregate asset volatility is given by:

$$\log(\sigma_{Y,A}(t)) = \frac{1}{N} \sum_{i=1}^{N} \left[ \log \left( \frac{E_{it}}{A_{it}} \right) + \log \left( \sigma_{E,A}^i(t) \right) \right]$$

where

- $A_{it}$ firm’s $i$ total asset value
- $E_{it}$ firm’s $i$ equity value
- $\sigma_{E,A}^i(t)$ firm’s $i$ aggregate equity volatility
## Forecasting Performance

<table>
<thead>
<tr>
<th></th>
<th>Real GDP Growth 4-Quarters Ahead Forecast Horizon</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FED</td>
<td></td>
<td>0.260**</td>
<td>-0.045</td>
<td>0.085</td>
<td>-0.072</td>
<td>-0.119</td>
</tr>
<tr>
<td>TSLO</td>
<td></td>
<td>0.491***</td>
<td>0.405***</td>
<td>0.429***</td>
<td>0.346***</td>
<td>0.349***</td>
</tr>
<tr>
<td>GZ</td>
<td></td>
<td>-0.510***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{Y,A}^{INV})</td>
<td></td>
<td></td>
<td></td>
<td>-0.356***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{Y,A}^{SPE})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td></td>
<td>0.17</td>
<td>0.32</td>
<td>0.26</td>
<td>0.34</td>
<td>0.36</td>
</tr>
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*** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \); standardized coefficients
Conclusion
Conclusion

- Aggregate asset volatility is key to account for joint dynamics of credit spreads and equity volatility.

- Aggregate asset volatility strongly forecasts economic activity and captures the informational predictive content of credit spreads.

- Measuring aggregate asset volatility does not require departures from Modigliani and Miller’s (1958) assumptions.
The Hamilton-Jacobi-Bellman equation in state $s$ (for $s = 1, \ldots, S$) is

$$0 = f(C, J(C, s)) + J_C(C, s)C\mu_Y(s) + \frac{1}{2} J_{CC}(C, s)C^2\sigma_Y^2(s)$$

$$+ \sum_{s' \neq s} \zeta_{s,s'}(J(C, s') - J(C, s)).$$

Conjecture that the solution for $J$ is

$$J(C, s) = \frac{(h(s)C)^{1-\gamma}}{1-\gamma}$$

and we get

$$0 = \rho \frac{1-\gamma}{1-\delta} h(s)^{\delta-\gamma} + \left[(1-\gamma)\mu_Y - \frac{1}{2}\gamma(1-\gamma)\sigma_Y^2(s) - \rho \frac{1-\gamma}{1-\delta}\right] h(s)^{1-\gamma}$$

$$+ \sum_{s' \neq s} \zeta_{s,s'}'(h(s')^{1-\gamma} - h(s)^{1-\gamma})$$

While no algebraic solution exists for this system of nonlinear equations, it is trivial to solve numerically for $h(s) \forall s \in \{1, \ldots, S\}$. 

Stochastic Discount Factor Solution (1/2)
Duffie and Skiadas (1994) show that the stochastic discount factor is equal to

$$\Lambda_t = \exp \left( \int_0^t f_J(C_u, J_u) du \right) f_C(C_t, J_t)$$

Thus we have

$$\Lambda_t = \exp \left( \int_0^t \frac{\rho(1-\gamma)}{1-\delta} \left( \frac{\delta - \gamma}{1-\gamma} h(s_u)^{\delta-1} - 1 \right) du \right) \rho H(s)^{\delta-\gamma} Y^{-\gamma}$$

Applying Ito’s formula with jumps, we get

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s) dt - \eta(s) dZ_t^A + \sum_{s' \neq s} \left( e^{\kappa(s, s')} - 1 \right) dM_{t}^{s,s'},$$

where

$$r(s) = -\frac{\rho(1-\gamma)}{1-\delta} \left[ \frac{\delta - \gamma}{1-\gamma} h(s)^{\delta-1} - 1 \right] + \gamma \mu_Y(s) - \frac{1}{2} \gamma(1+\gamma) \sigma_Y^2(s)$$

$$- \sum_{s' \neq s} \left( e^{\kappa(s, s')} - 1 \right)$$

$$\eta(s) = \gamma \sigma_Y(s)$$

$$\kappa(s, s') = (\delta - \gamma) \log \left( \frac{h(j)}{h(i)} \right)$$
This table displays the correlation between the first principal components of each variable averaged by rating class where

- $\log(cs)$ is log credit spread
- $\text{lev}$ is market leverage
- $\log(\sigma_E)$ is log equity return total volatility
The first principal component of credit spreads, leverage, and equity return volatility by rating classes explains 78% of the total variation.
Let $X_t$ be a discrete hidden random variable with $N$ possible values. We assume that $P(X_t|X_{t-1})$ is independent of time $t$, which leads to the definition of the time independent stochastic transition matrix $A = \{a_{ij}\}$ where $a_{ij} = P(X_t = j|X_{t-1} = i)$. The initial state distribution is given by $\mu_0$. The observation variables $Y_t$ can take one of $K$ possible values. The probability of a certain observation at time $t$ for state $j$ is given by

$$ \ell_j(y_t) = P(Y_t = y_t|X_t = j). $$

We will represent the observation density as follows: for every $y \in F$, we define the diagonal matrix $L(y)$ with nonzero elements $\{L(y)\}_{ii} = \ell_i(y)$. An observation sequence is given by $Y = (Y_0 = y_0, Y_2 = y_2, \ldots, Y_T = y_T)$. Thus we can describe a hidden Markov chain by $\theta = (A, L, \mu_0)$. The Baum-Welch algorithm finds a local maximum for $\theta^* = \arg \max_{\theta} P(Y|\theta)$. 

Baum-Welch Algorithm 1/3
Initiation Set $\theta = (A, L, \mu_0)$ with random initial conditions.

Forward procedure Let $\pi_{i,t} = P(Y_0 = y_0, ..., Y_t = y_t, X_t = i|\theta)$ be the probability of seeing the $y_0, y_1, ..., y_t$ and being in state $i$ at time $t$. First, we get

$$c_0 = 1'L(y_0)\mu_0,$$

$$\pi_0 = L(y_0)\mu_0/c_0.$$

Then for $k = 1, ..., N$

$$\tilde{\pi}_k = L(y_k)A'\pi_{k-1},$$

$$c_k = 1'\tilde{\pi}_k$$

$$\pi_k = \tilde{\pi}_k/c_k$$
Backward procedure Let $\beta_{i,t} = P(Y_{t+1} = y_{t+1}, \ldots, Y_T = y_T | X_t = i, \theta)$ that is the probability of the ending partial sequence $y_{t+1}, \ldots, y_T$ given starting state $i$ at time $t$. We calculate $\beta_{i,t}$ as

$$\beta_{N|N} = 1/c_N$$

then or $k = 1, \ldots, N$

$$\beta_{N-k|N} = A_L(y_{N-k+1})\beta_{N-k+1|N}/c_{N-k},$$

$$\pi_{N-k,N-k+1|N} = \text{diag}(\pi_{N-k})A_L(y_{N-k+1})\text{diag}(\beta_{N-k+1|N}),$$

$$\pi_{N-k|N} = \pi_{N-k,N-k+1|N}1,$$

where $\pi_{i,N-k|N} = P(X_t = i | Y, \theta)$ is the probability of being in state $i$ at time $t$ given the observed sequence $Y$ and the parameters $\theta$. We can now update $\theta$ as

$$P_{ij} = \frac{\sum_{k=1}^{N}(\pi_{k-1,k|N})_{ij}}{\sum_{k=1}^{N}(\pi_{k-1|N})_i},$$

$$\mu_0 = \pi_0|N,$$

and the parameters of $L(\cdot)$ as their empirical counterparts.
Unexpected Volatility Regime Changes

CBOE S&P500 3-Month Variance Futures

Average Equity Return Variance
Firms’ Asset Value

Value of firm’s assets given by

\[ V_t = y_t v(s_t), \]

where \( v(\cdot) \) is the state dependent price-earning ratio. Thus,

\[ \frac{dV_t}{V_t} = \mu_Y dt + \sigma_{Y,A}(s_t)dZ_t^A + \sigma_{Y,I}(s_t)dZ_t^I + \sum_{s_t \neq s_t^-} \left( \frac{v(s_t^-)}{v(s_t)} - 1 \right) dN_t^{(s_t^-},s_t), \]

where \( v(s_t^-)/v(s_t) \) represents jump in asset value from state \( s_t^- \) to state \( s_t \).
The first principal component explains 90% of the overall variation. Equity return volatility and leverage are also driven by a strong common factor (93% and 72%).
Co-movements in Credit Spreads, Leverage, and Equity Volatility

First Principal Components of Credit Spreads, Leverage, and Equity Return Volatility

The first principal component of credit spreads, leverage, and equity return volatility by rating classes explains 78% of the total variation.
System of Second Order Ordinary Differential Equations

\[ r_s D_s^H(y) = \mu_s \frac{\partial D_s^H(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 D_s^H(y)}{\partial y^2} + c + m(p - D_s^H(y)) \]

\[ + \sum_{s' \neq s} \zeta_{ss'} Q (D_{s'}^H(y) - D_s^H(y)) + \xi^H (D_s^L(y) - D_s^H(y)) \]

\[ r_s D_s^L(y) = \mu_s \frac{\partial D_s^L(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 D_s^L(y)}{\partial y^2} + c + m(p - D_s^L(y)) \]

\[ + \sum_{s' \neq s} \zeta_{ss'} Q (D_{s'}^L(y) - D_s^L(y)) + \lambda \beta (D_s^H(y) - D_s^L(y)) \]

\[ - \chi (B_s + N - P_s(y)) \]

\[ r_s E_s(y) = \mu_s \frac{\partial E_s(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 E_s(y)}{\partial y^2} + \exp(y) - (1 - \tau)c + m(D_s^H(y) - p) \]

\[ + \sum_{s' \neq s} \zeta_{ss'} Q (E_i(y) - E_s(y)) \]
Firms’ Asset Value Estimation

Match firm’s leverage with model-implied log asset return $y$ according to

$$lev = \frac{p}{p + E_s(y)},$$

where $p$ (principal) is the book value of outstanding debt. Thus, model-implied values of equity and debt are matched to their empirical counterparts.

I use 300,887 monthly observations of equity prices from CRSP and book value of debt from Compustat between 1973 and 2014. The model implied leverage distribution matches exactly the empirical leverage distribution in every month.
From Ito’s lemma, I can estimate firms’ asset volatility according to:

\[ \sigma_{E,A}^i(t|s) E^j_s \left( y^i_t \right) = E_{j,s'} \left( y^i_t \right) \sigma_{Y,A}(s), \]

\[ \sigma_{E,I}^i(t|s) E^j_s \left( y^i_t \right) = E_{j,s'} \left( y^i_t \right) \sigma_{Y,I}(s). \]

where

- \( y^i_t \) is the model-implied level of cash flows of firm \( i \) at time \( t \)
- \( E^j_s(y) \) is the equity value of firms of type \( j \)
- \( \sigma_{E,A}^i(t|s) \) is aggregate asset volatility of firm \( i \) at time \( t \) in state \( s \)
- \( \sigma_{E,I}^i(t|s) \) is idiosyncratic asset volatility of firm \( i \) at time \( t \) in state \( s \)
The bond recovery ratio in state $s$ is given by

$$\frac{(1 - \alpha(s))Vb(s)}{p}$$

where

- $p$ is the principal of the debt
- $Vb(s)$ is the value of firm’s assets at bankruptcy
- $\alpha$ is the fraction lost to bankruptcy costs
Aggregate and Idiosyncratic Equity Volatility

Idiosyncratic returns are constructed by estimating a factor model using all observations for that firm following:

$$r^i_t - r^f_t = \gamma^i_0 + F_t \gamma^i_t + \varepsilon^i_t,$$

where

- $r^i_t$ is the equity return from day $t - 1$ to $t$, including dividends of firm $i$
- $r^f_t$ is the 1-month treasury bill rate
- $F_t$ is the Fama and French (1992) and Carhart (1997) 4-factor model

Aggregate and idiosyncratic equity volatility is then given by:

$$\sigma^{i,E,A}(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} (F_k \hat{\gamma}_k^i)^2}$$

$$\sigma^{i,E,I}(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} (\hat{\varepsilon}_k^i)^2},$$

where $L_t$ is the last day in month $t$. 
Market leverage ratio of firm $i$ at time $t$ is defined as:

$$\text{lev}_{it} = \frac{\text{DLTT}_{it} + \text{DLC}_{it}}{\text{DLTT}_{it} + \text{DLC}_{it} + \text{CSHO}_{it} \times \text{PRCC}_{it}},$$

where

- DLTT is Compustat long-term debt
- DLC is Compustat debt in current liabilities
- CSHO is CRSP number of shares outstanding
- PRCC is CRSP stock price
Market Price of Aggregate Shocks and Systemic Volatility

**Sharpe Ratio of Claim to Aggregate Production**

**Systemic Volatility**
Value of firm’s assets given by

\[ V_t = y_t v(s_t), \]

where \( v(\cdot) \) is the state dependent price-earning ratio. Thus,

\[ \frac{dV_t}{V_t} = \mu_Y dt + \sigma_{Y,A}(s_t) dZ_t^A + \sigma_{Y,I}(s_t) dZ_t^I + \sum_{s_t \neq s_{t-}} (v(s_{t-})/v(s_t) - 1) dN_t^{(s_{t-},s_t)}, \]

where \( v(s_{t-})/v(s_t) \) represents jump in asset value from state \( s_{t-} \) to state \( s_t \).
I solve the model $F$ for two types $j$ of firms: investment- and speculative-grade.

$$F(y^i(t); s, x^j(s)) \rightarrow x^i(t|s) \rightarrow x^j(t|s) \rightarrow x^j(s)$$

where

$$x^j(t|s) = \frac{1}{N^j(t)} \sum_{i \in I^j(t)} x^i(t|s)$$

$$x^j(s) = \frac{\sum_{t=1}^{T} x^j(t|s) 1\{s_t = s\}}{\sum_{t=1}^{T} 1\{s_t = s\}}$$