Disentangling Credit Spreads and Equity Volatility

Job Market Paper

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What Drives Predictors of the Business Cycle?

Financial indicators are powerful predictors of economic activity.

- equity volatility and corporate bond credit spreads (Bloom 09, GZ 12)

No generally accepted understanding of what shocks drive these indicators.

- source of the predictive content

I propose a structural framework to quantify the drivers of financial indicators.

- dynamic capital structure model with liquidity frictions

Potential Driving Forces

To explain credit spreads and equity volatility, the model features shocks to

- firms' asset values
- firms' aggregate asset volatility
- firms' idiosyncratic asset volatility
- bankruptcy costs
- stochastic discount factor
- liquidity frictions

structurally estimated from 300,887 monthly firm-level observations of

- corporate bond credit spreads (Lehman/Warga and Merrill Lynch)
- equity prices and equity volatilities (CRSP)
- accounting statements (Compustat)
- bond recovery ratios (Moody)

in the U.S. from 1973 to 2014.

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External Validation

The model yields predictions for the levels and *joint* macrodynamics of

- corporate bond credit spreads
  - default risk
  - excess bond premium
  - bond bid-ask spreads
- equity volatility
  - aggregate equity volatility
  - idiosyncratic equity volatility
  - leverage

Results

(1) Model-implied financial indicators match historical counterparts.

(2) Shocks to firms' aggregate asset volatility are key for joint dynamics.

(3) Shocks to firms' aggregate asset volatility strongly predict economic activity.

Literature

Predictors of Real Economic Activity ▷ **source of the predictive content**

Gilchrist, Yankov, and Zakrajšek (2009); Stock and Watson (2012); Gilchrist and Zakrajšek (2012); Faust, Gilchrist, Wright, and Zakrajšek (2013); Caldara, Fuentes-Albero, Gilchrist, and Zakrajšsek (2016); Gilchrist and Zakrajšek (2012); Kelly, Manzo, and Palhares (2016); López-Salido, Stein, and Zakrajšek (2016)

Credit Spreads ▷ **structural decomposition**

Collin-Dufresne, Goldstein, and Martin (2001); Longstaff, Mithal, and Neis (2005); Hackbarth, Miao, and Morellec (2006); Almeida and Philippon (2007); David (2008); Chen, Collin-Dufresne, and Goldstein (2009); Bhamra, Kuehn, and Strebulaev (2010)

Equity Volatility ▷ **link with credit spreads**

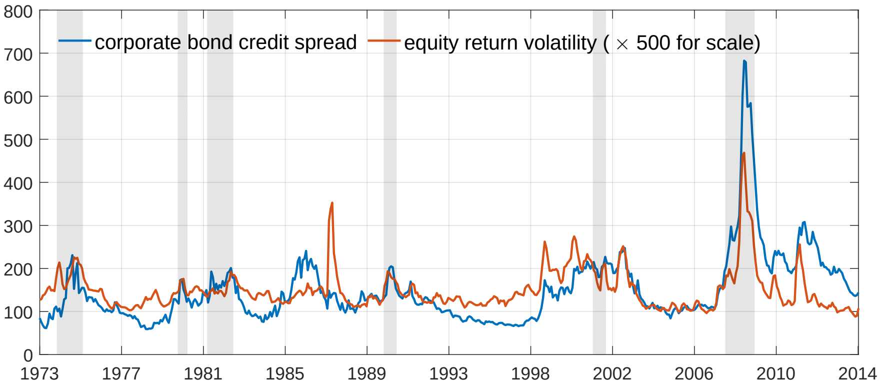
Schwert (1989), Campbell and Taksler (2003); Bloom (2009), Arellano, Bae, Kehoe (2012), Christiano, Motto, and Rostagno (2014); Atkeson, Eisfeldt, and Weill (2014); Jurado, Ludvigson, and Ng (2015); Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)

Structural Models of Default and Liquidity ▷ **empirical estimation of shocks**

Merton (1974), Leland (1994); Chen (2010); He and Milbradt (2014); Chen, Cui, He, and Milbradt (2016)

Macrodynamics of Credit Spreads and Equity Volatility

corr



Model

Model Overview

- firms have access to productive assets
- debt and equity as liabilities
- tax shield and bankruptcy costs
- stochastic discount factor
- liquidity as in Chen, Cui, He, and Milbradt (2016)

- structurally estimate shocks driving credit spreads and equity volatility

Firms' Assets

Type-j firm's asset cash flows follow a diffusion

$$\frac{dy_{it}}{y_{it}} = \mu^j_{Y,F}dt + \sigma^j_{Y,A}(s_t)dZ^A_t + \sigma^j_{Y,I}(s_t)dZ^I_{it}$$

asset value

Shocks on fundamental parameters follow a Markov chain $s_t \in \{1, \ldots, S\}$

- aggregate asset volatility $\sigma^j_{Y,A}(s_t)$
- idiosyncratic asset volatility $\sigma^j_{Y,I}(s_t)$

Equity and Debt

Equity holders earn

$$y_{it} - (1-\tau)c^j + m(D^j(y_{it}; s_t) - p)$$

where

- τ tax shield; c^j coupon payment; p principal; $1/m$ debt maturity
- $D^j(y_{it}; s_t)$ endogenous debt value
- equity holders optimally choose when to default

Upon bankruptcy, bondholders receive

$$(1 - \alpha^j(s_t))V^j(y_{it}; s_t)$$

where

- $\alpha^j(s_t)$ fraction lost to bankruptcy costs
- $V^j(y_{it}; s_t)$ value of firm's unlevered assets

asset value

Stochastic Discount Factor

Stochastic discount factor is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta(s_t)dZ_t^A + \sum_{s_t \neq s_{t-}} \Big(\kappa(s_{t-}, s_t) - 1\Big)\Big(dN(s_{t-}, s_t) - \zeta(s_{t-}, s_t)\Big)$$

where

- $r(s)$ the risk-free rate; $\eta(s)$ market price of risk
- $N(s, s')$ Poisson jumps; $\zeta(s, s')$ transition intensity
- $\kappa(s, s')$ jump-risk premium

$2S + (S^2 - S)/2$ parameters to calibrate for $r(s)$, $\eta(s)$, and $\kappa(s, s')$

Parametrization

Three assumptions similar to Chen (2010):

- Epstein-Zin preferences

more

- aggregate production:

$$\frac{dY_t}{Y_t} = \mu_Y(s_t)dt + \sigma_Y(s_t)dZ_t^A$$

- systemic volatility $\sigma_Y(s_t)$ and firms' aggregate asset volatility $\sigma_{Y,A}(s_t)$:

$$\sigma_{Y,A}(s_t) = \bar{\sigma}_{Y,A} + \theta\left(\sigma_Y(s_t) - \bar{\sigma}_Y\right)$$

With these assumptions, $5 + S$ parameters to calibrate:

- relative risk aversion γ; intertemporal substitution ψ; time discount rate ρ
- long-run volatility $\bar{\sigma}_Y$; sensitivity θ; growth rates μ_Y

Liquidity

- over-the-counter search frictions as in Chen, Cui, He, and Milbradt (2016)
- it takes time to find an intermediary
- investors hit by a liquidity shock bear a holding cost

$$\chi(N - P^j(y; s))$$

- Nash bargaining generates bid-ask spreads
- bid-ask spreads are endogenous

Solution

Finding firms' asset, equity, and bond prices

$$\left\{V^j(y;s), E^j(y;s), D^{H,j}(y;s), D^{L,j}(y;s)\right\}_{s\in S; j\in J}$$

system

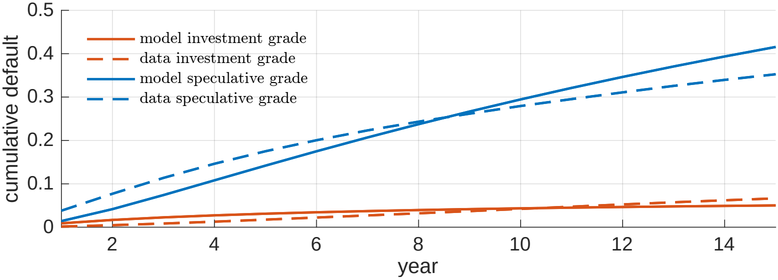
requires to solve a system of second order ordinary differential equations with endogenous default boundary conditions.

Calibration

Asset Return Drift

$$\frac{dy_{it}}{y_{it}} = \mu^j_{Y,F}dt + \sigma^j_{Y,A}(s_t)dZ^A_t + \sigma^j_{Y,I}(s_t)dZ^I_{it}$$

- drift $\mu^j_{Y,F}$ affects overall match between credit spreads and leverage
- calibrated to target the 8-10 years cumulative default rate from Moody



Parameters

Parametrization

relative risk aversion	γ	7.5	Bansal and Yaron (2004)
intertemporal substitution	ψ	1.5	Bansal and Yaron (2004)
time discount rate	ρ	0.02	Bansal and Yaron (2004)
systemic volatility	$\bar{\sigma}_Y$	0.0293	Bansal and Yaron (2004)
tax shield	τ	0.15	Graham (2003)
bargaining power	β	0.03	Feldhütter (2012)
meeting intensity	λ	50	Chen, Cui, He, and Milbradt (2015)
liquidity shock intensity	ξ	0.7	Chen, Cui, He, and Milbradt (2015)

Calibration

$\Delta\sigma_{Y,A}(s)/\Delta\sigma_Y(s)$	θ	6	average equity premium
holding cost slope	χ	0.06	average bid-ask spreads
holding cost intercept	N	0.12	average bid-ask spreads
average debt maturity	$1/m$	8	average maturity

Estimation

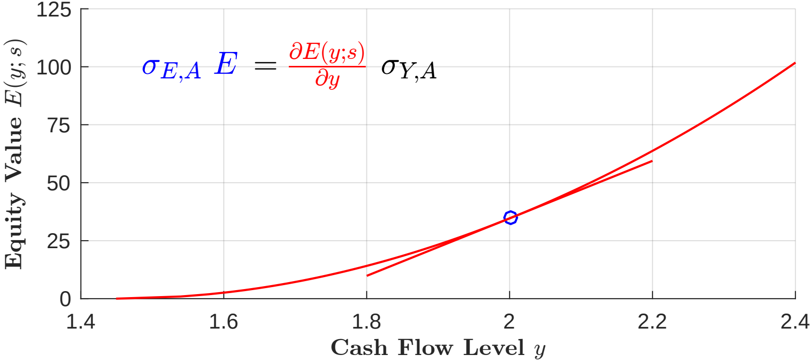
Firm-level Model Variables $x^i(t|s)$

I invert the model to infer firm-level model variables from observations.

observations		model variables	
▸ market equity/book debt	▸ data	▸ firms' asset value	▸ more
▸ aggregate equity volatility	▸ data	▸ aggregate asset volatility	▸ more
▸ idiosyncratic equity volatility	▸ data	▸ idiosyncratic asset volatility	▸ more
▸ bond recovery ratio	▸ data	▸ bankruptcy costs	▸ more

Mapping between Aggregate Equity and Asset Volatility

The model maps observations to model variables.



States Identification

comovements Baum-Welch

States estimated with Baum-Welch algorithm for hidden Markov models

(1) Initiate with values for the Markov chain $\mathcal{M} = \left\{ \sigma^j_{Y,A}(s), \sigma^j_{Y,I}(s), \zeta(s, s') \right\}$

(2) Solve the structural model and estimate $\mathcal{Y} = \left\{ \sigma^j_{Y,A}(t|s), \sigma^j_{Y,I}(t|s) \right\}$

(3) Maximize the likelihood of being in state s at time t given $\mathcal{Y}$

(4) Get new estimates of firms' aggregate and idiosyncratic asset volatilities

$$\sigma^j_{Y,A}(s) = \frac{\sum_{t=1}^{T} \sigma^j_{Y,A}(t|s) \mathbf{1}\left\{s_t = s\right\}}{\sum_{t=1}^{T} \mathbf{1}\left\{s_t = s\right\}} \quad \sigma^j_{Y,I}(s) = \frac{\sum_{t=1}^{T} \sigma^j_{Y,I}(t|s) \mathbf{1}\left\{s_t = s\right\}}{\sum_{t=1}^{T} \mathbf{1}\left\{s_t = s\right\}}$$

(5) Update transition intensities $\zeta(s, s')$ with historical transitions

(6) Iterate on 2-5 until convergence

Firms' Types

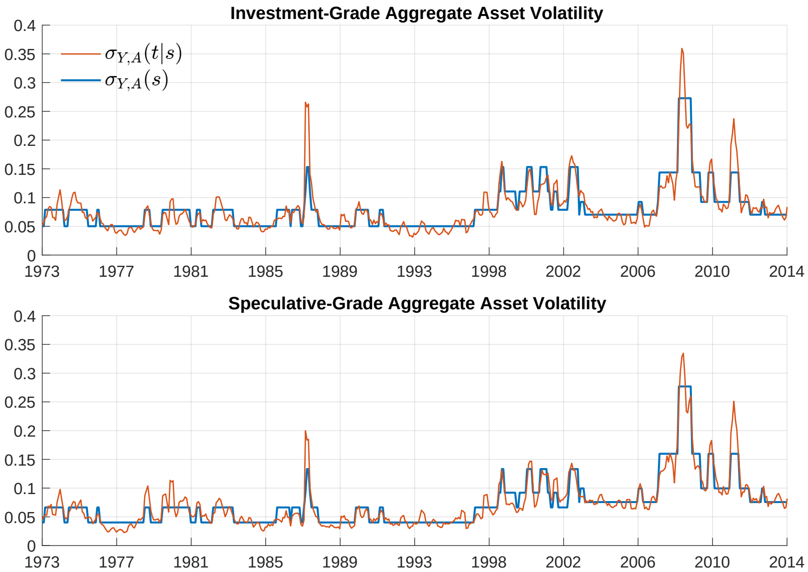
mapping

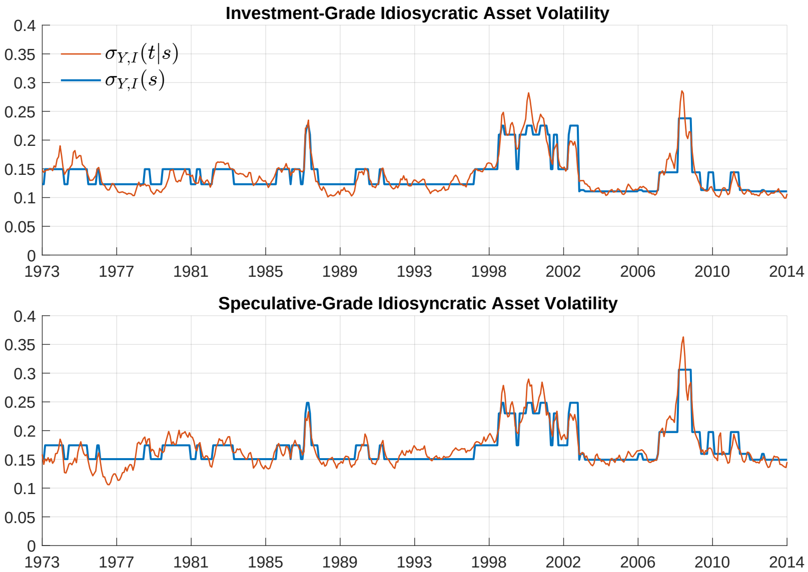
I solve the model for two types j of firms: investment- and speculative grade.

Thus, I structurally estimate

- firms' asset value y_{it}
- aggregate asset volatility $\sigma^j_{Y,A}(s_t)$
- idiosyncratic asset volatility $\sigma^j_{Y,I}(s_t)$
- bankruptcy costs $\alpha^j(s_t)$

for month t, firms i, firms' types j, and states s_t.


Investment-Grade Aggregate Asset Volatility
$\sigma_{Y,A}(t|s)$
$\sigma_{Y,A}(s)$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014
Speculative-Grade Aggregate Asset Volatility
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014


Investment-Grade Idiosycratic Asset Volatility
$\sigma_{Y,I}(t|s)$
$\sigma_{Y,I}(s)$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1973
1977
1981
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1989
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Speculative-Grade Idiosyncratic Asset Volatility
0.4
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0
1973
1977
1981
1985
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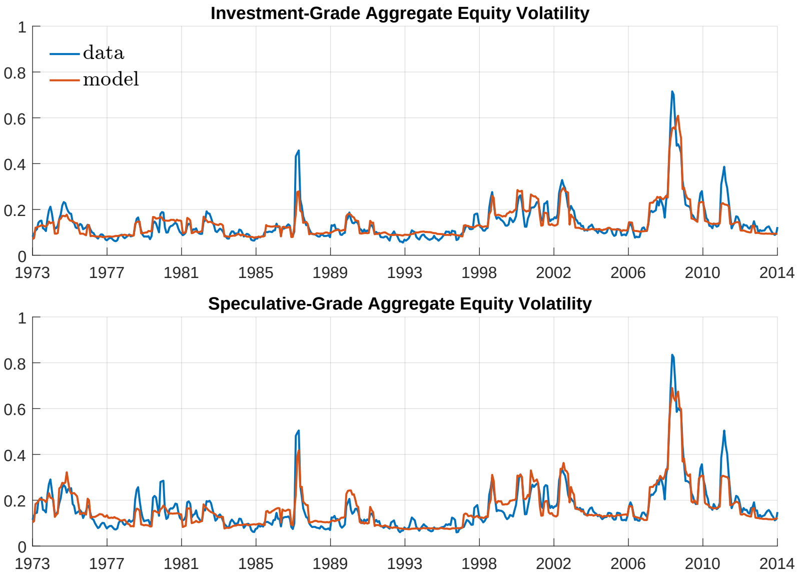
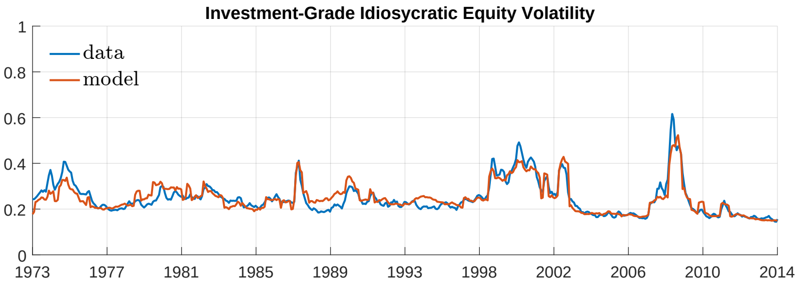
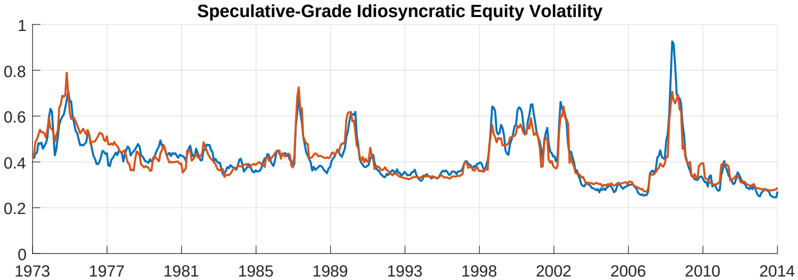
1998
2002
2006
2010
2014

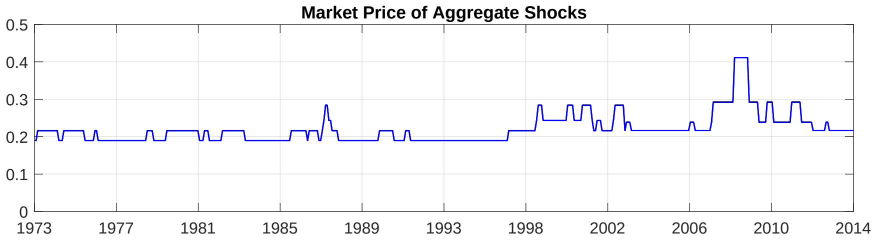

Investment-Grade Aggregate Equity Volatility
data
model
1
0.8
0.6
0.4
0.2
0
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014
Speculative-Grade Aggregate Equity Volatility
1
0.8
0.6
0.4
0.2
0
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014


Investment-Grade Idiosycratic Equity Volatility
data
model
0
0.2
0.4
0.6
0.8
1
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014


Speculative-Grade Idiosyncratic Equity Volatility
0
0.2
0.4
0.6
0.8
1
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014

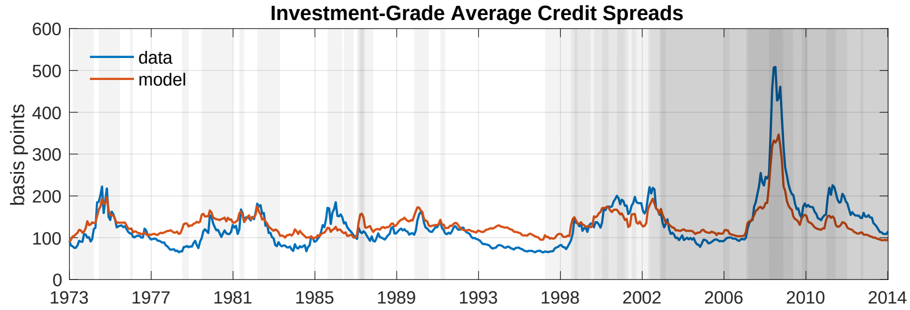
Equity and Bond Premia

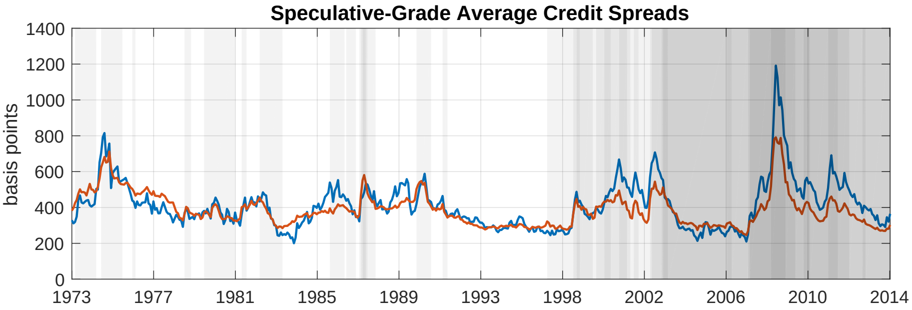
	equity premium			bond premium
	model	model	data	model
	1973–2014	1987–2014	1987–2014	1973–2014
investment-grade	520 bps	611 bps	622 bps	40 bps
speculative-grade	564 bps	656 bps		86 bps



sharpe ratio and systemic volatility

External Validation

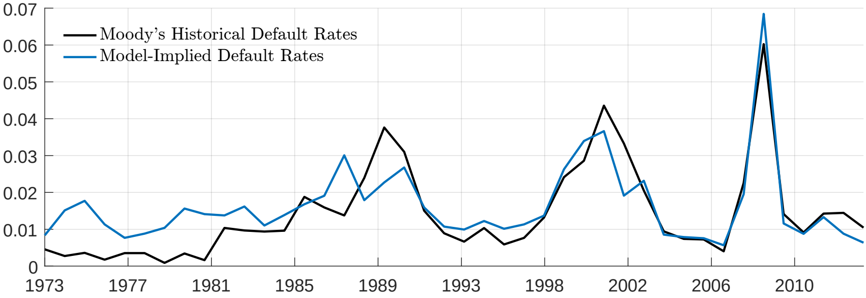

Investment-Grade Average Credit Spreads
600
500
400
300
200
100
0
basis points
data
model
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014


Speculative-Grade Average Credit Spreads
1400
1200
1000
800
600
400
200
0
basis points
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014

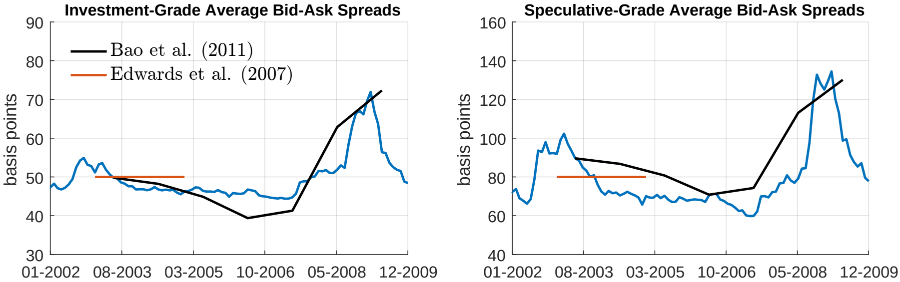
Goodness of Fit

	$R2(cs_t, \widehat{cs}_t)$	$R2(\Delta cs_t, \Delta\widehat{cs}_t)$
investment-grade firms	0.61	0.70
speculative-grade firms	0.71	0.74

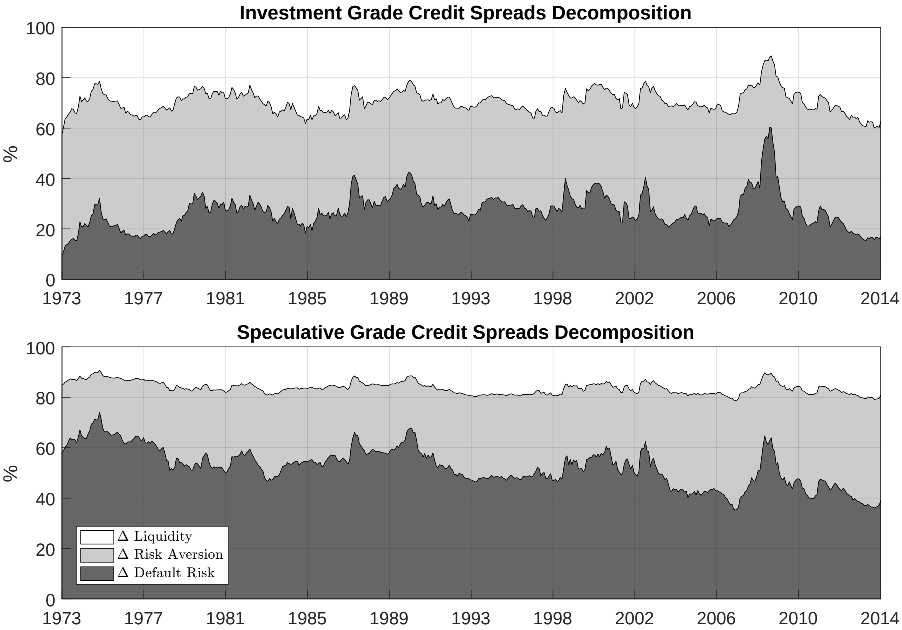
Goodness of Fit for Credit Spreads in Levels and First Differences The variable cs_t is the average of credit spreads within a rating class, while $\widehat{cs}_t$ corresponds to the model prediction. When taking first differences, the series are first averaged over each year.



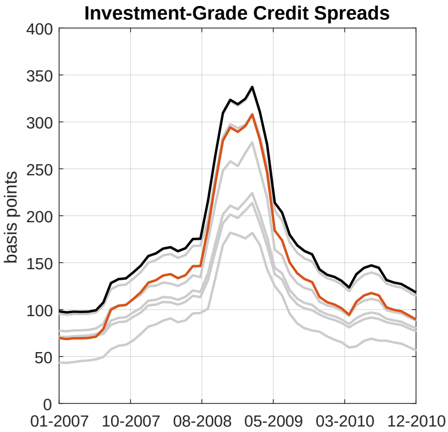
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
Moody's Historical Default Rates
Model-Implied Default Rates

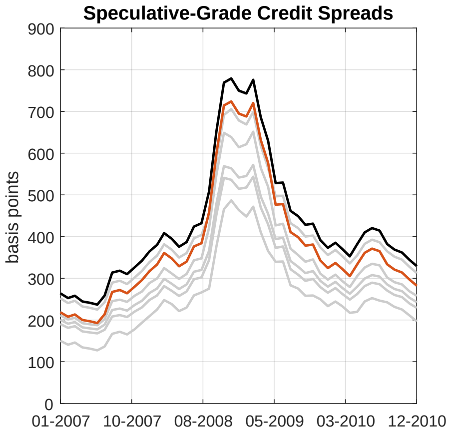

Investment-Grade Average Bid-Ask Spreads
Bao et al. (2011)
Edwards et al. (2007)
basis points
90
80
70
60
50
40
30
01-2002
08-2003
03-2005
10-2006
05-2008
12-2009
Speculative-Grade Average Bid-Ask Spreads
basis points
160
140
120
100
80
60
40
01-2002
08-2003
03-2005
10-2006
05-2008
12-2009

Decomposition

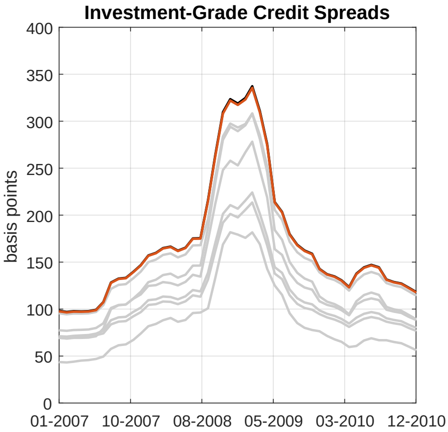

Investment Grade Credit Spreads Decomposition
100
80
60
40
20
0
%
1973
1977
1981
1985
1989
1993
1998
2002
2006
2010
2014
Speculative Grade Credit Spreads Decomposition
Δ Liquidity
Δ Risk Aversion
Δ Default Risk

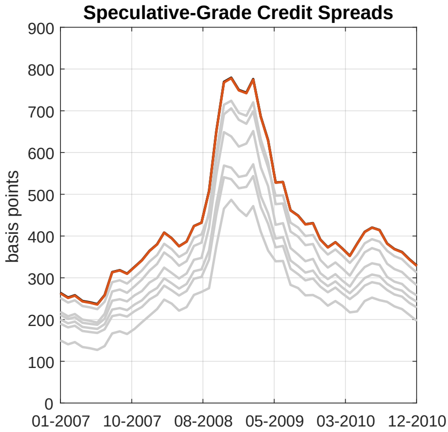
NO LIQUIDITY FRICTIONS



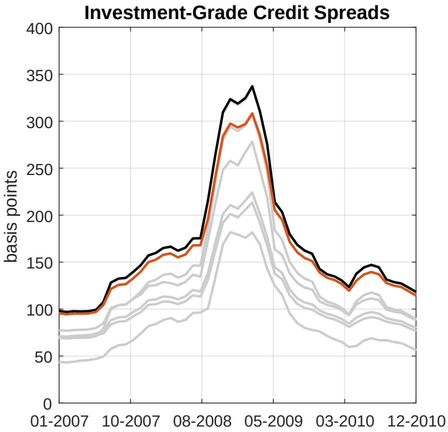


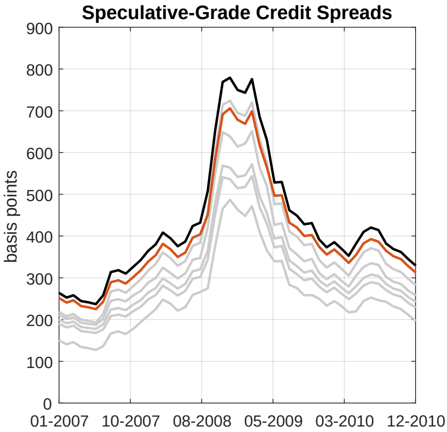
NO SHOCKS TO DEFAULT LOSSES



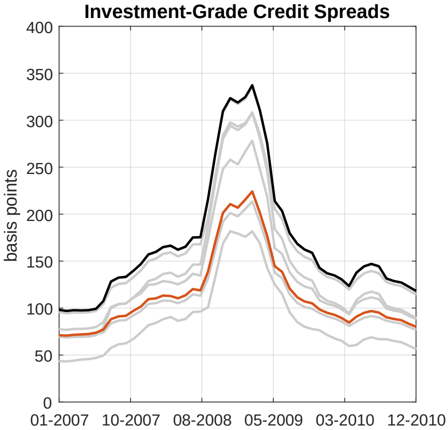


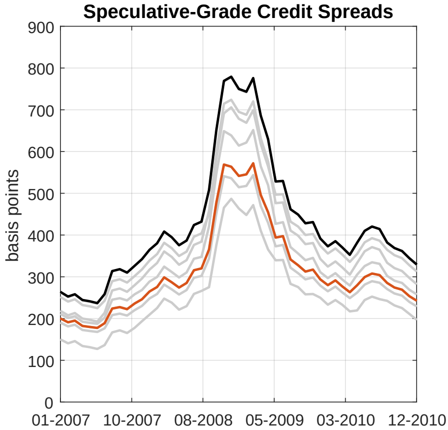
NO SHOCKS TO IDIOSYNCRATIC VOLATILITY



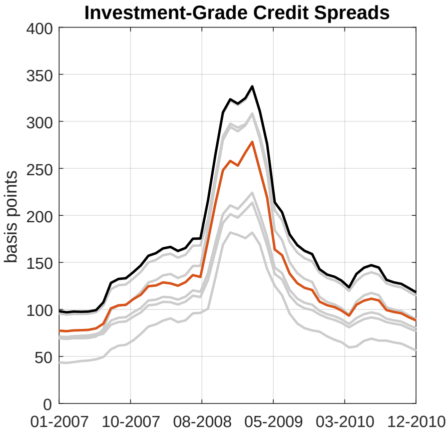


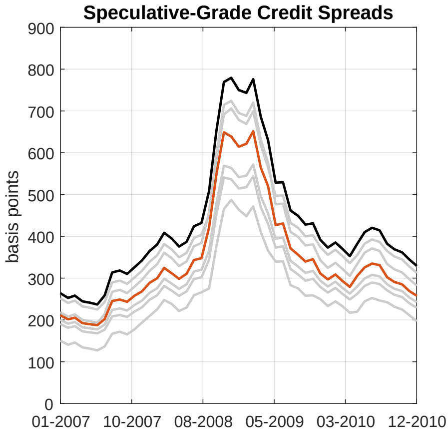
NO SHOCKS TO AGGREGATE VOLATILITY



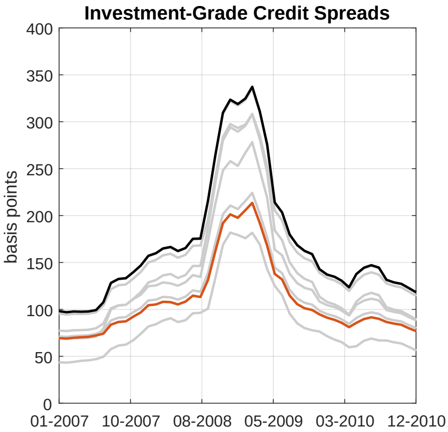


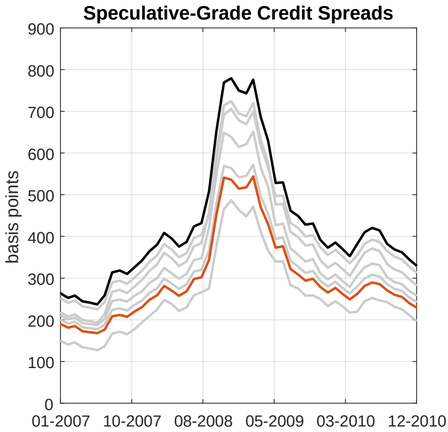
NO SHOCKS TO MARKET PRICE OF RISK



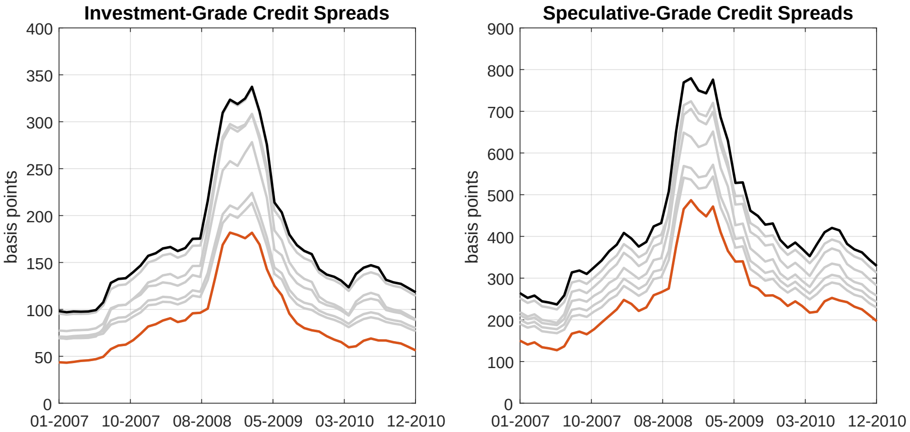


NO SHOCKS TO MARKET PRICE OF RISK AND AGGREGATE VOLATILITY

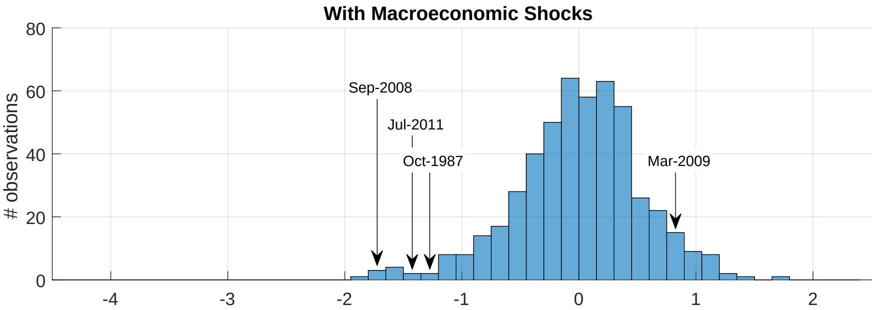


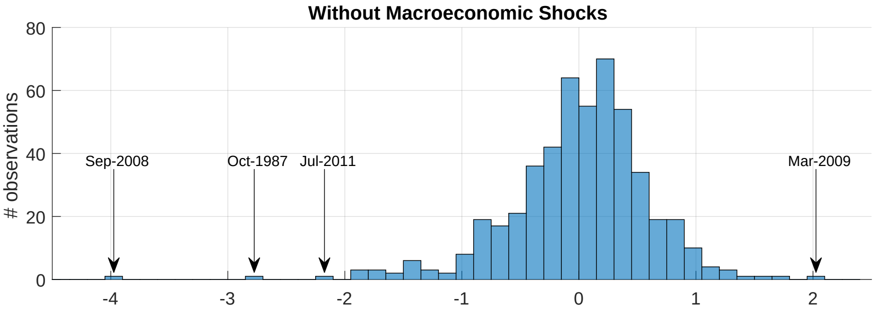


NO MARKOV STATES

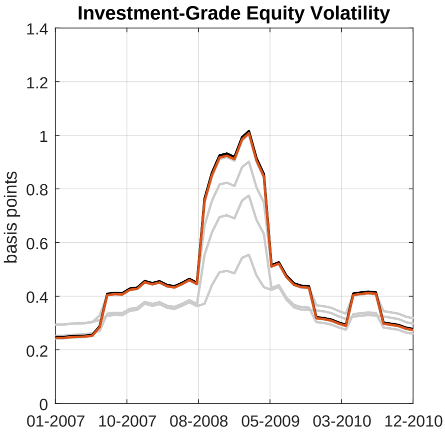


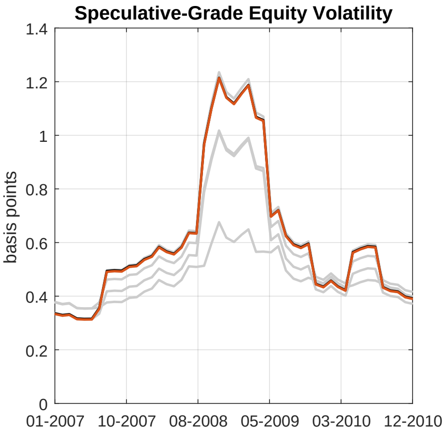
Histograms of Estimated Aggregate Shocks to Firms' Asset Values



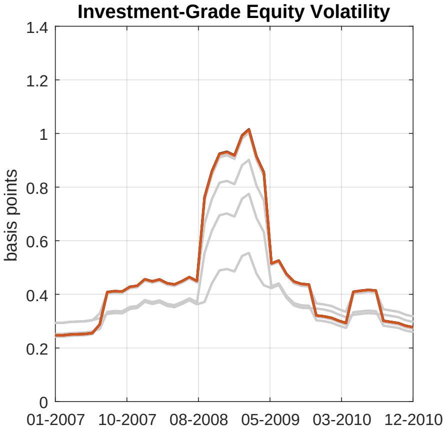


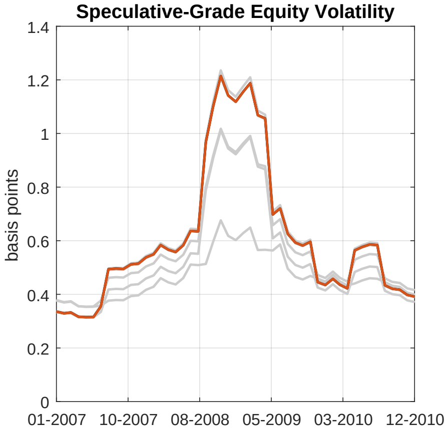
NO LIQUIDITY FRICTIONS



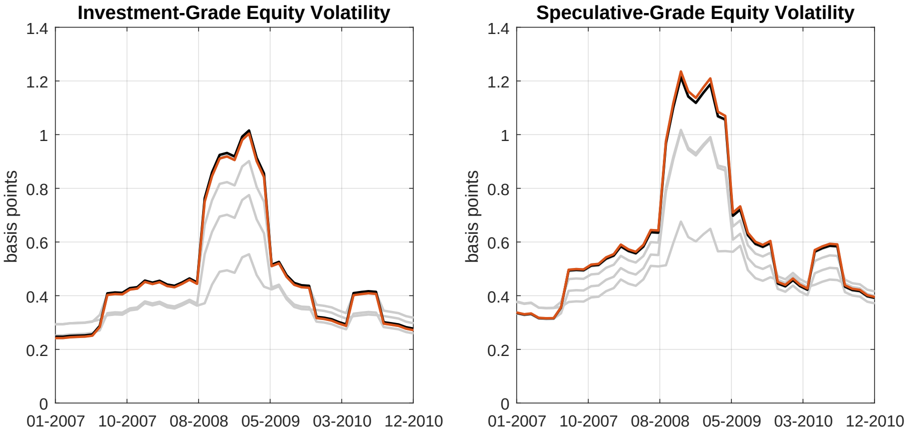


NO SHOCKS TO DEFAULT LOSSES

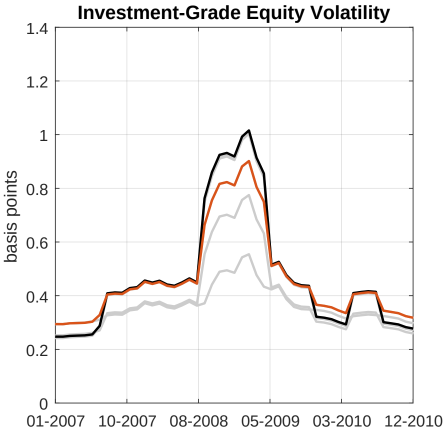


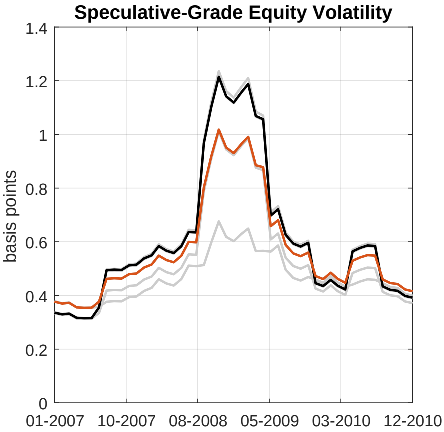


NO SHOCKS TO MARKET PRICE OF RISK

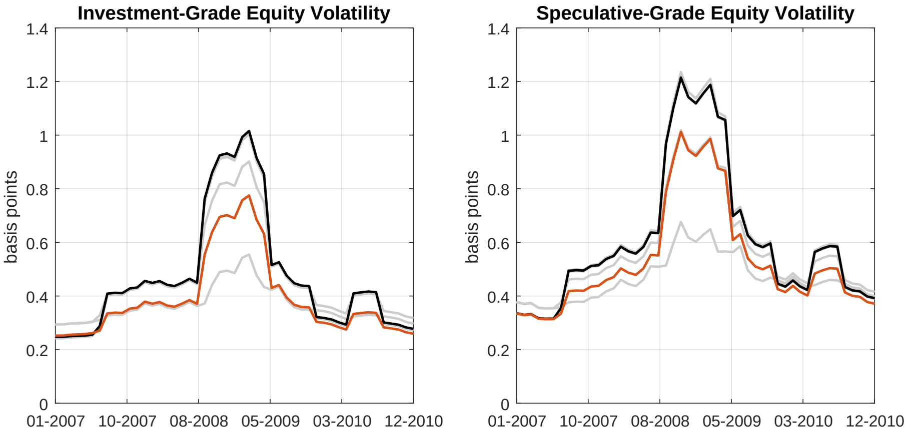


NO SHOCKS TO IDIOSYNCRATIC VOLATILITY

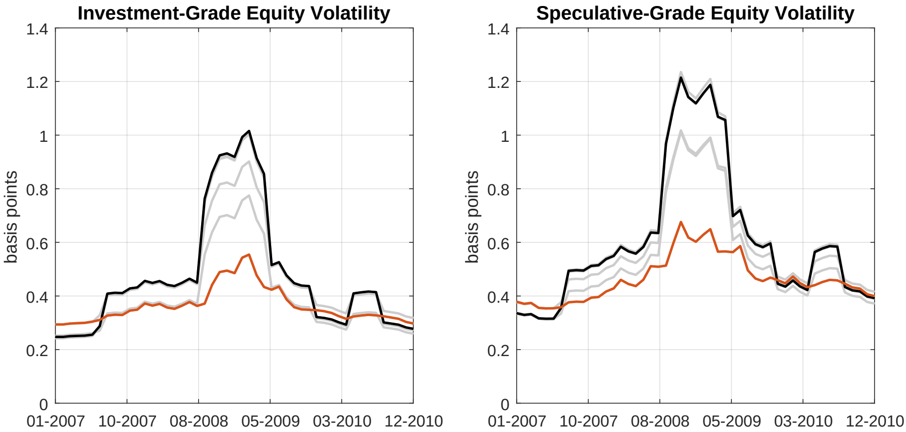




NO SHOCKS TO AGGREGATE VOLATILITY



NO MARKOV STATES



Forecasting

Approximation

An excellent approximation to firms' aggregate asset volatility is given by:

$$\log\left(\sigma_{Y,A}(t)\right) = \frac{1}{N}\sum_{i=1}^{N}\left[\log\left(E_{it}/A_{it}\right) + \log\left(\sigma^{i}_{E,A}(t)\right)\right]$$

where

- A_{it} firm's i total asset value
- E_{it} firm's i equity value
- $\sigma^{i}_{E,A}(t)$ firm's i aggregate equity volatility

Forecasting Performance

	Real GDP Growth 4-Quarters Ahead Forecast Horizon				
	(1)	(2)	(3)	(4)	(5)
FED	0.260**	-0.045	0.085	-0.072	-0.119
TSLO	0.491***	0.405***	0.429***	0.346***	0.349***
GZ		-0.510***			-0.263**
$\sigma_{Y,A}^{INV}$			-0.356***		
$\sigma_{Y,A}^{SPE}$				-0.514***	-0.339***
Adj. R^2	0.17	0.32	0.26	0.34	0.36

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$; standardized coefficients

Conclusion

Conclusion

- Aggregate asset volatility is key to account for joint dynamics of credit spreads and equity volatility.
- Aggregate asset volatility strongly forecasts economic activity and captures the informational predictive content of credit spreads.
- Measuring aggregate asset volatility does not require departures from Modigliani and Miller's (1958) assumptions.

Stochastic Discount Factor Solution (1/2)

back

The Hamilton-Jacobi-Bellman equation in state s (for $s = 1, \dots, S$) is

$$0 = f(C, J(C,s)) + J_C(C,s)C\mu_Y(s) + \frac{1}{2}J_{CC}(C,s)C^2\sigma_Y^2(s)$$

$$+ \sum_{s' \neq s} \zeta^{\mathcal{P}}_{s,s'}(J(C,s') - J(C,s)).$$

Conjecture that the solution for J is

$$J(C,s) = \frac{(h(s)C)^{1-\gamma}}{1-\gamma}$$

and we get

$$0 = \rho\frac{1-\gamma}{1-\delta}h(s)^{\delta-\gamma} + \left[(1-\gamma)\mu_Y - \frac{1}{2}\gamma(1-\gamma)\sigma_Y^2(s) - \rho\frac{1-\gamma}{1-\delta}\right]h(s)^{1-\gamma}$$

$$+ \sum_{s' \neq s} \zeta^{\mathcal{P}}_{s,s'}\left(h(s')^{1-\gamma} - h(s)^{1-\gamma}\right)$$

While no algebraic solution exists for this system of nonlinear equations, it is trivial to solve numerically for $h(s)$ $\forall s \in \{1, \dots, S\}$.

Stochastic Discount Factor Solution (2/2)

back

Duffie and Skiadas (1994) show that the stochastic discount factor is equal to

$$\Lambda_t = \exp\left(\int_0^t f_J(C_u, J_u)du\right) f_C(C_t, J_t)$$

Thus we have

$$\Lambda_t = \exp\left(\int_0^t \frac{\rho(1-\gamma)}{1-\delta}\left(\left(\frac{\delta-\gamma}{1-\gamma}\right)h(s_u)^{\delta-1} - 1\right)du\right)\rho H(s)^{\delta-\gamma}Y^{-\gamma}$$

Applying Ito's formula with jumps, we get

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s)dt - \eta(s)dZ_t^A + \sum_{s'\neq s}\left(e^{\kappa(s,s')} - 1\right)dM_t^{s,s'},$$

where

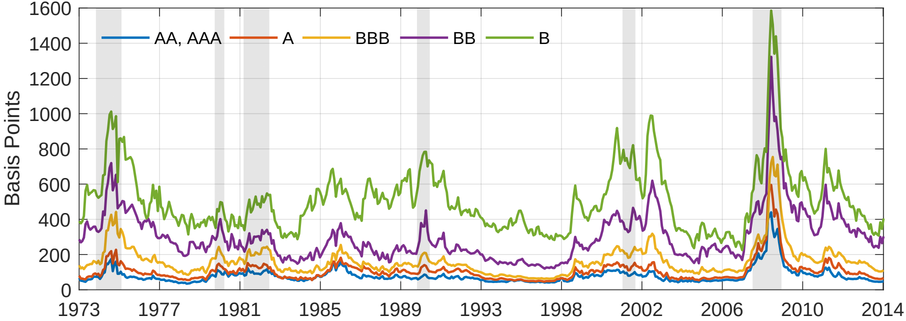
$$r(s) = -\frac{\rho(1-\gamma)}{1-\delta}\left[\frac{\delta-\gamma}{1-\gamma}h(s)^{\delta-1} - 1\right] + \gamma\mu_Y(s) - \frac{1}{2}\gamma(1+\gamma)\sigma_Y^2(s)$$

$$-\sum_{s'\neq s}(e^{\kappa(s,s')} - 1)$$

$$\eta(s) = \gamma\sigma_Y(s) \qquad\qquad \kappa(s,s') = (\delta-\gamma)\log\left(\frac{h(j)}{h(i)}\right)$$

Average Log Corporate Bond Credit Spread by Rating Class

back



First Principal Components Correlation Matrix

back

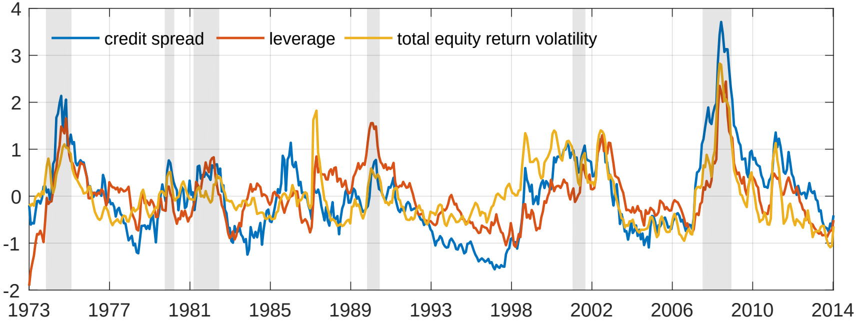
	$\log$(cs)	lev	$\log(\sigma_E)$
$\log$(cs)	1.00	0.71	0.74
lev	0.71	1.00	0.54
$\log(\sigma_E)$	0.74	0.54	1.00

This table displays the correlation between the first principal components of each variable averaged by rating class where

- $\log$(cs) is log credit spread
- lev is market leverage
- $\log(\sigma_E)$ is log equity return total volatility

Co-movements in Credit Spreads, Leverage, and Equity Volatility

back



First Principal Components of Credit Spreads, Leverage, and Equity Return Volatility

The first principal component of credit spreads, leverage, and equity return volatility by rating classes explains 78% of the total variation.

back

Let X_t be a discrete hidden random variable with N possible values. We assume that $P(X_t|X_{t-1})$ is independent of time t, which leads to the definition of the time independent stochastic transition matrix $\mathbf{A} = \{a_{ij}\}$ where $a_{ij} = P(X_t = j|X_{t-1} = i)$. The initial state distribution is given by $\boldsymbol{\mu}_0$. The observation variables Y_t can take one of K possible values. The probability of a certain observation at time t for state j is given by

$$\ell_j(y_t) = P(Y_t = y_t|X_t = j).$$

We will represent the observation density as follows: for every $y \in F$, we define the diagonal matrix $\mathbf{L}(y)$ with nonzero elements $\{\mathbf{L}(y)\}_{ii} = \ell_i(y)$. An observation sequence is given by $Y = (Y_0 = y_0, Y_2 = y_2, ..., Y_T = y_T)$. Thus we can describe a hidden Markov chain by $\boldsymbol{\theta} = (\mathbf{A}, \mathbf{L}, \boldsymbol{\mu}_0)$. The Baum-Welch algorithm finds a local maximum for $\boldsymbol{\theta}^* = \arg\max_\theta P(Y|\theta)$.

Baum-Welch Algorithm 2/3

back

Initiation Set $\boldsymbol{\theta} = (\mathbf{A}, \mathbf{L}, \boldsymbol{\mu}_0)$ with random initial conditions.

Forward procedure Let $\pi_{i,t} = P(Y_0 = y_0, ..., Y_t = y_t, X_t = i|\boldsymbol{\theta})$ be the probability of seeing the $y_0, y_1, ..., y_t$ and being in state i at time t. First, we get

$$c_0 = \mathbf{1}'\mathbf{L}(y_0)\boldsymbol{\mu}_0,$$

$$\boldsymbol{\pi}_0 = \mathbf{L}(y_0)\boldsymbol{\mu}_0/c_0.$$

Then for $k = 1, \ldots, N$

$$\widetilde{\boldsymbol{\pi}}_k = \mathbf{L}(y_k)\mathbf{A}'\boldsymbol{\pi}_{k-1},$$

$$c_k = \mathbf{1}'\widetilde{\boldsymbol{\pi}}_k$$

$$\boldsymbol{\pi}_k = \widetilde{\boldsymbol{\pi}}_k/c_k$$

Baum-Welch Algorithm 3/3

back

Backward procedure Let $\beta_{i,t} = P(Y_{t+1} = y_{t+1}, ..., Y_T = y_T | X_t = i, \boldsymbol{\theta})$ that is the probability of the ending partial sequence $y_{t+1}, ..., y_T$ given starting state i at time t. We calculate $\beta_{i,t}$ as

$$\boldsymbol{\beta}_{N|N} = \mathbf{1}/c_N$$

then or $k = 1, \ldots, N$

$$\boldsymbol{\beta}_{N-k|N} = \mathbf{AL}(y_{N-k+1})\boldsymbol{\beta}_{N-k+1|N}/c_{N-k},$$

$$\boldsymbol{\pi}_{N-k,N-k+1|N} = \text{diag}(\boldsymbol{\pi}_{N-k})\mathbf{AL}(y_{N-k+1})\text{diag}(\boldsymbol{\beta}_{N-k+1|N}),$$

$$\boldsymbol{\pi}_{N-k|N} = \boldsymbol{\pi}_{N-k,N-k+1|N}\mathbf{1},$$

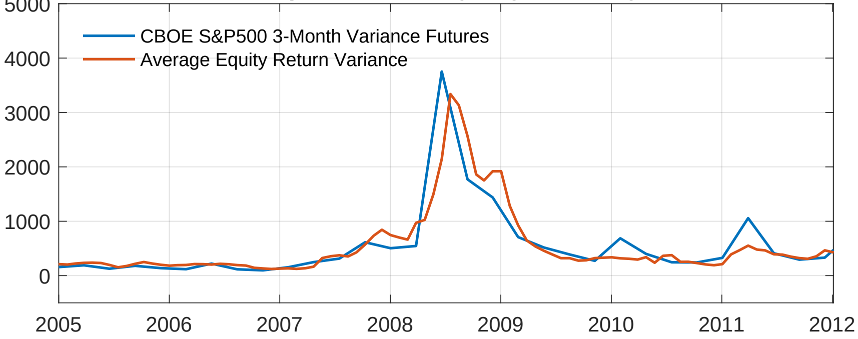
where $\pi_{i,N-k|N} = P(X_t = i|Y, \boldsymbol{\theta})$ is the probability of being in state i at time t given the observed sequence Y and the parameters $\boldsymbol{\theta}$. We can now update $\boldsymbol{\theta}$ as

$$\boldsymbol{P}_{ij} = \frac{\sum_{k=1}^{n} (\boldsymbol{\pi}_{k-1,k|N})_{ij}}{\sum_{k=1}^{N} (\boldsymbol{\pi}_{k-1|N})_i},$$

$$\boldsymbol{\mu}_0 = \boldsymbol{\pi}_{0|N},$$

and the parameters of $\mathbf{L}(\cdot)$ as their empirical counterparts.

Unexpected Volatitlity Regime Changes



Firms' Asset Value

back

Value of firm's assets given by

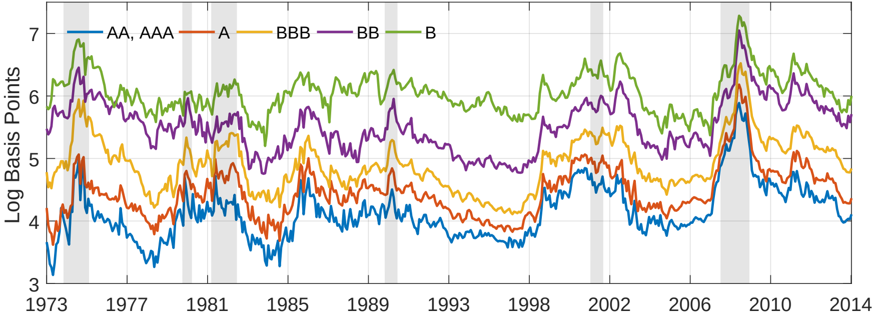
$$V_t = y_t v(s_t),$$

where $v(\cdot)$ is the state dependent price-earning ratio. Thus,

$$\frac{dV_t}{V_t} = \mu_Y dt + \sigma_{Y,A}(s_t) dZ_t^A + \sigma_{Y,I}(s_t) dZ_t^I + \sum_{s_t \neq s_{t-}} \left(v(s_{t-})/v(s_t) - 1 \right) dN_t^{(s_{t-}, s_t)},$$

where $v(s_{t-})/v(s_t)$ represents jump in asset value from state s_{t-} to state s_t

Strong Aggregate Common Factor



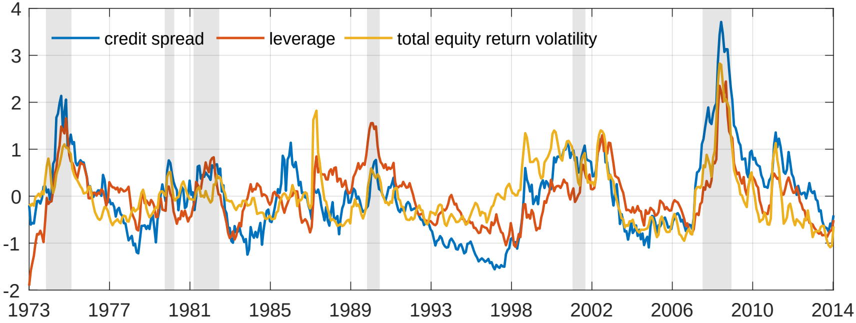
Average Log Corporate Bond Credit Spreads by Rating Class

The first principal component explains 90% of the overall variation. Equity return volatility and leverage are also driven by a strong common factor (93% and 72%).

bps

Co-movements in Credit Spreads, Leverage, and Equity Volatility

back



First Principal Components of Credit Spreads, Leverage, and Equity Return Volatility

The first principal component of credit spreads, leverage, and equity return volatility by rating classes explains 78% of the total variation.

System of Second Order Ordinary Differential Equations

back

$$r_s D_s^H(y) = \mu_s \frac{\partial D_s^H(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 D_s^H(y)}{\partial y^2} + c + m(p - D_s^H(y))$$

$$+ \sum_{s' \neq s} \zeta_{\mathcal{Q}}^{ss'} (D_{s'}^H(y) - D_s^H(y)) + \xi^H (D_s^L(y) - D_s^H(y))$$

$$r_s D_s^L(y) = \mu_s \frac{\partial D_s^L(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 D_s^L(y)}{\partial y^2} + c + m(p - D_s^L(y))$$

$$+ \sum_{s' \neq s} \zeta_{\mathcal{Q}}^{ss'} (D_{s'}^L(y) - D_s^L(y)) + \lambda\beta(D_s^H(y) - D_s^L(y))$$

$$- \chi(B_s + N - P_s(y))$$

$$r_s E_s(y) = \mu_s \frac{\partial E_s(y)}{\partial y} + 0.5\sigma_s^2 \frac{\partial^2 E_s(y)}{\partial y^2} + \exp(y) - (1-\tau)c + m(D_s^H(y) - p)$$

$$+ \sum_{s' \neq s} \zeta_{\mathcal{Q}}^{ss} (E_i(y) - E_s(y))$$

Firms' Asset Value Estimation

back

Match firm's leverage with model-implied log asset return y according to

$$lev = \frac{p}{p + E_s(y)},$$

where p (principal) is the book value of outstanding debt. Thus, model-implied values of equity and debt are matched to their empirical counterparts.

I use 300,887 monthly observations of equity prices from CRSP and book value of debt from Compustat between 1973 and 2014. The model implied leverage distribution matches exactly the empirical leverage distribution in every month.

Firms' Asset Volatility Estimation

back

From Ito's lemma, I can estimate firms' asset volatility according to:

$$\sigma^i_{E,A}(t|s)E^j_s\left(y^i_t\right) = E_{j,s'}\left(y^i_t\right)\sigma_{Y,A}(s),$$

$$\sigma^i_{E,I}(t|s)E^j_s\left(y^i_t\right) = E^{j,s'}\left(y^i_t\right)\sigma_{Y,I}(s).$$

where

- y^i_t is the model-implied level of cash flows of firm i at time t
- $E^j_s(y)$ is the equity value of firms of type j
- $\sigma^i_{E,A}(t|s)$ is aggregate asset volatility of firm i at time t in state s
- $\sigma^i_{E,I}(t|s)$ is idiosyncratic asset volatility of firm i at time t in state s

Bankruptcy Costs Estimation

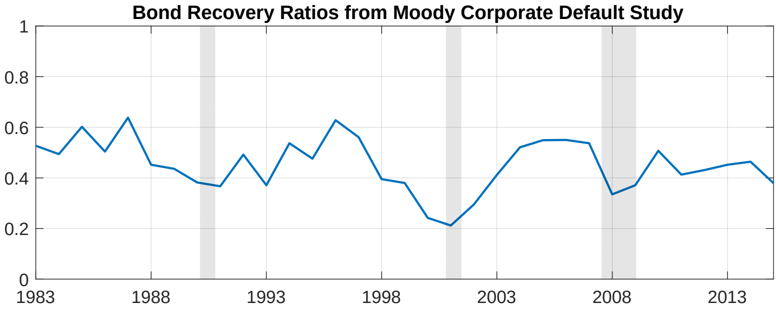
back

The bond recovery ratio in state s is given by

$$\frac{(1-\alpha(s))Vb(s)}{p}$$

where

- p is the principal of the debt
- $Vb(s)$ is the value of firm's assets at bankruptcy
- α is the fraction lost to bankruptcy costs



Aggregate and Idiosyncratic Equity Volatility

back

Idiosyncratic returns are constructed by estimating a factor model using all observations for that firm following:

$$r_t^i - r_t^f = \gamma_0^i + \mathbf{F}_t \boldsymbol{\gamma}^i + \varepsilon_t^i,$$

where

- r_t^i is the equity return from day $t-1$ to t, including dividends of firm i
- r_t^f is the 1-month treasury bill rate
- $\mathbf{F}_t$ is the Fama and French (1992) and Carhart (1997) 4-factor model

Aggregate and idiosyncratic equity volatility is then given by:

$$\sigma_{E,A}^i(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} \left(\mathbf{F}_k \widehat{\boldsymbol{\gamma}}^i\right)^2}$$

$$\sigma_{E,I}^i(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} \left(\widehat{\varepsilon}_k^i\right)^2},$$

where L_t is the last day in month t.

Market Value of Equity, Book Value of Debt, and Leverage

back

Market leverage ratio of firm i at time t is defined as:

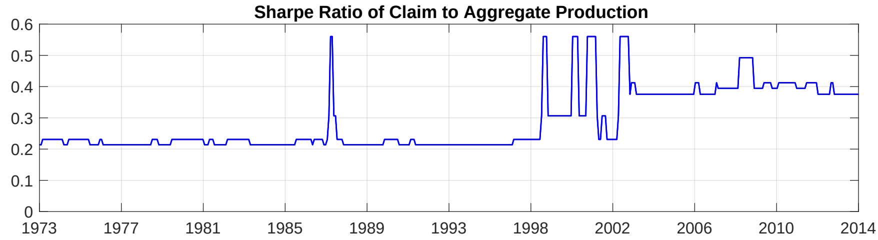
$$\mathsf{lev}_{it} = \frac{\mathsf{DLTT}_{it} + \mathsf{DLC}_{it}}{\mathsf{DLTT}_{it} + \mathsf{DLC}_{it} + \mathsf{CSHO}_{it} \times \mathsf{PRCC}_{it}},$$

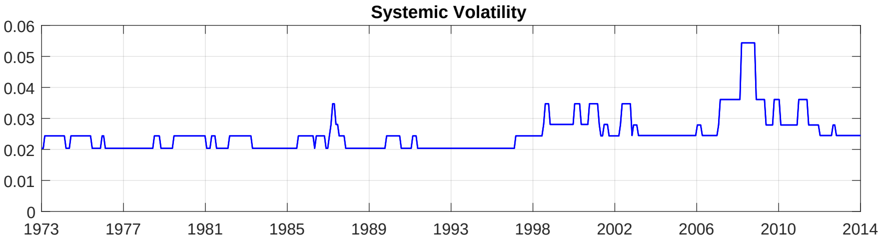
where

- DLTT is Compustat long-term debt
- DLC is Compustat debt in current liabilities
- CSHO is CRSP number of shares outstanding
- PRCC is CRSP stock price

Market Price of Aggregate Shocks and Systemic Volatility

back





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where $v(s_{t-})/v(s_t)$ represents jump in asset value from state s_{t-} to state s_t.

From Firm-Level Observations $y^i(t)$ to Model Variables $x^j(t|s)$

back

I solve the model F for two types j of firms: investment- and speculative-grade.

$$F(y^i(t); s, x^j(s)) \to x^i(t|s) \to x^j(t|s) \to x^j(s)$$

where

$$x^j(t|s) = \frac{1}{N^j(t)} \sum_{i \in I^j(t)} x^i(t|s)$$

$$x^j(s) = \frac{\sum_{t=1}^{T} x^j(t|s) \mathbf{1}\{s_t = s\}}{\sum_{t=1}^{T} \mathbf{1}\{s_t = s\}}$$