Can Stablecoins be Stable?

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Abstract

This paper proposes a framework to analyze the stability of stablecoins – cryptocurrencies designed to peg their price to a currency. We study the problem of a monopolist platform earning seignorage revenues from issuing stablecoins and characterize equilibrium stablecoin issuance-redemption and pegging dynamics, allowing for various degrees of commitment over the system’s key policy decisions. Because of two-way feedback between the value of the stablecoin and its ability to peg the currency, uncollateralized (pure algorithmic) platforms always admit zero price equilibrium. However, with full commitment, an equilibrium in which the platform maintains the peg also exists. This equilibrium is stable locally but vulnerable to large demand shocks. Without a commitment technology on supply adjustments, a stable solution may still exist if the platform commits to paying an interest rate on stablecoins contingent on its implicit leverage. Collateral and decentralizing stablecoin issuance help stabilize the peg.

Keywords: Stablecoins, Cryptocurrencies, Target Leverage, Dynamic Games, Coase Conjecture

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1 Introduction

A stablecoin is a cryptocurrency designed to maintain a stable value vis-à-vis an official currency. It aims to avoid a fundamental drawback of conventional cryptocurrency: being too volatile to be used as a means of payment or store of value. Stablecoins therefore allegedly combine the benefits of the blockchain technology with the stability of well-established currencies and have gained in popularity in the last couple of years, with their combined market capitalization growing from $3 billion in 2019 to $181 billion in April 2022. Confronted with the rapid growth of stablecoin platforms and multiple crashes (e.g., Terra in May 2022), legislators have become increasingly concerned about the financial stability risks posed by stablecoins and have introduced new regulatory initiatives to balance the perceived risks and benefits associated with this new technology.

Stablecoin protocols rely on a wide variety of pegging mechanisms to fulfill their promise of price stability: algorithmic supply adjustments (e.g., Terra), over-collateralization with dynamic liquidation (e.g., Frax), and decentralization of the issuance process (e.g., DAI). To this date, however, the academic literature provides little guidance about the stability of these tools and their optimal design. This paper aims to fill this gap by developing a general model of stablecoins to analyze the performance of various pegging mechanisms and assess their riskiness.

We propose a framework to study the dynamic problem of a stablecoin platform that caters to a time-varying demand from investors. Investors, who value price-stability, enjoy liquidity benefits from owning stablecoins when its price is pegged with respect to some unit of account. The platform acts as a monopolistic issuer and profits from these liquidity benefits while aiming at maintaining the peg. The existence of seigniorage revenues and the focus on price stability make a stablecoin platform akin to a central bank. Like a central bank who may overprint money, a stablecoin platform has a tendency to overissue stablecoins, which ultimately undermines the peg. A central bank’s ability to perform its tasks thus relies to a large extent on its credibility. The main technological proposition of stablecoins in this regard is the possibility to fully commit to specific key policies such as issuance and redemption, interest rates and fees, and collateral liquidation rules via smart

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2For instance, the US Congress is working on a STABLE (Stablecoin Tethering and Bank Licensing Enforcement) Act while in the UK, the Treasury has launched the “UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence”.
Our objective is to characterize the stablecoin price, the value of the platform’s equity shares—referred to in the crypto-space as governance tokens—and characterize conditions under which the peg holds and doesn’t. An equilibrium in our model has two components. The monopolistic platform chooses a dynamic issuance-repurchase policy, an interest policy paid in stablecoins and a collateralization policy. Investors price the stablecoin competitively given the liquidity benefits they derive from owning stablecoins and the interest paid by the platform. In our model the unique state variable is the demand ratio between current stablecoin demand and supply by the platform; the inverse of which represents the platform’s implicit leverage.

We first study stablecoin protocols that can fully commit to issuance-redemption and interest rate rules. In other words, all platform policies can be programmed ex-ante through credible smart contracts. This analysis provides an upper bound for the value of algorithmic stablecoin protocols that rely on programmable adjustments of the quantity of stablecoin such as Terra, NuBits, and Basis. We show that even under full commitment, there exists an equilibrium in which stablecoins and governance tokens are worth zero. This equilibrium always arises because both stablecoin dividends, that is, liquidity benefits and interest payments depend themselves on the value of stablecoins. As is known in other contexts, the self-referential value of money implies zero value fixed point.

We also show that a second equilibrium exists in which the peg is locally stable. In this equilibrium, the system generates seigniorage revenues and governance tokens have positive value. The platform maintains a constant demand ratio and sets interest payments to peg the price. To maintain the peg, the system reacts to a positive demand shock by creating new stablecoins and distributing them to governance token holders as seigniorage dividends. Conversely, it reacts to a negative demand shock by buying back stablecoins and, thereby, reducing supply. In a pure algorithmic setting, the platform finances these repurchase operations by issuing additional governance tokens and diluting equity.

We show, however, that even in this favorable equilibrium the platform cannot implement a strict peg over the entire state space. Like any financial institution, the platform is subject to limited liability as it cannot force stablecoin owners to finance any repurchase beyond the dilution of existing governance tokens. After a large negative demand shock, the value of future seigniorage revenues may be so low that the platform cannot finance a buyback and is unable to cut back supply enough to maintain the peg even with full dilution. The
peg is then broken as a too high stablecoin supply implies that market clears at a price below par. Although the peg is lost and governance tokens are worth zero, the stablecoin price may still be positive and fluctuates exogenously with demand as investors “hope for resurrection”. At some point, stablecoin demand may recover enough so that governance token holders can recapitalize the platform to repurchase the quantity of stablecoin that is necessary to re-establish the peg.

We then investigate the stability properties of a stablecoin scheme under a weaker form of commitment. In practice, some stablecoin schemes prefer to retain some ability to make discretionary changes in key parts of its algorithm to preserve adaptability to new market developments and technical issues. More precisely, we relax our initial assumption that all policies can fully be programmed via smart contracts and assume that the platform can commit to an interest rate rule while retaining flexibility over its issuance-redemption policy. For a constant interest rate rule, the leverage ratchet effect of Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2021) applies: it is never optimal for the system to reduce its leverage and the peg cannot be maintained. We find, however, that an equilibrium with local stability still exists if the interest rate payment decreases with the demand ratio. Such a rule penalizes over-issuance and forces the platform to implement repurchase. We stress that the strength of this punishment is purely endogenous as the interest is paid in stablecoins and the platform faces no direct issuance cost.

Next, we study how escrowing an external collateral asset on which smart contracts can be written—such as another crypto-currency—affects the system’s ability to maintain the peg. This design is common in practice, with many stablecoins such as DAI or Frax partly relying on external crypto-currency holdings to improve their stability. When the collateralization rate falls below a certain threshold, a smart contract triggers the liquidation of the platform. Imposing a minimum collateralization rate is a double-edged sword: On the one hand, it improves the stability of the stablecoin price as guarantees a residual value for stablecoin owners when the system liquidates its assets. On the other hand, locking crypto-assets in the platform is costly and future seigniorage revenues are lost when platform shuts down.

Last, we examine the stability of a stablecoin scheme that decentralizes the issuance and redemption of its stablecoin. This feature is present in DAI: a stablecoin that anyone with access to the Ethereum platform can mint freely. We find that this decentralization can act as an effective substitute for a commitment technology on stablecoin redemption and issuance. In this setting, investors acting as arbitrageurs prevents the price from moving
away from the peg by creating more (redeeming) stablecoins in reaction to a positive (negative) demand shock. Hence, decentralization allows the system to locally maintain the peg as in the full-commitment setting because the decisions affecting that system’s leverage have been externalized to agents that—unlike governance token holders—are not hurt by a reduction of leverage.

**Literature review**  Our paper contributes to an interdisciplinary literature on stablecoins. From the computer sciences literature, Klages-Mundt and Minca (2019, 2020) develop models featuring endogenous stablecoin price and an exogenous collateral and find deleveraging spirals and liquidation in a system with imperfectly elastic stablecoin demand. Gudgeon, Perez, Harz, Livshits, and Gervais (2020) simulate a stress-test scenario for a DeFi protocol and find that excessive outstanding debt and drying up of liquidity can lead the lending protocol to become undercollateralized. Our paper also relates to a descriptive literature on stablecoins (Arner, Auer, and Frost, 2020; Berentsen and Schär, 2019; Bullmann, Klemm, and Pinna, 2019; ECB, 2019; Eichengreen, 2019; G30, 2020). In closely-related contemporaneous work, Li and Mayer (2022) study the peg dynamics of stablecoin platforms under the assumption that stablecoins generate network externalities and the systems’ reserves are subject to stochastic shocks. Our paper differs by considering various commitment technologies and demand shocks that affect the system’s seigniorage and governance token value, allowing the study of fully algorithmic stablecoins such as Terra.

In studying the stabilization mechanisms across stablecoin types and the failure of governance incentives to recapitalize undercollateralized systems, our paper is connected to the corporate finance literature examining firm shareholders’ attitudes towards leverage. In Black and Scholes (1973) and Myers (1977) firm shareholders do not have incentives to voluntarily reduce leverage as this always implies a transfer of wealth to existing creditors. Admati, DeMarzo, Hellwig, and Pfleiderer (2018) generalize these findings to multiple asset classes of debt and with agency frictions and document a “leverage ratchet effect”, whereby shareholders never have any incentives to delever. DeMarzo and He (2021) also show delevering resistance effects in a dynamic setting, although in their model leverage mean-reverts to a target because of asset growth and debt maturity. Our paper contributes to this literature by considering cases in which the firm (stablecoin platform in our setting) can partially commit or decentralize the buybacks and coupon payment decisions through
a smart contract algorithm. These features can be seen as an extreme form of debt-convenants as studied in Smith and Warner (1979), Bolton and Scharfstein (1990), Aghion and Bolton (1992), and Jason Donaldson and Gromb (2020) in a continuous state-space.

More broadly, our paper contributes to the literature applying finance theory to model digital platforms and token valuations. While not mainly focusing on stablecoins, Cong, Li, and Wang (2020a) develop a continuous-time model of token-based platform economy with network effects and endogenous token price and also document conflicts of interests between platform owners and users, resulting in an under-investment outcome. Cong, Li, and Wang (2020b) build a dynamic asset pricing model with network effects and intertemporal linkages in endogenous token price and user adoption, and analyze the Markov equilibrium with platform productivity as the state variable.

2 General Environment

In this section, we describe our model of stablecoins. The central premise of our analysis is that investors enjoy utility benefits from holding stablecoins issued by the platform, as they would do for money or bank deposits. Our model also embeds investors’ preferences for stable means of payment. As a result, the stablecoin platform can generate seignoriage revenues if (but only if) it can maintain a peg between the stablecoin price and some target unit of account. We describe the formal building blocks of the model below.

2.1 Stablecoin Demand

Let \((\Omega, \mathcal{F}, \mathbb{P})\) be a probability space that satisfies the usual conditions. All agents are risk neutral with an exogenous discount rate of \(r > 0\). Time is continuous with \(t \in [0, \infty)\).

We consider a platform that issues stablecoins. Stablecoins are a liability of the platform that trade at (endogenous) price \(p_t\) expressed in the unit of account. The outstanding stock of these stablecoins at date \(t\) is \(C_t\). Stablecoins have value because investors enjoy direct utility from holding them: at date \(t\), holding stablecoins generates utility flow \(p_t u(A_t, p_tC_t)\) per unit with \(A_t\) an exogenous driver of stablecoin utility value. To fix ideas, one could interpret variable \(A_t\) as the value of some cryptoassets, which proxies for investors’ demand

\(^3\)Alternatively, we can interpret the model as written under a fixed risk-neutral measure that is independent of the stablecoin platform policies.
for alternative means of payment. The utility derived from holding stablecoins can be thought as a liquidity benefit users derive because stablecoins are a form of money. We denote the marginal utility benefit from holding stablecoins, or its convenience yield, as \( \ell(A_t, p_tC_t) \equiv u_c(A_t, p_tC_t) \) and make some restrictions on its properties.

**Assumption 1.** The convenience yield of stablecoins \( \ell(A, pC) \) is (i) positive and continuously differentiable in both arguments; (ii) strictly increasing in \( A \); (iii) bounded with \( 0 \leq \ell(A, pC) \leq r \); (iv) homogeneous of degree 0 and (v) equal to 0 if the stablecoin price \( p \) is not pegged to 1. Finally, (vi) the product of the convenience yield with the total value of stablecoins \( \ell(A, pC)pC \) is single-peaked with \( \lim_{x \to \infty} \ell(A, x)x = 0 \).

Property (i) rules out negative marginal utility from stablecoin holdings and ensures differentiability. Property (ii) states formally that the value of stablecoins increases with demand driver \( A_t \). Property (iii) and (iv) are technical assumptions ensuring respectively that the stablecoin price is well-defined and that the problem ultimately economizes on one state-variable. Property (v) states that stablecoin owners enjoy a liquidity benefit only if it is pegged to the unit of account. This assumption is meant to capture in a simple way a trust element whereby investors value the stablecoin as means of transactions to the extent that issuers can maintain a pre-announced peg. The peg at 1 is chosen for convenience and because it corresponds to market practice but our results do not depend upon it; only the real value of stablecoin holdings matters. Finally, Property (vi) will ensure that the optimal amount of stablecoins is interior. An example of a class of functions that satisfy Assumption 1 is \( \ell(A, C) = r \exp\left(-\alpha C/A\right) \) for \( \alpha > 0 \).

The cryptoasset value \( A_t \) that drives stablecoins’ demand has the following law of motion

\[
dA_t = \mu A_t dt + \sigma A_t dZ_t + A_t (S_t - 1) dN_t,
\]

where \( dZ_t \) is the increment of a standard Brownian motion and \( dN_t \) is a standard Poisson process with constant intensity \( \lambda > 0 \) adapted to \( \mathcal{F} \). The size of a downward jump, \(-\ln(S)\) is exponentially distributed with parameter \( \xi > 0 \) and the expected jump size is \( \mathbb{E}[S - 1] = -1/(\xi + 1) \). The Poisson process generates large negative shocks to stablecoin demand that can be thought of as speculative attacks. Overall, the expected growth rate

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4Our reduced-form specification can be microfounded assuming that stablecoins are essential to carry some transactions.

5We can relax this “extreme-peg” assumption by assuming that the liquidity benefit is still positive for small deviations from the peg, but decreasing in price volatility.
of stablecoin demand is given by \( \mu - \lambda / (\xi + 1) \), which we assume is lower than the discount rate \( r \). We use notation \( A_t^- \) to denote the cryptoasset’s value before a jump.\(^6\)

Finally, there exists a safe asset the platform can hold as collateral to back the issuance of stablecoins. This collateral trades in a competitive market at price \( p^k_t \) with

\[
d p^k_t = \mu^k p^k_t dt.
\]

This specification implies that collateral delivers a (safe) return \( \mu^k \) with \( \mu^k \leq r \). The difference between the discount rate and the return on collateral, \( r - \mu^k \) can be interpreted as a convenience yield enjoyed by collateral asset owners. As we will see, this feature generates a cost from holding collateral for the stablecoin platform.\(^7\) Examples of this asset include cash, government securities or bank deposits denominated in the target currency.

### 2.2 Platform Operation

We will analyze both a centralized and a decentralized platform. For clarity, we postpone the description of a decentralized platform to Section 5.

**Definition 1 (Centralized Platform Policies).** A sequence of policies for the platform is an issuance-repurchase policy \( \{dG_t\}_{t \geq 0} \), a coupon policy \( \{\delta_t\}_{t \geq 0} \) paid in stablecoins, with \( \delta_t > 0 \), its collateral purchase policy \( \{dM_t\}_{t \geq 0} \), and a stochastic default time \( \tau_D \).

The main platform policy is the issuance(-repurchase) policy \( \{dG_t\}_{t \geq 0} \). A positive (negative) value of \( dG_t \) corresponds to an issuance (repurchase) of stablecoins at price \( p_t \) at date \( t \). Second, the platform can pay a coupon as interest to stablecoin owners. As in practice, this interest is paid in stablecoins, not in the unit of account. A platform may hold collateral with \( dM_t \) the change in collateral value held by the platform at date \( t \).

There exists a useful analogy between the stablecoin platform and a central bank. When it issues stablecoins (\( dG_t > 0 \)), the platform receives a payment \( p_t dG_t \) from investors in the unit of account. Similarly, when it credits the account of a depository institution with

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\(^6\) \( A_t^- \) denotes the left limit \( A_t^- = \lim_{h \to 0^-} A_{t-h} \). Also note that \( A_t^- dt = A_t dt \) as the set \( \{T_k\}_{k \geq 1} \) of jump times has zero measure of length.

\(^7\) Our assumption of a safe collateral asset comes with some loss of generality because some stablecoin protocols are implicitly or explicitly backed by cryptoassets. In this case, the collateral price would likely be correlated with demand process \( A_t \). It is intuitive, however, that such correlation would reduce the usefulness of collateral as a hedge against demand fluctuations. From a technical standpoint, introducing correlation significantly complicates the analysis.
reserves, the central bank receives an asset in exchange. The stablecoin’s coupon policy whereby every stablecoin investor is credited with $\delta_t \geq 0$ units of free stablecoins per unit owned is akin to an interest payment on reserves. Finally, collateral holdings of the platform correspond to central bank’s holdings of foreign reserves.

We focus on a constant collateralization rule for the platform.

**Assumption 2.** The platform maintains a fixed collateralization ratio $\varphi$, that is,

$$K_t = \varphi C_t. \tag{3}$$

Assumption 2 states the platform must maintain a constant ratio $\varphi \in [0,1]$ between the value of its collateral and the par value of stablecoins. The case $\varphi = 0$ ($\varphi = 1$) corresponds to a so-called algorithmic stablecoin (narrow bank). Assumption 2 helps simplify our analysis in that it eliminates collateral as a state variable and is in line with actual stablecoin designs such as DAI. Although this collateralization rate $\varphi$ is fixed, the platform still has to choose a value for it at the initiation date 0 and does so to maximize its equity value.

**Law of Motions** The platform’s policies imply the following law of motions for the stock of stablecoin outstanding, $C_t$ and the value of its collateral, denoted $K_t$

$$dC_t = \delta_t C_t dt + dG_t, \tag{4}$$

$$dK_t = \mu^k K_t dt + dM_t \tag{5}$$

Equation (4) is the law of motion for stablecoins. The first term on the right-hand side captures the contribution of the coupon policy $\delta_t$ to stablecoin issuance. It must be treated separately from the active issuance component $dG_t$ because the coupon policy increases the stablecoin stock without compensation for the platform. Equation (5) is the law of motion for the collateral value. The first term on the right-hand side corresponds to passive changes in collateral value. The second term corresponds instead to active changes in value due to purchases or sales. The collateral policy $dM_t$ is fully determined by the issuance policy $dG_t$ and the coupon policy $\delta_t$ at date $t$ because $dK_t = \varphi dC_t$ under Assumption 2.8

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8Law of motion (5) can alternatively be written $dK_t = S_t^K dP_t^K + p_t^K dS_t^K$ with $S_t^K$ the quantity of collateral held by the platform. The term $dM_t$ in (5) corresponds to $p_t^K dS_t^K$. 

Jump Notation  There are both Brownian shocks and jumps to the value of cryptoassets in our model. We therefore also allow the platform’s policies to feature jumps. A jump represents a discrete, instantaneous change in a variable. We denote the value of a variable $X$ just before and after the jump by $X_{t-}$ and $X_t$, respectively.

2.3 Stablecoin Pricing and Platform’s Objective

Stablecoin Pricing  Investors price the stablecoin competitively taking as given the platform policies. They enjoy two income streams from holding stablecoins: the direct utility benefits when the price is pegged and coupon payments when the stablecoin platform pays interest, with respective value $\ell_t p_t$ and $\delta_t p_t$ per unit. Should the platform default, an instantaneously liquidation procedure applies in which stablecoin owners are treated as pari-passu creditors. They receive any platform’s collateral up to the parity value of stablecoins. At date $t$, the competitive stablecoin price given the platform’s continuation policies is thus

$$p_t = \mathbb{E}_t \left[ \int_t^{\tau_D} e^{-r(s-t)} \left( \ell_s + \delta_s \right) p_s ds + e^{-r(\tau_D-t)} \min \left\{ \frac{K_{\tau_D}}{C_{\tau_D}}, 1 \right\} \right].$$

(6)

Investors compute expected future cash flows by forming rational expectations over the platform’s policies from date $t$ onward.

Platform’s Objective  The platform starts with no stablecoin outstanding at date 0, that is, $C_0^- = 0$ and maximizes its value $E_0$, which is the sum of the issuance benefits net of the collateral purchases.

$$E_0 = \max_{\tau_D, \{\delta_t, dG_t, dM_t\}_{t \geq 0}} \mathbb{E}_0 \left[ \int_0^{\tau_D} e^{-rt} \left( p_t dG_t - dM_t \right) + e^{-r\tau_D} \max \left\{ 0, K_{\tau_D} - C_{\tau_D} \right\} \right]$$

(7)

where the price $p_t$ is given by equation (6). When the platform defaults, its shareholders enjoys the residual value of collateral, if any, after stablecoin owners have been paid at parity. As a monopolistic issuer, the platform has price impact. Hence, it pays the post-issuance (post-repurchase) price when it issues (repurchases) stablecoins. As in Admati,

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9Later, as the policies are Markov and thus only functions of the current values of the pay-off relevant states variables $(A_t, C_t)$, we substitute $\mathbb{E}_t[\cdots]$ by $\mathbb{E}[\cdots|A_t, C_t]$. 

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DeMarzo, Hellwig, and Pfleiderer (2018); DeMarzo and He (2021), this feature is important because it weakens the platform’s incentives to buy back blocks of debt.

A platform’s ability to implement at date $t > 0$ a policy chosen ex-ante at date 0 depends on its commitment power. A key technological proposition of stablecoins is that rules and procedures can be programmed in advance through algorithms, so-called *smart contracts*. In many cases, however, platforms retain some flexibility over parts of the algorithm for technical maintenance, future adaptability, or to decrease vulnerability to hacking. To reflect these concerns and to capture heterogeneity in smart contracts’ credibility and transparency, we analyze optimal policies under varying degrees of commitment.

### 2.4 Discussion of the Environment

**Platform Fees** For simplicity, we will assume that the platform coupon policy is never negative $\delta_t < 0$, that is, the platform never levies fees on stablecoin users. Doing so simplifies the analysis as only buy-backs can be used to reduce stablecoin supply. This assumption also corresponds to the practice of the main stablecoin platforms. I.e., Terra famously subsidized platform usage by paying an annual interest rate of 20%, DAI’s interest rate is typically fluctuating between 1% and 7%, and Tether does not pay any interest nor levies fees.

**Peg vs. Redemption Rights** In our model, the platform does not provide redemption rights to investors. Instead, investors must trade in a competitive market to exchange their stablecoins for the unit of account and the platform administer the peg through supply adjustments. To the extent that the platform maintains the peg, however, investors are effectively guaranteed a fixed exchange rate between stablecoins and the unit of account.

**Platform Competition** We focus on the analysis of an economy featuring a single stablecoin platform. In practice, several stablecoins competes to cater to investors’ demand for alternative means of payment. Although, we refrain from modeling competition and entry of platforms for parsimony, one can interpret the platform’s convenience yield as investors’ residual demand for a platform’s stablecoins after accounting for supply from other platforms. All that is needed is that the platform enjoys some market power, which would arise naturally with payment network effects as in Cong, Li, and Wang (2020a).
3 Credible Smart Contracts

In this section, we analyze the problem of a stablecoin platform that can commit to all future policies. A platform with full commitment can be viewed as a stablecoin protocol with credible smart contracts governing all policies including issuance and repurchase of stablecoins. The analysis under full commitment provides minimal necessary conditions for a stablecoin platform to have positive value and to be able to maintain its peg.

For this analysis, the only constraint on the platform’s policy choices at date 0 is that its equity cannot become negative at some future date $t$, that is, limited liability applies. To clearly highlight the role of this constraint, we first consider a benchmark with unlimited liability in Section 3.1 and then reintroduce limited liability in Section 3.2.

Before proceeding, note that policies such that the platform never defaults are without loss of generality under full commitment. Implementing default cannot expand the set of outcomes when the platform can commit. We can thus set $\tau_D = \infty$ in this section.

3.1 Unlimited Liability Benchmark

We first assume the platform’s equity value may become negative. The platform chooses a stablecoin issuance-redemption policy $\{d_{G_t}\}_{t \geq 0}$, an interest policy $\{\delta_t\}_{t \geq 0}$ and a collateralization rate $\phi$ to maximize the value of the platform at date 0 given by

$$E_0 = \max_{\phi, \{\delta_t, d_{G_t}\}_{t \geq 0}} \mathbb{E} \left[ \int_0^\infty e^{-rt} (p_t d_{G_t} - d_M_t) \middle| A_0, C_0 = 0 \right]$$ (8)

subject to (6), (4) and (3).

The platform’s payoff is the net present value of issuance proceeds net of collateral purchase costs. Equation (4) is the law of motion for stablecoins implied by the issuance policy and the initial condition $C_0 = 0$. Equation (6) is the competitive pricing function for stablecoins at any date $t$, given policies chosen by the platform for dates $\tau \geq t$.

Our first result is that even under full commitment, there exists an equilibrium with zero stablecoin price and zero platform value if the platform does not hold collateral.

**Proposition 1.** For an uncollateralized platform with $\phi = 0$, there always exists a zero-price equilibrium in which $p_t = 0$, $\forall t \geq 0$. 
The zero-price equilibrium arises because there is no anchor between the stablecoin and the unit of account for an uncollateralized platform. In particular, the coupon is paid in stablecoins, not in the unit of account. To see why a zero-price equilibrium exists, suppose the price is indeed 0. Then both components of the stablecoin dividend in the pricing function (6) are equal to 0. Stablecoin owners enjoy no liquidity benefit because the price is not pegged to 1 and the real coupon \( p\delta \) is also worth 0 even if the platform promises a very large nominal coupon payment \( \delta \). Finally, without collateral, the price is not supported by an external asset. Hence, the stablecoin price is equal to zero and the platform has no value because it captures no stablecoin issuance benefits.

Proposition 1 shows that unbacked stablecoins, like any fiat money, is fragile: stablecoins may be worth zero even when issuance and repurchase are fully programmable. Having shown this result, we now consider equilibria with positive stablecoin value if any. Under full commitment and with unlimited liability, there exists an equilibrium in which the stablecoin has value and the platform enjoys seigniorage revenues.

**Proposition 2.** With full commitment and unlimited liability, the equilibrium with positive stablecoin price features a target demand ratio \( A_t/C_t = A_t/C_{ul}(A_t) = a^*_{ul} \) for all \( t \) with

\[
C^*_{ul}(A) = \arg \max_C \{ \ell(A,C)C \}. \tag{9}
\]

The coupon policy at demand ratio \( a^*_{ul} \) is \( \delta^* = r - \ell(a^*_{ul}) \) to peg the stablecoin price to 1 and is not determined otherwise. The platform sets collateralization ratio \( \varphi^* = 0 \).

As we show formally in the Appendix, the platform value is the present value of liquidity benefits enjoyed by investors net of the collateral holding costs,

\[
E_0 = \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \ell(A_t,C_t)C_t 1_{(p_t=1)} + (\mu^k - r)\varphi C_t \right) dt \left| A_0 \right. \right],
\]

with \( \ell(A,C)C \) the instantaneous total seigniorage revenues for the platform when the price is pegged. This equivalence is intuitive because the platform captures all gains from trade. Maximizing the platform value \( E_0 \) with unlimited liability thus becomes a static optimization problem to the extent the platform can maintain the peg. In this case, the optimal collateralization rate is \( \varphi^* = 0 \) because holding collateral is costly. Given current demand \( A \), an interior optimum \( C^*_{ul}(A) \) for stablecoin supply exists under Assumption 1.
Homogeneity of the liquidity benefit, $\ell(A, C)$ further implies that $C^*_{ul}(A)$ is linear in $A$, and we call $a^*_{ul}$ the target demand ratio.

The need to maintain the peg, $p_t = 1$ in order to capture liquidity benefits determines the platform’s coupon policy. In equilibrium, the demand ratio $a_t$ is constant, so we only need to specify $\delta^* \equiv \delta(a^*_{ul})$. It is easy to verify that the peg holds when $\delta^*$ is given as in Proposition 2 because for all $t$, we then have

$$p_t = \frac{\ell(a^*_{ul}) + \delta^*}{r} = 1.$$  \hspace{1cm} (10)

Proposition 2 implies that the platform issues (buys back) stablecoins when demand $A_t$ increases (decreases) in order to implement its target demand ratio. This optimal policy reflects the common practice of algorithmic supply adjustment by many stablecoin platforms. In our benchmark, the platform can always perform these adjustments and, as a result, always maintains the peg. However, as we show next, the mere introduction of limited liability jeopardizes the platform’s ability to always maintain the peg even under full commitment.

### 3.2 Limited Liability

The full commitment policy with unlimited liability requires the platform to conduct large stablecoin repurchases when the underlying cryptoasset value drops in order to restore an optimal demand ratio. For a large drop, however, the repurchase cost might exceed the post-repurchase platform value. In practice, the platform would then be unable to finance the repurchase by issuing new equity even if it fully dilutes current equity.

From this point, we assume that policies must satisfy limited liability. That is, the platform’s equity value must be positive at all times.\textsuperscript{10} In other words, no smart contract may impose actions such that the platform’s continuation value is negative. The value of the platform under limited liability constraint at date $t$, $E_t \geq 0$ can be derived at each

\textsuperscript{10}The limited liability assumption has a similar interpretation as for limited liability companies: a claimant to the company (platform) cannot expect to recover more than the value of assets belonging explicitly to the company (platform), thereby protecting the private wealth of the shareholders. For regular companies, this feature derives from a limited liability contractual arrangement, whereas for stablecoin platforms, it is the consequence of the platform policy and asset accessibility. Only escrowed collateral can effectively be accessed at liquidation.
point in time through the same steps as Proposition 2:

\[
E_t = E \left[ \int_t^\infty e^{-r(s-t)} \left( \ell(A_s, C_s)C_s + (\mu^k - r)\varphi C_s \right) ds \bigg| A_t, C_t = 0 \right] - (p_t - \varphi)C_t^\tau \geq 0. \tag{11}
\]

The first term in (11) is the total platform value from date \(t\) onward as in (10) at date 0. The second term, \((p_t - \varphi)C_t^\tau\), captures the net value of outstanding debt. Note that this term is zero at date 0 as \(C_0 = 0\), so that it does not appear in (10). The equity value of the platform at time \(t\) is thus equal to the value of a new platform starting with zero stablecoins net of the cost of repurchasing all outstanding stablecoins. This net cost is given by \(p_t - \varphi\) because buying back one stablecoin frees up collateral value \(\varphi\). Equation (11) therefore implies a role of collateral in relaxing the limited liability constraint. We formalize this intuition below.\(^{11}\)

First we show that for a large enough negative demand shock, the policy we defined under the unlimited liability benchmark in Proposition 2 end up violating the constraint (11). Heuristically, after a negative demand shock to demand, the platform would have to repurchase a large stock of stablecoins to implement target \(a^\star\). Though, if the shock is large enough, this cost exceeds the present value of future convenience yield. Hence, in that case, the platform’s equity value would then become negative if the platform were to implement the policy in Proposition 2. At that point, the platform is unable to finance the stablecoin buybacks necessary to maintain the peg.

To analyze the platform’s problem under full commitment and limited liability—problem (8) with the additional constraint (11)—we focus the set of policies with the following structure:

**Definition 2.** A Flexible Target Markov Policy (FTMP) is given by coupon policy \(\delta_t = \delta(A_t, C_t) = \delta(a_t)\) with \(a_t = \frac{A_t}{C_t^\tau}\) and issuance policy

\[
dG(A_t, C_t^\tau) = \begin{cases} 
G(A_t, C_t^\tau)dt & \text{if } a_t < \bar{a}, \\
\frac{A_t}{a^\star} - C_t^\tau & \text{if } a_t \geq \bar{a}
\end{cases}
\tag{12}
\]

\(^{11}\)The equity value decomposition in (11) does not imply that the platform must repurchase all stablecoins before issuing new ones. It simply breaks down any policy into two steps which happen simultaneously at the same price: (i) repurchase all outstanding stablecoins \(C_{t-}\), and (ii) issue new stablecoins to the new level, \(C_t\).
where the issuance policy over \([0, \bar{a}]\) is smooth of order \(dt\) following the definition in Demarzo and He (2021).

A Flexible Target Markov Policy (FTMP) has two components. First policies are Markov in the sense that they only depend on the value of current state variables \(A_t\) and \(C_t\) as opposed to the entire history of exogenous shocks. This memoryless property considerably simplifies our analysis in the presence of the constraint (11). Our focus on FTMP is restrictive and conservative in the sense that we are ruling out non-Markov equilibrium mechanism such as ones that would rely on policies punishing deviations from the peg. Second, an FTMP is characterized by a target region \([\bar{a}, \infty]\) in which the platform always implements a constant demand ratio \(a^* \geq \bar{a}\). The value of \(\delta\) for \(a \in [\bar{a}, \infty] \setminus \{a^*\}\) is irrelevant because the platform never stays in these states. In region \([0, \bar{a}]\), however, the platform abandons the target and switches to a smooth issuance policy of order \(dt\) according to which the platform never adjusts quantities in discrete blocks. This feature helps ensure that limited liability can be satisfied after a large negative shock to demand (low \(a_t = \frac{A_t}{C_t}\)). We show below that this assumption is not particularly restrictive as all optimal Markov policies are FTMPs in most of our specifications. For example, the policy from Proposition 2 is an FTMP with \(\bar{a} = 0\).

Given the set of policies considered, we can define functions \(E(A, C)\) and \(p(A, C)\) for the platform’s equity and the stablecoin price, now omitting the time index. Due to the homogeneity of the problem, the ultimate state variable for equity and price is \(a = A_C\) and we also define \(e(a) \equiv E(A, C)/C\) and \(p(a) \equiv p(A, C)\) where \(e(a)\) is the platform’s equity value per stablecoin outstanding.

To solve for these two variables, we first guess and verify that, in a FTMP, the price function satisfies \(p(a) = 1\) for \(a \in [0, \bar{a}]\) and \(p(a) < 1\) for \(a \in [0, \bar{a}]\), that is, investors enjoy liquidity benefits only in the target region. We then characterize the optimal repurchase policy in the region in which the peg is lost.

**Lemma 1.** For \(a \leq \bar{a}\), an optimal FTMP under commitment and limited liability satisfies

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12The general problem is not standard because limited-liability constraints (11) are forward-looking, which means equity value \(E_t\) is not the solution to a standard Hamilton-Jacobi-Bellman (HJB) equation. Techniques developed by Marcet and Marimon (2019) do not apply to our problem; the additional complexity comes from the term \((p_t - \varphi)C_t\) on the right-hand side of (11) as a state variable \(C_t\) multiplies forward-looking variable \(p_t\), which depends on all future policy choices. Our focus on Markovian policies ensure the equity value and the stablecoin price solve HJB equations.
that is, the platform does not pay coupon when the peg is lost and it uses all collateral proceeds to repurchase stablecoins. Under repurchase policy (13), $e(a) = 0$ for all $a \in [0, \bar{a}]$.

The intuition for this result is as follows. As shown by (10), the platform’s value rests on its ability to capture investors’ liquidity benefits. As a result, the platform seeks to minimize the time it will spend in region $[0, \bar{a}]$ where the peg is lost ($p(a) < 1$) and investors enjoy no such benefit. To increase $a_t = \frac{A_t}{C_t}$ when $a_t \in [0, \bar{a}]$, stablecoin issuance should be minimized in this region. This involves paying no coupon to investors and using returns on collateral to buy back stablecoins. To see why the latter condition yields equation (13), observe that each stablecoin is backed by collateral value $\varphi$ that grows at rate $\mu_k$. The buyback payment for a stablecoin is $p - \varphi$ because buying back a stablecoin frees up collateral value $\varphi$. Hence, the maximum rate at which the platform can repurchase stablecoins is given by (13). The fact that the platform’s equity value is equal to 0 when the peg is lost is intuitive. Given the platform’s objective to maximize time spent in the peg region $[\bar{a}, \infty)$, it should reduce any slack in the limited liability constraint in the region in which the peg is lost.

We make use of Lemma 1 to solve for the target demand ratio $a^\star$, the lower bound $\bar{a}$ of the target region and the coupon policy at the target demand ratio, $\delta^\star \equiv \delta(a^\star)$. This second step requires characterizing the equilibrium price $p(a)$ and the equity value $e(a)$ over the state space $[0, \infty]$.

**Lemma 2.** For an optimal FTMP under full commitment, the following statements hold

1. The platform’s equity value is characterized by the following two equations

$$e(a) = \begin{cases} 
0 & \text{if } a \leq \bar{a}, \\
\left[ e(a^\star) + (p(a^\star) - \varphi) \right] \frac{a}{a^\star} - (p(a^\star) - \varphi) & \text{if } a \geq \bar{a},
\end{cases}$$

(14)

$$(r + \lambda - \mu)e(a^\star) = \mu_k \varphi + \mu(p(a^\star) - \varphi) - \delta^\star p(a^\star) + \lambda \mathbb{E}[e(Sa^\star)].$$

(15)
2. The coupon policy maintaining the peg at parity $p(a^*) = 1$ in target region $[\bar{a}, \infty)$ is
\[
\delta^* = r - \ell(a^*) + \lambda (1 - \mathbb{E}[p(a^* S)]) .
\] (16)

3. The stablecoin price is $p(a) = 1$ for $a \in [\bar{a}, \infty)$ while for $a \leq \bar{a}$, it solves
\[
(r + \lambda)p(a) = (\mu - g(a))ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)],
\] (17)
where $g(a)$ is given by (13).

Consider first the platform’s equity value. We already show in Lemma 1 that the equity value is zero when the peg is lost. In the peg region $[\bar{a}, \infty)$, the platform issues or repurchases stablecoins to maintain a constant demand ratio $a^*$. The second line of equation (14) can be easily understood start from the definition of equity value at $a^*$:
\[
E(A, C) = E(A, C^*(A)) + (p(a^*) - \varphi)(C^*(A) - C)
\] (18)
where $p(a^*) = p(A, C^*(A))$ and dividing each side by the stablecoin quantity $C$. Equation (15) is the Hamilton-Jacobi-Bellman (HJB) equation for equity value at the constant demand ratio $a^*$. Equity holders receive two cash flows: interest on collateral and expected issuance proceeds, respectively $\mu k \varphi$ and $\mu (p(a^*) - \varphi)$ per unit of collateral. Importantly, expected issuance proceeds are positive if demand $A_t$ grows on expectation ($\mu > 0$) and if the platform is less than fully collateralized ($\varphi < 1$).

The second part of Lemma 2 characterizes the coupon policy necessary to maintain the peg in region $[\bar{a}, \infty)$. In the absence of Poisson shocks ($\lambda = 0$), equation (16) is the same as in Proposition 2 with unlimited liability. Large negative demand shocks force the platform to abandon the peg, in which case the stablecoin price drops below 1. This effect requires the platform to pay a larger coupon in order to compensate for this expected price devaluation as is apparent in the last term of equation (16).

Finally, the third part of Lemma 2 characterizes the equilibrium price dynamics in the region $[0, \bar{a}]$ where the peg is lost. The optimal repurchase policy derived in Lemma 1 enters HJB equation (17) because it governs the rate at which the demand ratio $a_t$ increases in region $[0, \bar{a}]$. As shown by (13), the stablecoin repurchase rate $g$ depends itself on the price.
3.3 Optimal Platform Design

Given Lemma 2, we may now solve for the optimal platform design under full commitment but limited liability. The platform chooses policy parameters \( \{a^*, \pi\} \) so as to maximize its date-0 value,

\[
E_0 = E(A, C^*(A_0)) + (p(a^*) - \varphi)C^*A_0 = A_0 \frac{e(a^*) + p(a^*) - \varphi}{a^*},
\]

which is comprised of the sum of date-0 issuance gains proportional to \( p(a^*) - \varphi \) and the post-issuance equity value. Note that the remaining policy parameters are characterized by Lemma 1 and 2. Moreover, Lemma 2 provides an explicit solution for \( e(a^*) \) as a function of policy parameters \( \{a^*, \pi\} \). Solving for \( p(a^*) \) analytically is however not possible in the general case because of the feedback loop in dynamic price equation (17) via the repurchase decision \( g(a) \) given by (13). Two special cases of interest allow for an explicit characterization of the platform’s policy choice: the uncollateralized platform \( (\varphi = 0) \) and the fully-collateralized platform \( (\varphi = 1) \). We make use of these two extreme cases to characterize the effect of collateralization on platform stability and then provide numerical results for the general case.

3.4 Purely-Algorithmic Platforms

Consider an uncollateralized platform with \( \varphi = 0 \). Equation (13) then shows that the optimal repurchase policy is \( g = 0 \) in the smooth region \([0, \pi]\). This result is intuitive because the platform receives no collateral proceeds to finance stablecoin repurchases in that case. Consequently, only a positive exogenous shocks to demand could push the platform back above \( \pi \) to recover the peg. This feature allows us to provide an explicit solution for \( p \) over the whole state space and thus to characterize the optimal platform policy choice.

Proposition 3 (Purely-Algorithmic Protocol Equilibrium). If an equilibrium with positive stablecoin value exists for a fully-uncollateralized platform \( (\varphi = 0) \), the following results apply:

1. The region \([0, \pi]\) in which the peg is lost is non-empty and the equilibrium stablecoin
price is given by 

\[ p(a) = \left( \frac{a}{\bar{a}} \right)^{-\gamma}, \text{ for } a \leq \bar{a} \text{ where } \gamma < -1 \] is the unique negative root of

\[ r + \lambda = -\mu \gamma + \frac{\sigma^2}{2} (1 + \gamma) \gamma + \frac{\lambda \xi}{\xi - \gamma}. \tag{20} \]

2. The optimal policy \( \theta_0^* \) is characterized by Lemma 1 and 2 and \( \bar{a} \leq a^* \) that solve

\[ \max_{\{\bar{a}, a^*\}} \frac{e(a^*) + p(a^*)}{a^*} = \frac{\ell(a^*)}{a^* \left[ r + \frac{\lambda \xi}{\xi + 1} - \mu + \left( \frac{\lambda \xi}{\xi + 1} - \frac{\lambda \xi}{\xi - \gamma} \right) \left( \frac{a^*}{\bar{a}} \right)^{-(\xi + 1)} \right]} \tag{21} \]

subject to \[ [e(a^*) + p(a^*)] \frac{\bar{a}}{a^*} = 1 \tag{22} \]

An uncollateralized platform loses the peg after a large enough negative demand shock; even if it can commit to an issuance policy. When demand drops, the platform would like repurchase a sufficient amount of stablecoins to maintain the peg. For a large enough drop, however, the net present value of liquidity benefits is so low that the value of equity is zero. In that case, repurchase cannot be financed through equity dilution and the peg is lost. This dynamics can be observed in the crash of the two algorithmic stablecoins Terra and NuBits for which the market capitalization of their governance tokens fell to zero at the time of loosing the peg (see Online Appendix). In the region \([0, \bar{a}]\) in which the peg is lost, the stablecoin price remains strictly positive although investors enjoy no liquidity benefit since \( p(a) < 1 \). The stablecoin value is then driven entirely by the probability that the demand ratio \( a_t \) exogenously reaches the peg threshold \( \bar{a} \). The speed of this process depends on the value of the root \( \gamma \).

The second part of Proposition 3 characterizes the optimal policy choice of an uncollateralized platform under limited liability. Given that \( e(a) = 0 \) for all \( a \leq \bar{a} \) and \( e(a) \) is linear and increasing for \( a \in [\bar{a}, \infty) \), limited liability holds everywhere if \( e(\bar{a}) = 0 \), which is constraint (22). The platform’s objective is once again to maximize the present value of liquidity benefits, in this case given by Equation (21). This expression shows that the platform’s effective discount rate now depends on its policy choices \((\bar{a}, a^*)\). Given \( \bar{a} > 0 \), a lower value of \( a^* \) increases the probability to lose the peg, which raises the platform’s discount rate. While the platform would like to set \( \bar{a} = 0 \) to minimize this effect, this choice would violate limited liability constraint (22), which confirms \( \bar{a} > 0 \). This effect implies that the target demand ratio \( a^* \) is higher than its counterpart under unlimited
liability, $a_{ul}^*$. The intuition behind this result is that reducing stablecoin issuance protects the platform against large negative demand shocks.

### 3.5 Fully-Collateralized Platforms

We now turn to the analysis of fully-collateralized stablecoin platforms and characterize its equilibrium prices.

**Proposition 4 (Fully-Collateralized Protocol Equilibrium).** If an equilibrium with positive stablecoin value exists for a fully collateralized platform, the following results apply

1. The peg is always maintained, that is, $\bar{a} = 0$.
2. The optimal FTMP is given by Lemma 1 and 2, $\bar{a} = 0$ and $a^* > a_{ul}^*$ that solves

$$\max_{a^*} \frac{e(a^*)}{a^*} = \frac{\ell(a^*) + \mu^k - r}{a^*(r - \mu + \lambda/(\xi + 1))}$$  \hspace{1cm} (23)

The key difference in Proposition 4 relative to Proposition 3 is that the peg is never lost with a fully collateralized platform, that is, $\bar{a} = 0$ with $\varphi = 1$. This result follows directly from observing limited liability constraint (11). With $p(a^*) = 1$ and $\varphi = 1$, the net value of outstanding stablecoins is zero. Intuitively, with a fully-collateralized platform any stablecoin repurchase is fully financed through collateral holdings, which means no equity dilution is ever required to pay for a repurchase. Hence, the limited liability constraint does not affect the ability of the platform to perform these operations and maintain the peg.\(^\text{13}\)

Consequently, the optimization problem (23) is similar to that of a fully-collateralized platform under unlimited liability. In that case, however, the platform held no collateral because it is costly ($\mu^k < r$) and large stablecoin repurchases could be financed by letting equity become negative. With limited liability instead, the platform must hold collateral to maintain the peg at all times. The comparison between target ratios $a^*$ and $a_{ul}^*$ reflects this additional collateral cost. Under limited liability, the platform issues less stablecoins.

\(^{13}\)The result holds in part because the collateral asset is uncorrelated with the demand process $A_t$. If it were (positively) correlated, the value of collateral holdings would decrease when demand drops. In this case, collateral available to finance a repurchase may fall short of the repurchase cost. Unless collateral is fully correlated with the demand process, however, the qualitative effect that collateral relaxes limited liability constraint (11) remains.
because the net liquidity benefit per stablecoin accounting for the collateral cost is typically lower than under unlimited liability as $\ell + \mu^k - r \leq \ell$ given our assumption that $\mu^k \leq r$.

3.6 Existence Conditions

Proposition 3 and 4 characterize a platform’s optimal policy given that an equilibrium with positive stablecoin value exists. Now we provide existence conditions for both cases.

Proposition 5 (Existence Condition). Given collateralization ratio $\varphi \in \{0, 1\}$ a stablecoin platform with positive value exists under full commitment only if:

$$\lim_{a \to \infty} l(a) \geq \begin{cases} r - \mu + \frac{\lambda}{\xi+1} & \text{if } \varphi = 0 \\ r - \mu^k & \text{if } \varphi = 1 \end{cases} \quad (24)$$

We derive the existence conditions in Proposition 5 from imposing condition $e(a^*) \geq 0$. This condition implied by $e(\bar{a}) \geq 0$ and $a^* \geq \bar{a}$ is necessary and sufficient for an equilibrium with positive stablecoin value to exist under limited liability.

In the fully-collateralized case ($\varphi = 1$), the existence condition states that the liquidity benefit captured by the platform $\ell(a^*)$ must exceed the collateral holding cost $\mu^k - r$. As discussed above, the collateral holding cost can be interpreted as a liquidity benefit from holding the underlying asset, which the platform forgoes when using the asset as collateral. Condition (24) states that issuing stablecoins that are fully backed by collateral can only be profitable if the former commands larger liquidity benefits.

In the uncollateralized case ($\varphi = 0$), condition (24) is necessary but not sufficient. We provide a necessary condition in the Appendix. To obtain some intuition about the condition, let’s consider the case $\bar{a} = 0$. The expression (21) then gives a total platform value at the target as approximately

$$\left[ e(a^*) + p(a^*) \right] \frac{A_0}{a^*} \approx \frac{\ell(a^*)}{\ell + \frac{\lambda}{\xi+1} - \mu} \frac{A_0}{a^*}, \quad (25)$$

per unit of stablecoin. Given that $p(a^*) = 1$, condition (24) must hold for equity value $e(a^*)$ to be positive. Issuance gains at date 0 should be less than 100% of the gains from trade, given by the right-hand-side of (25), as otherwise the post-issuance equity value of the platform would be negative. Consequently, the platform’s equity is positive only if the
growth rate of stablecoin demand, \( \mu - \frac{\lambda}{\xi+1} \) and the liquidity benefits are high enough. In other words, an uncollateralized platform exists only if demand for the stablecoin keeps growing over time.

**Corollary 1.** An uncollateralized platform value can exist only if stablecoin demand keeps growing, that is, if \( \mu - \frac{\lambda}{\xi+1} \geq 0 \).

Corollary 1 follows from Proposition 5 and Assumption 1 stating that the marginal liquidity benefit is no larger than the discount rate \((\ell < r)\). Stablecoin demand must grow over time for an uncollateralized stablecoin platform to have any value. Without this growth component, platform owners would prefer to default immediately after the initial stablecoin issuance because all gains from trade would have been realized already. In that sense, this result relates to existing argument on algorithmic stables, which are portrayed as “Ponzi Schemes.” If the growth rate of the demand for the stablecoin were to unexpectedly and permanently fall to zero, the equity value of the platform would also fall to zero so that it would lose its peg permanently.

### 3.7 Numerical Example

We provide a numerical example illustration under full commitment in Figure 1. As we showed, the peg can be maintained in all circumstances only under full commitment. The first panel shows that limited liability protects equity holders, as their equity value is always positive after large negative shocks. From an ex-ante perspective, however, the inability to conduct large repurchases lowers the initial platform value. In both cases, the initial platform value is given by the rightmost panel of Figure 1 taking \( a \to \infty \).

### 4 Non-Programmable Issuance

In this section, we analyze the centralized platform’s problem under weaker forms of commitment. We assume that a subset of policies cannot be fully programmed via smart contracts at date 0. We maintain commitment with respect to the coupon policy \( \{\delta_t\}_{t \geq 0} \) and the minimum collateralization rule \( \varphi \) chosen at date 0 but the platform now chooses its issuance policy sequentially at every date \( t \geq 0 \). Limited liability still applies: the platform
can avoid paying coupon or violate its collateralization rule by defaulting. \footnote{As described below, some degree of commitment is necessary for an equilibrium with positive stablecoin value to exist. See Appendix V for a proof of this claim.}

In line with our previous analysis, we assume that the coupon policy $\delta$ is Markov in that it only depends on state variables $(A_t, C_t)$, and more precisely, on the demand ratio $a_t = \frac{A_t}{C_t}$. In what follows, we first refine our equilibrium concept under partial commitment and then highlights how a commitment problem arises in our environment. Finally, we show how the commitment problem can be addressed by a centralized platform.

## 4.1 Equilibrium Concept under Partial Commitment

We refine our equilibrium concept under limited commitment by introducing the concept of a Markov Perfect Equilibrium (MPE), defined with respect to the “natural” state variables of our economy $(A_t, C_t)$. In a MPE, the platform’s issuance policy and the stablecoin pricing function depend only on $(A_t, C_t)$, as opposed to the entire history of shocks. We first define a MPE and then discuss the merits and limitations of this concept.

**Definition 3.** Given a coupon policy $\delta(A, C)$ homogeneous of degree 0 and a collateralization ratio $\varphi \in [0, 1]$, a Markov equilibrium is given by an equity owner value function $E(A, C)$, a stablecoin pricing function $p(A, C)$, an issuance policy $dG(A, C)$, and an optimal default policy $\tau_D$ such that the issuance policy $dG$ and default policy $\tau_D$ maximize the
platform’s equity value in any state \((A,C)\), that is,

\[
E(A,C) = \max_{\tau_D, d\tilde{G}} \mathbb{E}_t \left[ \int_t^{\tau_D} e^{-r(s-t)} \left( p_s d\tilde{G}_t - \varphi dC_t \right) \middle| A_t = A, C_t = C \right],
\]

(26)
given coupon policy \(\delta\), the law of motion for stablecoins (4) and debt pricing function

\[
p(A,C) = \mathbb{E}_t \left[ \int_t^{\tau_D} e^{-r(s-t)} (\ell_s + \delta_s)p_s ds + e^{-r(\tau_D-t)} \min\{\varphi, 1\} \middle| A_t = A, C_t = C \right].
\]

(27)

Optimality criterion (26) states that the issuance policy must be sequentially optimal under limited commitment. In a Markov equilibrium, this means the policy must be optimal in any state \((A,C)\). Sequential optimality under limited commitment thus imposes additional constraints on the platform’s problem, which is to maximize value at date 0, that is, in state \((A_0, 0)\). In writing Equation (26) for the equity value above, we simplified equation (6) by substituting for the collateral purchase policy \(dM_t = \varphi dC_t\) implied by collateralization rule (3) and letting the platform’s value in default be 0 because the collateral value is less than the par value of stablecoins \(K_t = \varphi C_t \leq C_t\).

Debt pricing equation (27) embeds the restriction imposed by Markov perfection. The stablecoin price may depend only on current state variables \((A,C)\), not on the history of past issuance decisions. This restriction implies investors may not “punish” the platform for deviating. Suppose the platform announces policy \(\{d\tilde{G}_t\}_{t \geq 0}\) but deviates to \(d\tilde{G}_\tau \neq dG_\tau\) at some date \(\tau > 0\). The new price faced by the platform may change only because state variables \((A,C)\) change after \(d\tilde{G}_\tau\), not because \(d\tilde{G}_\tau\) is a deviation. If instead we let the stablecoin price depend on the entire history of actions, investors could explicitly punish the platform following a deviation using so-called grim-trigger strategies. For instance, investors could refuse to buy stablecoins, thereby setting the price to 0. We use Markov Perfection to discipline out-of-equilibrium behavior but also for realism because disperse investors would find difficult to coordinate and commit to grim-trigger strategies.\(^{15}\)

\(^{15}\)See Malenko and Tsoy (2020) who consider such explicit punishments in a related dynamic leverage choice problem for firms.
4.2 Issuance and Default Policy under Limited Commitment

The first step of the analysis is to characterize an equilibrium stablecoin issuance policy \(d_G\) and default policy \(\tau_D\) under partial commitment. Remember that under full commitment, we assumed \(d_G\) belonged to the class of targeted Markov policies. Under limited commitment, however, sequential optimality puts stronger requirements and the equilibrium policy which allow us to show it must belong to this class.

**Proposition 6 (Equilibrium Issuance Policy).** *For an optimal coupon policy \(\delta\) chosen at date 0, if an equilibrium exists, the equilibrium issuance policy \(d_G\) under limited commitment belongs to the class of targeted Markov policies introduced in Definition 4. The platform never defaults.*

Proposition 6 builds on a series of Lemma relegated to the proof in the interest of space. We provide some high-level description of these intermediate steps and refer the interested reader to the Appendix for details. We first establish that the equilibrium equity function is weakly convex and the stablecoin price is weakly increasing as a function of the demand ratio \(a\). Following arguments from DeMarzo and He (2021), we then show that the equilibrium issuance policy is smooth (features jumps) on intervals for which the equity value is strictly convex (increasing). Next, we show that if the coupon policy \(\delta\) is chosen optimally at date 0, the issuance policy is smooth over \([0, \bar{a}]\) for some \(\bar{a} \geq 0\) and features a jump to some target demand ratio \(a^*\). Hence, as the platform does not default, the equilibrium issuance policy is a targeted markov policy.

As mentioned above, the platform does not default, even under limited commitment. This result because the platform is never forced to make payments in the unit of account. It can always choose not to repurchase debt if it is too costly and the interest payment is made in stablecoins not in the unit of account. As the platform can issue stablecoins at no cost, it may not gain from defaulting and thus always prefers to hope for resurrection.

For the first result, observe that optimality of the issuance policy implies

\[
E(A, C) \geq E(A, C') + (p(A, C') - \varphi)(C' - C).
\]

When the state variable is \((A, C)\), the platform can perform a discreet issuance \(C' - C\) at the post-issuance price \(p(A, C')\) for any \(C'\). It would then enjoy the net issuance proceeds, \((p(A, C') - \varphi)(C' - C)\) and the post-issuance equity value \(E(A, C')\). By optimality of the
platform’s issuance decisions, inequality (22) must hold. If \( E \) is strictly convex in \( C \), it can further be shown that inequality (22) is strict which means any discrete stablecoin issuance or repurchase is dominated. Hence, the issuance policy should be smooth.

The second result from Lemma 10 is a consequence of the “leverage ratchet effect” as in DeMarzo and He (2021) who study a dynamic debt issuance problem. If the equilibrium issuance policy is smooth, platform owners may not capture any of the liquidity benefits from stablecoin issuance. This leverage ratchet effect implies that the protocol value in any equilibrium is weakly negative because no liquidity benefit is captured and holding collateral is costly. Furthermore, the platform would be unable to maintain a peg, which implies the stablecoin would have no value in equilibrium.

The second preliminary result is the counterpart of Lemma 10. It shows that if the equity value is linear over some segment, the equilibrium issuance policy must feature jumps.

Finally, we show that the equilibrium policies do not include default.

Lemma 3. In equilibrium, the platform never defaults, that is, \( \tau_D = \infty \).

Lemma 10 and 9 restrict the set of issuance policies compatible with an equilibrium with strictly positive stablecoin value. They show that the issuance policy must feature jumps, but leave open the possibility that there could be several non-overlapping regions with discrete issuance or repurchase. We can show however that the equilibrium issuance policy must be part of the class of stable issuance policies.

The first standard step of the proof consists in showing that the equilibrium equity value must be weakly convex. It implies that on any segment, the equity value is either strictly convex or linear. We then show that the only possibility is that the equity value is strictly convex close to 0 and there is only one linear segment \([\bar{a}, \infty)\). This second step requires using the optimality of a coupon policy chosen at date 0. In other words, there could be coupon policies and associated MPEs in which the properties of Proposition 6 do not hold but our argument then shows that these coupon policies may not be optimal. Given these two steps, the result follows from Lemma 10 and 9.

Proposition 6 also

Knowing that a MPE with positive stablecoin value must feature a stable debt policy, we can provide conditions on the equilibrium equity value and the stablecoin price.

Proposition 7. Let \( \delta(a) \) be an optimal coupon policy chosen at date 0. A non-zero MPE
induced by this coupon policy satisfies the following properties

1. Equity value $e(a)$ is strictly convex in $a$ for $a \in [0, \overline{a}]$ and linear for $a \geq \overline{a}$ with

   $$(r + \lambda)e(a) = -\delta(a)p(a) + \mu e'(a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)], \quad \forall a \leq \overline{a}.$$ 

2. The stablecoin price $p(a)$ is strictly increasing (equal to 1) for $a \in [0, \overline{a}]$ ($a \geq \overline{a}$) with

   $$(r + \lambda)p(a) = \delta(a)p(a) - (g(a) + \delta(a) - \mu)ap'(a) + \frac{\sigma^2}{2} a^2 p''(a) + \lambda \mathbb{E}[p(Sa)], \quad \forall a \leq \overline{a}.$$ 

3. In the smooth region $[0, \overline{a}]$, the platform’s issuance rate is equal to

   $$g(a) = \frac{\delta'(a)p(a)}{p'(a)}$$

   and the platform’s equity value is the same as if it issued no debt.

4. The coupon policy must satisfy

   $$\delta(a) \geq (r + \lambda) + \lambda \mathbb{E}[e(Sa)] - (\ell(a^*) + \lambda \mathbb{E}[e(Sa^*) + p(Sa^*)]) \frac{a}{a^*} \quad \text{for all } a \geq \overline{a}.$$ 

We show in the proof of Proposition 6 that a stable debt issuance policy is only compatible with equity value and stablecoin price functions that satisfy Conditions 1 and 2. When the platform is in the peg region, it implements a constant demand ratio $a^*$. In this region, from any debt level $C$ the platform issues a discrete block of stablecoins $C^*(A) - C$ at price $p(A, C^*(A))$. Hence, equity value is given by

$$E(A, C) = E(A, C^*(A)) + p(A, C^*(A))(C^*(A) - C)$$

$$e(a) = \left[ e(a^*) + p(a^*) \right] \frac{a}{a^*} - p(a^*)$$

which implies $e(a)$ is linear in $a$. As before, the platform sets the coupon policy $\delta(a^*)$ to ensure $p(a^*) = 1$ as otherwise stablecoin owners would not enjoy liquidity benefit.

In the smooth region, that is, when $a$ falls below $\overline{a}$, the platform abandons the peg. The stablecoin price is strictly positive and increasing in the smooth region although stablecoin owners do not currently enjoy liquidity benefits. As the peg may be restored following a series of positive demand shocks, however, future stablecoin owners will enjoy liquidity
benefits, which supports the price today. Equations (??) and (??) are the Hamilton-Jacobi-Bellman equations in the smooth region for equity value and stablecoin price respectively.

Equilibrium requirement 3 states that the platform is indifferent between issuing stablecoins and staying idle in the smooth region. A smooth debt policy can be optimal only if the return to issuance is zero. This result is similar to the leverage ratchet effect of DeMarzo and He (2021) in a similar context. Because the platform can freely and continuously issue stablecoins, it is unable to capture any issuance benefit under limited commitment because it competes against its future self, a version of the Coase (1972) problem for monopolists. Unlike in DeMarzo and He (2021), however, our equilibrium also features a peg region in which the platform does enjoy issuance benefits.

The result that the platform is indifferent about stablecoin issuance in the smooth region does not mean that the platform does not issue stablecoin in this region. Stablecoin issuance may be necessary to support the equilibrium price. Equation (??) shows that the issuance policy is determined by the endogenous price function and the coupon policy. The steepest the price function (high \( p'(a) \)), the largest the platform’s price impact and the lower issuance or repurchase is. The platform issues (repurchases) if \( \delta'(a) > 0 \) (\( \delta'(a) < 0 \)), that is, it the ex-post equilibrium issuance policy leans against the coupon policy chosen ex-ante. In particular, if the coupon decreases with \( a \), \( \delta'(a) > 0 \), so as to penalize low demand ratios, the platform repurchases debt to avoid the increase in coupon payments.

Finally, Criterion 4 is a condition on the coupon payment policy such that no deviation to a smooth debt policy is preferred to the equilibrium policy in the target region. To get some intuition about this condition, consider a demand ratio \( a = \frac{A}{C} \in [\pi, a^\star] \) in the peg region for which the platform is supposed to jump to \( a^\star \). Instead of the equilibrium policy, suppose the platform deviates by not jumping and then revert back to the equilibrium policy after an interval \( dt \). We show that this deviation is unprofitable if\(^1\)\(^1\)

\[
(r + \lambda)(C - C^\star(A)) \leq \delta(a)C - \delta(a^\star)C^\star(A) + \lambda(E[E(SA, C^\star)] - E[E(SA, C)]). \tag{29}
\]

The left-hand-side of (29) is the gain from postponing the repurchase, equal to the effective

\(^1\)A good analogy is with a mixed-strategy Nash equilibrium in which a player is indifferent between options but one specific randomization should be chosen to support the equilibrium.

\(^1\)The same condition applies if instead \( a \geq a^\star \) but in this case the platform is supposed to issue stablecoins rather than repurchase them, which means the commitment problem does not bind.
interest rate $r + \lambda$ multiplied by the equilibrium repurchase quantity $C - C^\star$ at equilibrium price of 1. The right-hand side of (29) corresponds to the punishment from the deviation. The first term is the difference in coupon payments evaluated at the stablecoin price of 1. The second term is the benefit from implementing a larger demand ratio $a^\star$ vs. $a$ which protects against large negative demand shocks. Condition (28) follows directly from (29).

The platform chooses the coupon policy at date 0 under the constraint $\delta(a) \geq 0$. It is thus always possible to specify a coupon policy such that (28) is satisfied because the coupon policy on $a \in [\bar{a}, \infty) \setminus \{a^\star\}$ does not impact equilibrium objects. Indeed, the optimal issuance policy is to jump to $a^\star$ when $a \in [\bar{a}, \infty)$, which implies states $a \in [\bar{a}, \infty) \setminus \{a^\star\}$ are not visited in equilibrium. Hence, given equilibrium objects, one can always set $\delta$ on $[\bar{a}, \infty) \setminus \{a^\star\}$ so as to satisfy (28). The coupon policy on $[\bar{a}, \infty) \setminus \{a^\star\}$ plays the role of an out-of-equilibrium threat to discourage deviations in the target region.

Having characterized the MPE without commitment to the issuance policy, we may now analyze the optimal coupon policy $\delta(a)$ chosen to maximize the date-0 platform value. At date 0, the platform takes as given the equilibrium played by its future selves who have full discretion over the repurchase-issuance policy. Hence, unlike in the full commitment case, the coupon policy plays a new role: it can help discipline the platform in the future to act in its own interest at date 0. To simplify the analysis, we assume the coupon policy is fixed in the smooth region, similar to the restriction imposed under full commitment.

**Assumption 3.** The coupon policy in the smooth region is such that $\delta(a) = \hat{\delta}$ for $a \leq \bar{a}$.

Assumption 3 simplifies the problem because we can provide analytical functional forms for the MPE equity value and price in the smooth region. Setting $\delta(a) = \hat{\delta}$ in HJB equations (??) and (??), we can guess and verify the following functional forms

$$e(a) = \begin{cases} \sum_{k=1}^{3} c_k (a/\bar{a})^{-\gamma_k} & \text{if } a < \bar{a}, \\ (e^\star + 1) a/a^\star - 1 & \text{if } a \geq \bar{a}, \end{cases}$$

(30)

$$p(a) = \begin{cases} \sum_{k=1}^{3} b_k (a/\bar{a})^{-\gamma_k} & \text{if } a < \bar{a}, \\ 1 & \text{if } a \geq \bar{a}, \end{cases}$$

(31)
where $\gamma_k$s are roots of the characteristic equation

$$r + \lambda - \delta = - (\mu - \delta)\gamma + \frac{\sigma^2}{2} (1 + \gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \quad (32)$$

We may now characterize the optimal choice of the platform at date 0. Under Assumption 3 optimization problem over the coupon policy can be thought as a choice over $\theta_0 = \{\delta, \delta^*, \pi, a^*\}$. The platform chooses the optimal policy $\theta_0^*$ to maximize the platform’s value at date 0 subject to the requirement that the stable issuance policy with parameters $\theta_0^*$ is part of an MPE. We show that the optimization problem can be characterized as follows.

**Lemma 4 (Optimal Policy).** The optimal date-0 policy $\theta_0^*$ for a centralized uncollateralized platform is the solution to the following maximization problem

$$E_0/A_0 = \max_{(\pi, a^*, \delta)} \frac{\ell(a^*)/a^*}{r + \frac{\lambda}{\xi + 1} - \mu + \left(\frac{\lambda\xi}{\xi + 1} - \frac{\lambda\xi}{\xi - \gamma}\right) \frac{(a^*)}{\pi}^{-(\xi + 1)}}, \quad (33)$$

subject to

$$\frac{\gamma}{1 + \gamma} \frac{1}{a} = \frac{\ell(a^*)/a^*}{r + \frac{\lambda}{\xi + 1} - \mu + \left(\frac{\lambda\xi}{\xi + 1} - \frac{\lambda\xi}{\xi - \gamma}\right) \frac{(a^*)}{\pi}^{-(\xi + 1)}}, \quad (34)$$

$$\pi \in [0, a^*] \quad (35)$$

with $\gamma$ the only negative root of equation (32).

Observe first that $\delta^*$ does not appear in the maximization program. As before, the only role of this parameter is to ensure the price is pegged at one in the target region, that is, $p(a^*) = 1$. We thus substituted for the corresponding value of $\delta^*$ to write the platform’s objective function (33). The second constraint is the smooth-pasting condition for the equity value between the target and the smooth regions. This constraint reflects the requirement that the issuance policy must be optimal ex-post for the platform. In particular, the platform chooses optimally to switch from the peg to the smooth region. This feature generates condition (77). We show in the proof of Lemma 4 that this constraint implies that limited liability is satisfied for all values of $a$. Finally, the last constraint must be satisfied for a MPE to exist by definition of a stable debt policy.

Unlike in the full commitment case, a precise analytical characterization of the solution is difficult. The proof of Lemma 4 reports partial characterization for some cases. In the main text, we provide a numerical illustration of the dominant MPE in Figure 2.
For each panel, the solution without commitment is compared to the solution where the only constraint is limited liability. We show that under limited commitment, the protocol chooses a lower target inverse supply $a^*$ and it abandons the peg for a higher value of $a$. These differences are reflected in a lower value of equity and of total protocol value compared to the commitment case.

Figure 2: Solution with commitment and limited liability (black) and without commitment (blue). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(a) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

4.3 Centralized Platform with Collateral

[Preliminary.]

In this section, we consider a minimum collateralization rule as a tool to improve the stability of a centralized stablecoin protocol. This analysis sheds lights on crypto-collateralized protocols, for which we provide a balance sheet representation in Figure ???. Unlike an uncollateralized protocol with no asset, a collateralized protocol holds the cryptoasset on its balance sheet to back stablecoin issuance. The difference between the market value of the cryptoasset and the par value of stablecoins is akin to overcollateralization. Overcollateralization creates a buffer in case the collateral asset suddenly loses its value.

In the following section, we solve for equilibrium issuance, collateralization, and default strategies $dG$, $dM$, and $\tau_D$ given commitment to the interest policy $\delta(a)$ and a minimum collateralization rule $K = \varphi C$. The collateralization rule forces the protocol to maintain a minimum ratio between its cryptoasset holdings and the stock of stablecoins issued. The protocol shuts down automatically when the collateralization rule, given by $K_t \leq K$, is breached, which happens at stochastic time $\tau_K$.

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4.4 Stability Benefits of Collateral

In this section, we characterize the optimal collateral policy of the protocol. The collateral policy \( dM \) specifies the change in the value of collateral held by the protocol as a function of current state variables \( K_t \), \( C_t \), \( A_t \). The law of motion for collateral is given by (5).

Our first result is that in equilibrium, the protocol’s equity value is linear in the amount of collateral it holds.

**Lemma 5.** In any MPE in \((A,C,K)\), \( E(A,C,K) \) is linear and increasing in \( K \). Hence,

\[
\forall K, K' \geq \varphi C, \quad E(A,C,K) = E(A,C,K') + K - K'.
\]

The intuition for Lemma 5 is that the protocol can freely add or remove collateral to the platform subject to the collateralization constraint, and the protocol has no influence on the cryptoasset price. These features imply that the equilibrium equity value is linear in the value of collateral held by the platform.\(^\text{18}\) To form intuition about (36), it is useful to remember the optimality decision for debt, given by equation (9) in Proposition 7. The derivative of the equity value with respect to stablecoin issuance is equal to the price of stablecoins. Equation (36) shows the same equation holds with respect to collateral with the only difference that the collateral price is exogenously given.

Lemma 5 motivates the following conjecture for the optimal collateral policy. For any value \((A,C)\) of cryptoasset value and stablecoin stock, there exists a collateral target such that the protocol buys/sells collateral discretely to reach that target from any level \( K \geq \varphi C \). Formally, we define the following class of policies

**Definition 4 (Class of Targeted Collateral Policies).** A collateral policy is part of the class of targeted collateral policies \( \mathbb{T} \) if it is characterized by a lower bound \( \bar{a} \) and a collateral target function \( k^*(a) \) such that, denoting \( k \equiv \frac{K}{C} \),

\[
dM(a_t, k_t, C_t) = \begin{cases} 
0 & \text{if } a_t < \bar{a}, \\
(\varphi - k_t)C_t & \text{if } \bar{a} \leq a_t \leq \bar{a}, \\
(k^*(a_t) - k_t)C_t & \text{if } a_t > \bar{a}.
\end{cases}
\]

Equation (36) shows that if the protocol considers a discrete collateral change from any

\(^{18}\)The cryptoasset price is not equal to one, but \( K_t \) is the value of collateral held by the protocol, as opposed to the amount of collateral. Hence, a change from \( K \) to say \( K' > K \) costs \( K' - K \) to the platform.
level \( K_t \), any adjustment is weakly optimal by the linearity of the equity value function. In particular, an adjustment to some specified target \( K^*(A, C) \) is optimal. To characterize the target \( k^*(a) = \frac{K^*(A, C)}{C} \), we provide conditions such that there is no smooth deviation - of the order \( dt \) - from the discrete adjustment policy. We obtain the following characterization

**Lemma 6 (Equilibrium Collateral Policy Characterization).** A targeted policy is part of a MPE with positive stablecoin price if

\[
\lambda \frac{\partial E_t[E(SA, C, SK)]}{\partial K}_{K=K^*(A, C)} = r + \lambda - \mu
\]

Any equilibrium collateral policy is part of the class of targeted collateral policies.

Condition (??) captures the trade-off that pins down the optimal collateral target \( K^*(A, C) \). The right-hand-side is the marginal cost of adding one unit of cryptoasset as collateral for the protocol. It is equal to the effective opportunity cost, \( r + \lambda \), of the protocol minus the growth rate of the cryptoasset \( \mu \). As explained before, this difference can be interpreted as the cost of locking up the cryptoasset as collateral. The left-hand-side of (??) is the marginal benefit of adding one unit of collateral evaluated at the target \( K^*(A, C) \). Collateral protects the protocol against the risk that a negative Poisson shock to the cryptoasset triggers a breach of the minimum collateral requirement. The right-hand-side is indeed proportional to the probability of such a shock. Intuitively, if the protocol sets collateral equal to the minimum requirement \( \varphi C \), any negative discrete shock triggers a shutdown. Setting too high a collateral target is not optimal either as the protocol finds it optimal not to fight some very negative Poisson shocks under limited liability.

5 Decentralized Protocols

[To be completed.]

In this section, we adapt our general model to account for decentralized stablecoin protocols. Such protocols—with Dai as its most prominent example—allow for the decentralized creation of new stablecoins by anyone with enough collateral. To do so, individual investors have to lock some collateral asset in a smart contract generated by the protocol—a *vault*—and can issue some stablecoins against it. Once the stablecoins are sold to outside investors,
the vault represents for its owner a leveraged position in the collateral asset. Moreover, the newly issued stablecoins are not tied to a particular vault and are fully indistinguishable from other stablecoins—i.e., decentralized stablecoins are perfectly fungible. Vault owners can unlock their collateral assets by repurchasing and “burning” enough stablecoins to liquidate the vault. The system’s stability, therefore, relies on providing the right set of incentives for individual investors to adopt prudent risk management practices and not to over-extend the supply of stablecoins. As in the centralized case, the protocol also issues governance tokens with voting rights on the system’s key parameters and claims to the system’s seigniorage revenues.

5.1 Tokens Valuation

In a decentralized crypto-collateralized protocol, individual vaults indexed \( i \in [0, 1] \) are created using collateral value \( K^i_t \) in exchange for a quantity \( C^i_t \) of stablecoins with price \( p_t \). When the loan is repaid by the vault owner, the stablecoin is “burned” and removed from the supply. As for centralized crypto-collateralized protocols, a vault with the value of its collateral \( K^i_t \) below the threshold \( \varphi C^i_t \) is liquidated. In such a case, the vault owner receives the value of the collateral after repayment of the stablecoins, if any. As there is no heterogeneity across infinitesimal vault owners, the state variables of the system is given by total stock of collateral, stablecoins, and the market capitalization of crypto-assets: \( K_t = \int K^i_t di, \ C_t = \int C^i_t di, \) and \( A_t \).

We differentiate vault-specific variables from their aggregate counterparts by the superscript \( i \). Thus, the value of a vault (decentralized equity) can be written as

\[
V(A_t, C_t, C^i_t, K^i_t, K_t^i) = \max_{\tau^i, \delta^i, G^i_t, M^i_t} \mathbb{E}_t \left[ \int_{\tau^i}^{t} e^{-r(s-t)} \left( \delta_s G^i_s - dM^i_s \right) + e^{-r(\tau^i-t)} \max\{K^i_{\tau^i} - C^i_{\tau^i}, 0\} \right]
\]

such that

\[
dC^i_t = s_t C^i_t dt + dG^i_s, \quad \text{and} \quad dK^i_t = \mu K^i_t dt + \sigma K^i_t dZ_t + K^i_t (S_{N_t} - 1) dN_t + dM^i_t.
\]

Fees accrue through time between the different investors according to predetermined parameters. Stablecoin holders receive the convenience yield \( \ell C \) as well as some interest rate paid in stablecoins by the governance system \( \delta \). Besides paying this interest to stablecoin holders, the governance system charges vault owners the stability fee—denoted \( s \).
result, the net nominal spread earned by the governance token is \((s - \delta)C\). Whenever a vault is liquidated, the platform must dilute the equity of the system to sustain the loss at time \(t\) given by \(\int \min\{K_i^t - C_i^t, 0\} \mathbf{1}\{\tau_i = t\} di\). Thus, an equity token value is given by

\[
E(t, C_t, A_t, K_t) = \max_{\tau, s, \delta} \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} \left( (s - \delta)p_s - \int \min\{K_i^s - C_i^s, 0\} \mathbf{1}\{\tau_i = s\} di \right) ds \right].
\]

Finally, the price of one stablecoin per unit of debt is given by

\[
p(t, C_t, K_t) = \mathbb{E} \left[ \int_t^\tau e^{-r(s-t)} (\ell(A_s, C_s) + \delta_s p_s) ds + e^{-r(\tau-t)} \min\{K_t/\tau_t, 1\} \right].
\]

5.2 Arbitrage

Given any amount of stablecoin issued by an individual vault \(C_i\), vault owners have the option to adjust to \(\tilde{C}_i\) by issuing \(\tilde{C}_i - C_i\) at the price of \(p(A, C, K)\). Therefore, the value of the vault must be at least as high as the value of the vault after the adjustment:

\[
V(A, C, C_i, K, K_i) \geq V(A, C, \tilde{C}_i, K, \tilde{K}_i) + p(A, C, K)(\tilde{C}_i - C_i).
\]

Contrarily to the centralized setting, the price of the stablecoin is not a function of leverage of the individual vault and the previous equation must hold with equality. The same argument holds for the collateral and we get that the value of a vault must be linear in \(C_i\) and \(K_i\):

\[
V(A, C, C_i, K, K_i) = V(A, C, \tilde{C}_i, K, \tilde{K}_i) + p(A, C, K)(\tilde{C}_i - C_i) - (\tilde{K}_i - K_i).
\]

for all and \(K_i \geq \varphi C_i \geq 0\) and \(\tilde{K}_i \geq \varphi \tilde{C}_i \geq 0\). Furthermore, because the value of creating an empty vault must be 0, otherwise there are arbitrage opportunities, we get

\[
V(A, C, C_i, K, K_i) = K_i - p(A, C, K)C_i. \tag{37}
\]

This has non-trivial consequences. Whenever the discounted value of owning a vault deviates from that equality, arbitrage opportunities arises to either create or burn vaults and stablecoins. By adjusting the stability fee \(s\), the maker can incentivize arbitrageurs to adjust the supply to a desired target. As atomistic agents, vault owners cannot capture any value from the option to default and do not internalize the impact of their issuance on
the stablecoin price.

Another consequence of condition (37) is that the minimum collateral requirement must be at least as high as the value of stablecoin issued: $\varphi \geq 1$. Otherwise, a vault owner might issue more stablecoin than the value of the collateral and dispose of the vault. In the following lemma, we establish that if the minimum collateralization rate is not too high, it is never optimal to inject more collateral than the strict minimum to protect the vault against unintended liquidation due to large negative demand shocks.

Lemma 7. If

$$\varphi \leq 1 + \frac{1}{\xi + 1},$$

then $K^i(A, C, K, C^i) = \varphi C^i$.

Thus, we can simplify the notation by defining the value of a vault per unit of stablecoin issued:

$$V(A, C, C^i, K(A, C), K^i(A, C, C^i)) \equiv v(a)C^i = (\varphi - p(a))C^i. \quad (38)$$

If the stability fee $s(a)$ is too high or the price of the stablecoin $p(a)$ is too low such that $v(a) < \varphi - p(a)$, vault owners will be able to make arbitrage profits by purchasing and burning stablecoins to get back the collateral in their vault. Equity holders then internalize that any deviation from the pair of stability fee $s(a)$ and interest payment $\delta(a)$ such that the arbitrage condition (38) is not satisfied triggers immediate changes in the supply of stablecoins. In the following proposition, we characterize the unique MPE policies for a decentralized platform.

Proposition 8 (Targeted MPE). The unique MPE policies $s(a_t)$ and $\delta(a_t)$ are such that $d\mathcal{G}_t = \int d\mathcal{G}_t^i di$ is given by

$$d\mathcal{G}(a_t, C_t^*) = \begin{cases} 0 & \text{if } a_t < \bar{a}, \\ C^*(A_t) - C_t^* & \text{if } a_t \geq \bar{a}. \end{cases}$$

where $C^*(A_t) \equiv A_t/a^*$ is defined by

$$C^*(A) = \arg \max \{ \ell(A, C)C - (r + \lambda - \mu)\varphi C + \lambda\mathbb{E}[S\varphi C - C] \}. \quad (39)$$
At $a^*$, the policies are given by

$$s(a^*) = \mu \varphi - (r + \lambda)(\varphi - 1) + \lambda \mathbb{E}[\max(0, S_\varphi - 1)]$$

and

$$\delta(a^*) = r - \ell(a^*).$$

The value of equity is given by

$$E(A) = \frac{\ell(a^*) - (r + \lambda - \mu)\varphi + \lambda \mathbb{E}[S_\varphi - 1]}{a^*} \frac{A}{r - \mu}. $$

The key insight is that in the presence of arbitrageurs, equity holders are able to target an optimal ratio $a^*$ with the stability fee and the interest payment policies without incentives to deviate. Any deviation from the equilibrium policies $s(a^*)$ and $\delta(a^*)$ triggers an immediate adjustment of the supply of stablecoins to a suboptimal level. Thus, a decentralized platform does not require commitment to any of its policies to enforce a stable equilibrium.

6 Conclusion

In this paper, we propose a general model of stablecoins and examine the merits and vulnerabilities of various stabilization mechanisms. Our analysis highlights that, although local equilibria are feasible, platforms that are only partially collateralized are always vulnerable to large negative demand shocks. This result holds even in a full commitment case. Platforms that do not have full commitment over their issuance-redemption rule may still also feature a locally stable equilibrium given a convex coupon issuance policy. The decentralization of a platform acts as a substitute for a commitment technology. Overall, our work has practical implications for the design and regulation of stablecoins.
References


Appendices

A Proof of Proposition 2

Substituting for $d\mathcal{G}_t = dC_t - \delta_t C_t dt$, the objective function can be written

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[ \int_0^\infty e^{-rt} \left( p_t dC_t - \delta_t p_t C_t dt + \mu^k \varphi C_t dt - \varphi dC_t \right) \right]$$

Integrating the terms in $dC_t$ by parts, we obtain

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[ \left( p_t - \varphi \right) C_t e^{-rt} \right]_{0}^{\infty} - \int_0^\infty e^{-rt} C_t \left( dp_t - r(p_t - \varphi) dt + \delta_t p_t dt - \mu^k \varphi \right)$$

$$= \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[ \int_0^\infty e^{-rt} \left( \ell(A_t, C_t) \mathbb{1}_{\{p_t = 1\}} + (\mu^k - r) \varphi \right) C_t dt \right], \quad (40)$$

To obtain the second line, we guess and verify that $\lim_{t \to \infty} \mathbb{E}_0[(p_t - \varphi) C_t e^{-rt}] = 0$. We use the pricing equation (6) to substitute for $dp_t - (r - \delta) p_t dt$ within the expectation.

Equation (40) shows that setting $\varphi = 0$ is optimal. Second $\delta_t$ is only determined to the extent that it maintains the price peg. Assuming such coupon policy can be chosen, the platform’s problem is static and the optimal issuance rule is such that $C_t$ maximizes $\ell(A_t, C_t) C_t$. By Property (iii) in Assumption 1, this maximizer exists, is unique, and is given by (9). The fact that $C^*_u(T) = \frac{A_T}{\delta^*_u}$ is linear in $A$ follows from Assumption 1. Finally, our conjecture $\lim_{t \to \infty} \mathbb{E}_0[(p_t - \varphi) C_t e^{-rt}] = 0$ and the fact that the objective function is bounded follows from the fact that $A_T$ grows at a rate inferior to $r$. Finally, the coupon policy must be such that $p_t = 1$ for all $t$, which holds with $\delta(a^*_u) = r - \ell(a^*_u)$.

To conclude, the optimal issuance-repurchase policy $\{d\mathcal{G}_t\}_{t \geq 0}$ features a jump from 0 to $C^*_u(A_0)$ at date 0 and is such that $d\mathcal{G}_t + \delta_t C_t dt = dA_t$ for $t > 0$.

B Proof of Lemma 1

For ease of notation, we write $a^*$ instead of $a^*_T$ in this proof. We guess and verify that $p(a) = 1$ if and only if $a \in [\bar{a}, a^*]$ and $p(a) < 1$ otherwise. This implies liquidity benefits are enjoyed by the stablecoin owners only when $a \in [0, \bar{a}]$. The first step of the analysis is
to characterize recursive equations for the different value functions.

**Step 1. Total Platform Value**

Consider first the net platform value $F$. Suppose first $a = A/C > \bar{a}$. In this case $F$ only depends on $A$, not on the outstanding stock of stablecoins $C$ and we denote $\bar{F}(A)$ to avoid confusion. Let $\tau_S$ denote the first (stochastic) time when a shock $S \leq a/a^\star$ hits. We have

$$\bar{F}(A_0) = E_{\tau_S} \left[ \int_0^{\tau_S} e^{-rt} \left( \ell(A_t, C^\star(A_t))C^\star(A_t) + \varphi(\mu^k - r)C^\star(A_t) \right) dt + e^{-r\tau_S} E \left[ F(SA_{\tau_S}, C^\star(A_{\tau_S})) \mid S \leq \frac{\bar{a}}{a^\star} \right] \right]$$

Given values for $(a^\star, \bar{a})$, maximizing value $\bar{F}(A_0)$ consists in maximizing the second term of the equation above. We thus explicit the dynamic equation for $F(A, C)$ in the region where $a = A/C \in [0, \bar{a}]$. For a given $a \in [0, \bar{a}]$, denote $\tau(a)$ the first stochastic time when $a_t = \bar{a}$. We have

$$F(A_0, C_0) = E_{\tau(a_0)} \left[ \int_0^{\tau(a_0)} e^{-rt}(\mu^k - r)C_t dt + e^{-r\tau(a_0)} \bar{F}(A_{\tau(a_0)}) \right],$$

subject to (1), $dC = (\delta C + G)dt$.

The dividend flow for the total platform is negative in the region $[0, \bar{a}]$. Hence, maximizing $F(A, C)$ in region $[0, \bar{a}]$ and thus $\bar{F}(A)$ amounts to minimizing the expected time $\tau(a)$ from any given point $a$. Given the policies in $[0, \bar{a}]$ in (12), we have

$$E \left[ \frac{da_t}{a_t} \right] = \left( \mu - \frac{\lambda}{\xi + 1} \right) dt - (\delta + G/C)dt.$$  

Hence the platform should seek to minimize $\delta$ and $G$ subject to the constraint that equity $E(A, C)$ remains positive for $A/C \in [0, \bar{a}]$. We determine below lower bounds on $\delta$ and $G$ compatible with this constraint.

**Step 2. HJB for equity value**

In the next step, we derive the recursive equation for the equity value in order to pin down the minimum value of $G$ such that limited liability holds in region $[0, \bar{a}]$. In doing so,
we guess and verify that it holds for $[\overline{a}, \infty]$. Adapting Equation (7), we have

$$E(A, C) = (p(A, C) - \varphi)Gdt$$

$$+ (1 - rdt)(1 - \lambda dt)\mathbb{E}[E(A + dA, C + dC) + \mu^k Kdt + \varphi(C + Gdt) - \varphi(C + dC)]$$

$$+ (1 - rdt)\lambda dt \mathbb{E}[E(SA, C)]$$

where the terms within the second expectation operator are the difference between the passive and active increases in collateral value and the new required collateral value given the new amount of stablecoins $C + dC$. Using Ito’s Lemma for the term $E(A + dA, C + dC)$ above and keeping only terms of order $dt$, we obtain the following HJB

$$(r + \lambda)E(A, C) = (p(A, C) - \varphi)G + \mu A E_A(A, C) + \frac{\sigma^2}{2} E_{AA}(A, C)$$

$$+ (\delta C + G) E_C(A, C) + (\mu^k - \varphi)C + \lambda \mathbb{E}[E(SA, C)].$$

(41)

We rewrite the equation above as a functional equation for $e(a) = E(A, C)$.

$$e' = \frac{E(A, C)}{C}.$$ With $E_A(A, C) = e'(a)$, $E_{AA}(A, C) = e''(a)$, $E_C(A, C) = e(a) - ae'(a)$, and $g \equiv G/C$, we get

$$(r + \lambda)e(a) = (p(a) - \varphi)g + \mu ae'(a) + \frac{\sigma^2}{2} e''(a) + (\delta + g)(e(a) - ae'(a)) + (\mu^k - \delta)\varphi + \lambda \mathbb{E}[e(Sa)]$$

(42)

It follows from the equation above that the minimum value of $g$ such that $e(a) \geq 0$ for all $a \in [0, \overline{a}]$ is given by

$$g(a) = -\frac{\mu^k - \delta}{p(a) - \varphi} \varphi.$$ Hence is is optimal to set $\delta = 0$ for $a \leq \overline{a}$ and $g$ as in (13). This concludes the proof.

C Proof of Lemma 2

Step 1. Equity Value

The fact that equity value is equal to 0 in region $[0, \overline{a}]$ is shown in Lemma 1. Consider now interval $[\overline{a}, \infty)$. As argued in the main text, by definition of a policy in (12), equation
(18) must hold. We can rewrite this relationship as follows

\[ Ce(a) = C^*(A)e(a^*) + (p(a^*) - \varphi)(C^*(A) - C) \]

Dividing both terms by \( C \) and using \( C^*(A) = \frac{A}{a^*} \) by definition of \( a^* \), we obtain equation (15).

We are thus left to derive the HJB for the equity value at demand ratio \( e(a^*) \). The recursive equation is the following.

\[
E(a^*C_-, C_-) = (1 - r dt)(1 - \lambda dt)E \left[ E(a^*C_+ + dA, C_+ + dC) + \mu^k K dt - \varphi dC \middle| dN_t = 0 \right] \\
+ (1 - r dt)\lambda dt E \left[ E(Sa^*C_-, C_-) \middle| dN_t = 1 \right].
\]  

(43)

where the term in the first line corresponds to the case in which the adjustment in demand \( A_t \) is smooth \((dN_t = 0)\) while the second term corresponds to the case in which demand is hit by a Poisson shock \((dN_t = 1)\). The term \( \mu^k K dt - \varphi dC \) corresponds to the change in collateral value.

We develop the first term corresponding to Brownian shocks. Remember that in region \([\bar{a}, \infty)\), we have

\[
E(A, C) = C^*(A)e(a^*) + (p(a^*) - \varphi)(C^*(A) - C) = \left[ e(a^*) + p - \varphi \right] \frac{A}{a^*} - (p - \varphi)C
\]

Hence, given that \( dC = \delta^* C dt \), we obtain the following relationship by Ito’s Lemma.

\[
E \left[ E(a^*C_+ + dA, C_+ + dC) \middle| dN_t = 0 \right] = E(a^*C_-, C_-) + \mu \left[ e(a^*) + p - \varphi \right] C^*(A) dt - (p - \varphi)\delta^* C^*(A) dt
\]

Overall, we obtain

\[
E \left[ E(a^*C_+ + dA, C_+ + dC) + \mu^k K dt - \varphi dC \middle| dN_t = 0 \right] = e(a^*)C^*(A) + \mu \left[ e(a^*) + p(a^*) - \varphi \right] C^*(A) dt \\
- p(a^*)\delta^* C^*(A) dt + \mu^k \varphi C^*(A) dt
\]

Hence, keeping only terms of order at least \( dt \) in equation (43) and dividing by \( C^*(A) \), we obtain

\[
e(a^*) = e(a^*) + \left( -(r + \lambda) e(a^*) + \mu \left[ e(a^*) + p - \varphi \right] - p(a^*)\delta^* + \mu^k \varphi + \lambda E[e(Sa^*)] \right) dt
\]

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which simplifies to equation (15).

We can further solve for \( e(a^*) \) by computing the term \( \mathbb{E}[e(Sa^*)] \) in (15). Using (14), we get

\[
\mathbb{E}[e(Sa^*)] = \int_0^{\ln(a^*/\pi)} e(e^{-s}a^*)\xi e^{-\xi s}ds
\]

\[
= \int_0^{\ln(a^*/\pi)} [(e(a^*) + p(a^*) - \varphi)e^{-s} - p(a^*) + \varphi]\xi e^{-\xi s}ds
\]

\[
= \frac{\xi}{\xi + 1} \left( 1 - \left( \frac{a^*}{a} \right)^{-(\xi + 1)} \right) (e(a^*) + p(a^*) - \varphi) - \left( 1 - \left( \frac{a^*}{a} \right)^{-\xi} \right) (p(a^*) - \varphi)
\]

Plugging this equation in (15), we get

\[
\left( r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda \xi}{\xi + 1} \left( \frac{a^*}{a} \right)^{-(\xi + 1)} \right) e(a^*) = \left( \mu^k - \mu + \frac{\lambda}{\xi + 1} - \lambda \left( \frac{a^*}{a} \right)^{-\xi} + \frac{\lambda \xi}{\xi + 1} \left( \frac{a^*}{a} \right)^{-(\xi + 1)} \right) \varphi
\]

\[
+ p(a^*) \left( \mu^k - \frac{\lambda}{\xi + 1} - \frac{\lambda \xi}{\xi + 1} \left( \frac{a^*}{a} \right)^{-(\xi + 1)} + \lambda \left( \frac{a^*}{a} \right)^{-\xi} \right)
\]

(44)

**Step 2. Stablecoin price and \( \delta^* \)**

We determine the stablecoin price in the target region \([\bar{a}, \infty)\). From any point in this region, the platform jumps discretely to \( a^* \). Hence, the equilibrium price is \( p(a) = p(a^*) \) when \( a \in [0, \bar{a}] \). Next we determine the value of \( \delta^* \) such that \( p(a^*) = 1 \). The dynamic equation for the price is as follows

\[
p(A, C^*(A)) = (\delta^* + \ell(A, C^*(A)))p(A, C^*(A))dt + (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C^*(A) + dC)]
\]

\[
+ (1 - rdt)\lambda dt \mathbb{E}[p(SA, C)]
\]

Given that the price is constant over \([\bar{a}, \infty)\), we can substitute \( \mathbb{E}[p(A + dA, C^*(A) + dC)] \) with \( p(A, C^*(A)) \). Keeping only terms of order \( dt \), we obtain

\[
(r + \lambda)p(a^*) = (\delta^* + \ell(a^*))p(a^*) + \lambda \mathbb{E}[p(Sa^*)]
\]

Setting \( p(a^*) = 1 \) and solving for \( \delta^* \) we obtain (16).
Finally, we characterize the price dynamics in region $[0, \bar{a}]$. The price equation is

$$p(A, C) = (1 - r dt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C + dC)] + (1 - r dt)\lambda dt\mathbb{E}[p(SA, C)]$$  \hspace{1cm} (45)$$

When $a \in [0, \bar{a}]$, stablecoin owners enjoy no cash flow because the platform optimally sets $\phi = 0$ and liquidity benefits are equal to 0 because the price is not pegged to 1 since $p(a) < 1$. Using $dC = gC dt$, the first term on the right-hand side can be expanded using Ito’s Lemma:

$$\mathbb{E}[p(A + dA, C + dC)] = p(A, C) + p_A(A, C)\mu A dt + \frac{\sigma^2}{2} A^2 p_{AA}(A, C) dt + p_C(A, C) gC dt$$

$$= p(a) + \mu a p'(a) dt + \frac{\sigma^2}{2} a^2 p''(a) dt - gap'(a) dt$$  \hspace{1cm} (46)$$

To obtain the second line, we used the homogeneity of degree 0 of the price function, that is, $p\left(\frac{A}{\bar{a}}\right) \equiv p(A, C)$ to replace $p_A(A, C) = \frac{1}{\bar{a}} p'(a)$, $p_{AA}(A, C) = \frac{1}{\bar{a}^2} p''(a)$ and $p_C(A, C) = -\frac{A}{\bar{a}^2} p'(a)$. Plugging (46) into (45) and keeping only terms of order $dt$ we obtain

$$0 = -(r + \lambda)p(a) + (\mu - g)ap'(a) + \frac{\sigma^2}{2} a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]$$

which is equivalent to equation (17). This concludes the proof.

### D Proof of Proposition 3

**Step 1. Price function**

Our conjecture for the pricing function is

$$p(a) = \begin{cases} 
\sum_{k=1}^{3} b_k a^{-\gamma_k} & \text{if } 0 \leq a < \bar{a}, \\
1 & \text{if } a \geq \bar{a}.
\end{cases}$$

The issuance policy in region $[0, \bar{a}]$ is given by (13), that is, $g = 0$ when $\varphi = 0$. We first derive conditions on $\{\gamma_k\}_{k=1,3}$ such that HJB equation (17) is satisfied by our guess. We
have

\[ p'(a) = -\sum_{k=1}^{3} b_k \gamma_k a^{-(\gamma_k+1)}, \]

\[ p''(a) = \sum_{k=1}^{3} b_k \gamma_k (\gamma_k + 1) a^{-(\gamma_k+2)}, \]

\[ \mathbb{E}[p(Sa)] = \int_{0}^{\infty} p(e^{-s}a)\xi e^{-\xi s} ds = \int_{0}^{\infty} \sum_{k=1}^{3} b_k e^{s\gamma_k} a^{-\gamma_k} \xi e^{-\xi s} ds = \sum_{k=1}^{3} \frac{b_k \xi a^{-\gamma_k}}{\xi - \gamma_k}. \]

Replacing into (17) and equalizing terms proportional to \( a^{-\gamma_k} \), we obtain that for each \( k \in \{1, 2, 3\} \), \( \gamma_k \) must be a root of characteristic equation (20). The roots of this third-order polynomial are

\[ \gamma_k = -\frac{1}{2t_1} \left( t_2 + \zeta \nu R + \frac{\Delta_0}{\zeta \nu R} \right) \]

where

\[ \Delta_0 = t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27t_1^2 t_4, \]

\[ R = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = -1 + \sqrt{-3}, \quad \nu = \{0, 1, 2\}, \]

\[ t_1 = -\frac{\sigma^2}{2}, \quad t_2 = \mu + \frac{\sigma^2}{2} (\xi - 1), \quad t_3 = -\mu \xi + \frac{\sigma^2}{2} \xi + r + \lambda, \quad t_4 = -r \xi. \]

According to Descartes’ rule of sign, this polynomial has 2 positive roots and 1 negative root. Because the price must be bounded below by 0, the coefficients \( b_k \) corresponding to positive roots must be 0. We now call \( \gamma \) the negative root of this polynomial. It can be shown that \( \gamma < -1 \).

The price function is thus given by \( p(a) = ba^{-\gamma} \) for \( a \in [0, \bar{a}] \). To determine \( b \), we use the continuity of \( p \) at \( \bar{a} \). Setting \( p(\bar{a}) = 1 \) yields \( b = \bar{a}\gamma \).

**Step 2. Maximization Problem**

We now show that the maximization problem of the platform at date 0 is given by (21).
Rewriting equation (44), we obtain

\[ e(a^*) + p(a^*) = \frac{r - \delta^* + \lambda \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi}}{r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda \xi}{\xi + 1} \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi+1}} \]

We are left to substitute for \( \delta^* \) thanks to equation (16). We have

\[
\delta^* = r - \ell(a^*) + \lambda \left( 1 - E[p(a^*)S] \right),
\]

\[
= r - \ell(a^*) + \lambda \left[ \int_0^{\ln(a^*/\bar{\pi})} \xi e^{-\xi s} ds + \int_{\ln(a^*/\bar{\pi})}^{\infty} \left( \frac{a^*}{\bar{\pi}} \right)^{-\gamma} e^{s \gamma} e^{-\xi s} ds \right],
\]

\[
= r - \ell(a^*) + \lambda \left[ 1 - \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi} \right] - \lambda \frac{\xi}{\xi - \gamma} \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi}.
\]

Substituting for \( \delta^* \) into (47) and setting \( p(a^*) = 1 \), we obtain

\[
e(a^*) + p(a^*) = \frac{\ell(a^*) + \lambda \xi \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi}}{r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda \xi}{\xi + 1} \left( \frac{a^*}{\bar{\pi}} \right)^{-\xi+1}}
\]

Simple computations show this equation is equivalent to (21) if (22) holds, which we show below.

We are left to derive liability constraint (22). From Lemma 1, we have \( e(a) = 0 \) for all \( a \in [0, \bar{\pi}] \) and from Lemma 2, \( e(a) \) strictly increases with \( a \) for \( a \in [\bar{\pi}, \infty] \). Hence, limited liability holds for all \( a \) if \( e(\bar{\pi}) = 0 \). Using equation (14) with \( \varphi = 0 \) and \( p(a^*) = 1 \), this condition writes

\[
\left[ e(a^*) + p(a^*) \right] \frac{\bar{\pi}}{a^*} - 1 = 0,
\]

which is equivalent to (22). This concludes the proof.

### E Proof of Proposition 4

We first show that \( \bar{\pi} = 0 \) when \( \varphi = 1 \). This result follows from equation (14) in Lemma 2. Setting \( \varphi = 1 \) and \( p(a^*) = 1 \), it is clear that \( e(\bar{\pi}) \geq 0 \) for all \( a \geq \bar{\pi} \) if \( e(a^*) \geq 0 \). This latter condition is verified later in the existence result of Proposition 5.
For the second part of the proof, we rewrite Equation (44) with \( \bar{a} = 0 \) to obtain

\[
e(a^*) = e(a^*) + p(a^*) - 1 = \frac{\mu k - \delta^*}{r - \mu + \frac{\lambda}{\xi+1}}.
\]

Substituting for \( \delta^* \) thanks to equation (48), which becomes \( \delta^* = r - \ell(a^*) \) in this case, we obtain equation (23). This concludes the proof.

\section{Proof of Proposition 5}

Consider first the case \( \varphi = 0 \). Proposition ?? shows that an equilibrium with positive stablecoin value exists if there exist \((\bar{a}, a^*)\) with \( \bar{a} \leq a^* \) such that condition (22) holds. Using equation (??) to substitute for \( e(a^*) + p(a^*) \), this condition holds if there exists \( a^* \) and \( x \in [0,1] \) such that

\[
\ell(a^*)x - c - bx^{\xi+1} \geq 0, \quad \text{with} \quad c \equiv r + \frac{\lambda}{\xi+1} - \mu, \quad b \equiv \frac{\lambda \xi}{\xi+1} - \frac{\lambda \xi}{\xi - \gamma}.
\] (50)

To derive implications from this condition define \( H : x \mapsto \frac{x}{c + bx^{\xi+1}} \) and let \( x_{max} \) be the global maximum of \( H \) on \( [0,1] \). We have

\[
H'(x) \propto c - b \xi x^{\xi+1},
\]

which is strictly decreasing with \( x \) because \( b > 0 \) since \( \gamma < -1 \). Two cases are then possible. Either \( H'(1) = c - \xi b \geq 0 \) and \( x_{max} = 1 \) or \( H'(1) < 0 \) and \( x_{max} = \left( \frac{c}{b} \right)^{\frac{1}{\xi+1}} \) so that overall \( x_{max} = \min \left\{ 1, \frac{c}{b} \right\}^{\frac{1}{\xi+1}} \) and, for a given \( a^* \), a necessary condition for the desired equilibrium to exist is

\[
\ell(a^*) \geq \frac{c + bx_m^{\xi+1}}{x_m} = \frac{c + b \min \left\{ 1, \frac{c}{b} \right\}^{\frac{1}{\xi+1}}}{\min \left\{ 1, \frac{c}{b} \right\}^{\frac{1}{\xi+1}}}.
\] (51)

A necessary condition for (51) to hold is \( \ell(a^*) \geq c \) as stated in Proposition 5.

Consider now case \( \varphi = 1 \). According to Proposition 4, a solution exists if there exists \( a^* \) such that \( \ell(a^*) + \mu k - r \geq 0 \). Given that \( \ell \) is strictly increasing, this condition can hold
only if (24) holds. This concludes the proof.

G Proof of Proposition 6

We first states a series of Lemma and prove them at the end of this section. The argument for the absence of default follows from the discussion in the main text.

Lemma 8. The equity value \( e(a) \) is weakly convex, continuously differentiable, and stablecoin price function \( p(a) \) is continuous and increasing.

Lemma 9. If the equity value \( e(a) \) is linear over some interval \([a_L, a_U]\), the equilibrium issuance policy features a target demand ratio \( a^{\text{jump}} \in [a_L, a_U] \) such that the issuance policy for any \( a \in [a_L, a_U] \) is to jump at \( a^{\text{jump}} \).

Lemma 10. If \( e \) is strictly convex over some interval \([a_L, a_U]\), the equilibrium debt policy is smooth in that region. Furthermore, there is no MPE with positive stablecoin price if the equilibrium issuance policy is smooth everywhere.

Proposition 6 is then a corollary of the next result.

Lemma 11. If the coupon policy is optimally chosen at date 0, there exists \((\bar{a}, a^*)\) such that the equilibrium issuance policy is smooth over \([0, \bar{a}]\) and features a jump at \( a^* \) when \( a \in [0, \bar{a}] \).

We now provide a proof for these lemma.

Proof of Lemma 8. These properties follow from Lemma A.1 in DeMarzo and He (2021).

Proof of Lemma 9. We first show that if the equity value \( e \) is linearly increasing in \( a \) over some segment \([a_L, a_U]\), the equilibrium issuance policy cannot be smooth over this interval. We then show that for any such interval \([a_L, a_U]\), there is a single jump point.

Step 1 Non-smooth issuance The proof is by contradiction. Suppose \( dG_t = G(a)dt \) over \([a_L, a_U]\) with \( g(a) \equiv G(a)/C \) the stablecoin issuance rate per unit of stablecoins. With a smooth debt policy, \( dG_t = g_tC_tdt \) where \( g_t = G_t/C_t \), use equation (41) to rewrite the
The HJB equation governing stablecoin issuance as follows

\[(r + \lambda)e(a) = \max_{g(a)} \left\{ g(a)(p(a) - \varphi) + \mu ae'(a) + (\mu - \delta(a))\varphi \\
+ (g(a) + \delta(a))(e(a) - e'(a)a) + \sigma^2 \frac{a^2}{2} e''(a) + \lambda \mathbb{E}[e(Sa)] \right\}. \tag{52} \]

A smooth debt policy is optimal, if the first-order condition with respect to \(g\) is satisfied, that is, if

\[p(a) - \varphi = e'(a)a - e(a). \tag{53} \]

The assumption that \(e(a)\) is linear in \(a\) further implies that \(p'(a) = e''(a)a = 0\) and we denote \(p(a) = p\) in what follows. Hence, equation (52) simplifies to

\[(r + \lambda)e(a) = \mu k \varphi - \delta(a)p + \mu ae'(a) + \lambda \mathbb{E}[e(Sa)]. \tag{54} \]

We now establish a contradiction between equation (53) and (54) when \(e(a)\) is linear. Taking the first-order-derivative with respect to \(a\) of the terms in (54), we obtain

\[(r + \lambda)e'(a) = -\delta'(a)p + \mu e'(a) + \lambda \mathbb{E}[e'(Sa)S]. \tag{55} \]

The HJB equation for the stablecoin price is given by

\[(r + \lambda)p(a) = \ell(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \sigma^2 \frac{2}{a^2} p''(a) + \lambda \mathbb{E}[p(Sa)]. \tag{56} \]

which, for a constant \(p(a) = p\), simplifies to

\[(r + \lambda)p = \ell(a) + \delta(a)p + \lambda \mathbb{E}[p(Sa)]. \tag{57} \]
Combining equations (54), (55) and (55), we obtain

$$0 = (r + \lambda)(p(a) - \varphi + e(a) - e'(a)a) = \ell(a) + \delta(a)p + \lambda \mathbb{E}[p(Sa)] - (r + \lambda)\varphi + \mu k \varphi - \delta(a)p + \mu ae'(a)$$

$$+ \lambda \mathbb{E}[e(Sa)] + \delta'(a)ap - \mu ae'(a) - \lambda \mathbb{E}[e'(Sa)Sa],$$

$$= (\mu^k - r)\varphi + \ell(a) + \delta'(a)ap(a) + \lambda \mathbb{E}[p(Sa) - \varphi + e(Sa) - e'(Sa)Sa],$$

$$= (\mu^k - r)\varphi + \ell(a) + \delta'(a)ap(a). \quad (58)$$

The last equality follows from equation (58). We proved this relationship for segments where the equilibrium issuance policy is smooth. For segments over which the issuance policy features jumps, equation (58) shows that for any $a, a'$ in this segment, we have

$$e(a') = [e(a) + p - \varphi] \frac{a'}{a} - (p - \varphi) \quad (59)$$

Taking the first-order derivative with respect to $a'$ and then setting $a' = a$ we obtain equation (58).

We now establish a contradiction. Suppose first $a_L = 0$. Then, if $p \neq 1$ and thus $l(a) = 0$, it is immediate that equations (58) and (57) are inconsistent. If instead $p = 1$, these two equations imply that

$$(\mu^k - r)\varphi + l(a) - l'(a)a = 0$$

which means $\ell$ should have a specific form.

Suppose now $a_L > 0$. Suppose first $p(a) = p \neq 1$ in which case $\ell(a) = 0$ by definition. Equations (58) and (57) then imply that

$$\delta'(a) = \frac{r - \mu k}{ap} \varphi = -\lambda \mathbb{E}[Sp'(Sa)] \quad (60)$$

This equation may not hold because $r > \mu^k$ while $p' \geq 0$ by Lemma 8. Finally, suppose $p(a) = 1$. Equations (57) and (58) imply together that

$$\frac{\ell(a) - \ell'(a)a}{a} = \lambda \mathbb{E}[p'(Sa)S]. \quad (61)$$
We have

$$\mathbb{E}[p'(Sa)S] = \int_0^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds$$

$$= \int_{\ln(a/a_L)}^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds = \kappa \left( \frac{a}{a_L} \right)^{-(\xi+1)}$$

where $\kappa \equiv \int_0^\infty p'(e^{-s}a_L)\xi e^{-s(\xi+1)}ds$ is a positive constant. To obtain the second line, we used the fact that $p$ is constant over $[a_L, a_U]$. Thus, we must have

$$\ell(a) = \ell'(a)a + \lambda \kappa \left( \frac{a}{a_L} \right)^{-(\xi+1)}$$

for $a \in [a_L, a_U]$. A general solution to this equation is of the form

$$\ell(a) = \alpha a + \beta + f a^{-\xi-1}$$

with $f \geq 0$. Hence, assuming the issuance policy is smooth pins down a function form for $\ell$. This leads to a contradiction because $\ell(\cdot)$ is an exogenous function in this problem.

**Step 2 Single jump point** We now show there can only be one jump point $a_{jump} \in [a_L, a_U]$ if $e$ is linear over $[a_L, a_U]$. Suppose there are two such jump points (the argument generalizes for more jump points) labeled $a_{jump}^1$ and $a_{jump}^2$. Then, the single-peak property in Assumption 1 ensures there must be one jump point, say, $a_{jump}^1$ for which liquidity benefits $l(a)/a \ast A$ are larger than at $a_{jump}^2$. Hence, to maximize its date-0 value, the platform would strictly prefer jumping to $a_{jump}^1$ from any point in $[a_L, a_U]$ rather than to $a_{jump}^2$.

We are left to show that jumping to $a_{jump}^1$ instead of $a_{jump}^2$ is compatible with the equilibrium issuance policy. By Lemma 10 and 9, the issuance policy features jumps on $[a_L, a_U]$ only if equity value is linear and price is constant. Hence, from any state $a$ with jump point $a_{jump}^2$, we have

$$e(a) = \left[ e(a_{jump}^2) + p(a_{jump}^2) - \varphi \right] \frac{a}{a_{jump}^2} = \left[ e(a_{jump}^1) + p(a_{jump}^1) - \varphi \right] \frac{a}{a_{jump}^1}$$

Hence, jumping to $a_{jump}^1$ is also an optimal equilibrium issuance policy. This equality simply reflects the fact that the platform is indifferent ex-post between all points in $[a_L, a_U]$. 

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At date-0, however, the platform would choose jump point $a^1_{jump}$ as the sole jump point.

**Proof of Lemma 10.** We first show that if the equity value is strictly convex in $C$ over some interval, the issuance policy is smooth in this region. Given any debt level $\hat{C}$, equity holders have the option to adjust the stock of stablecoins to $C$ by issuing $C - \hat{C}$ at the price of $p(A, C)$. Therefore, by optimality of the debt issuance policy, the equity value at $\hat{C}$ must satisfy

$$E(A, \hat{C}) \geq E(A, C) + p(A, C)(C - \hat{C}).$$

(62)

To show that discrete repurchases are suboptimal, we prove that inequality (62) is strict if the equity value is strictly convex with respect to its second argument. Suppose to the contrary there exists $C' \neq C$ such that $E(A, C') = E(A, C) + p(A, C)(C - C')$. By strict convexity of $E$, we get that for all $x \in ]0, 1[

$$E(A, xC + (1 - x)C') < xE(A, C) + (1 - x)E(A, C') = E(A, C) + (1 - x)p(A, C)(C - C').$$

(63)

Using then condition (62) for $\hat{C} = xC + (1 - x)C'$, we obtain

$$E(A, xC + (1 - x)C') \geq E(A, C) + (1 - x)p(A, C)(C - C'),$$

which is a contradiction with (63). Thus, it must be that

$$E(A, C') > E(A, C) + p(A, C)(C - C').$$

Hence, any discrete issuance with $|C - C'| > 0$ would be suboptimal for shareholders, that is, the debt policy must be smooth everywhere $E$ is strictly convex in $C$, which

Second, we show that there cannot be an equilibrium with positive platform value and a smooth debt policy for all $a$. For the equilibrium issuance policy to be smooth it must be that equation (53) holds. The platform starts at date 0 if liquidity benefits can be captured in equilibrium. Two cases are possible given that $p$ is weakly increasing with $a$. First, there exists an interval $[a_L, a_U]$ over which the price is constant with $p(a) = 1$. Equation (53) then implies that $e$ is linear. We can then use Lemma 9 to show that the equilibrium debt policy features jump, a contradiction. The second case is that of a single point $\hat{a}$ for which
$p(\hat{a}) = 1$ and such that the platform spends strictly positive time at $\hat{a}$. Such a feature requires the platform to perform a control at $\hat{a}$. However DeMarzo and He (2021) show that such a policy cannot be part of an equilibrium in a region where the equity value is strictly convex.

Proof of Lemma 11. From Lemma 8, we know that since the equity value $e(a)$ is weakly convex, there must be a strictly ordered sequence $\{a^{(n)}\}_{n \geq 0}$ such that $a^{(0)} = 0$ and $\lim_{n \to \infty} a^{(n)} = \infty$ such that on each segment $[a^{(n)}, a^{(n+1)}]$, $e$ is either strictly convex or linear, with different convexity on two consecutive segments.

Second, we show that there is at least 1 segment with $e$ strictly convex (possibly empty), and one segment with $e$ linear. Equity value cannot be linear on segment $[0, a^{(1)}]$ unless $\varphi = 1$. The existence of a segment over which $e$ is linear follows from Lemma 10. Indeed, if no such segment exists, $e$ is strictly convex which implies the equilibrium policy is smooth leading to a contradiction. To prove the second point, suppose $e$ is linear on $[0, a^{(1)}]$. By Lemma 9 and 9, there must be $a^{jump} \in [0, a^{(1)}]$ such that the issuance policy is to jump at $a^{jump}$ from any point in $[0, a^{(1)}]$. For the jump to $a^{jump}$ to be part of the equilibrium issuance policy, it must be that for any $a \in [0, a^{(1)}]$

$$e(a) = \left[ e(a^{jump}) + p(a^{jump}) - \varphi \right] \frac{a}{a^{jump}} - (p(a^{jump}) - \varphi),$$

with $p(a^{jump})$ constant over $[0, a^{(1)}]$ and $p(a^{jump}) > \varphi$ unless $\varphi = 1$. Hence, when $a \to 0$ limited liability is violated except in the case $\varphi = 1$. This proves the equity value is strictly convex over $[0, a^{(1)}]$ unless $\varphi = 1$. In that case, the equilibrium may feature an equity value linear for all $a$.

The last step of the proof is to show there exists $\bar{a}$ such that the equity value is strictly convex over $[0, \bar{a}]$ and linear over $[\bar{a}, \infty)$. The characterization of the equilibrium issuance policy as a targeted Markov policy then follows from Lemma 10, 9 and 9. Let $\delta(a)$ be an interest policy that induces a non-zero MPE with issuance policy $d\hat{G}$ such that there exists a segment $[a^{(2)}, a^{(3)}]$ over which $e$ is strictly convex. Call it the original (coupon) policy for short. We want to show that there exists an alternative coupon policy $\tilde{\delta}(a)$ that induces a Markov equilibrium with issuance policy $d\tilde{G}$ such that $e$ has the desired properties and the date-0 platform value if strictly higher.
We first construct an alternative policy and its induced equilibrium. Let \( a^* \) be the target value in the first linear region \([a^{(1)}, a^{(2)}]\) for equity in the equilibrium induced by the original policy. Construct the alternative policy and the induced equilibrium as follows. Set \( \tilde{\delta}(a) = \delta(a) \) and \( \tilde{d}\tilde{G}(a, C) = dG(a, C) \) for all \( a \leq a^* \). For \( a \leq a^* \), set \( \tilde{d}\tilde{G}(a, C) = A/a^* - C \). As states with \( a > a^* \) are visited with zero probability, we can set \( \tilde{\delta}(a) = \delta(a) \) for these states. Furthermore, conjecture a linear equity value \( \hat{e} \) and a constant price \( \hat{p} \) for all \( a \in [a^{(1)}, \infty) \).

Next, we argue that the issuance policy \( d\tilde{G}(a, C) \) is an equilibrium policy induced by the alternative coupon policy \( \tilde{\delta}(a) \). The subspace \([0, a^*]\) is absorbing for the equilibrium induced by both policies because there are only downward jumps to \( A \) and the equilibrium issuance policy is such that \( a \leq a^* \) when \( a \in [a^{(1)}, a^{(2)}] \). Hence, the fact that \( dG(a, C) \) for \( a \in [0, a^{(2)}] \) is an equilibrium issuance policy induced by the original coupon policy implies that \( d\tilde{G}(a, C) \) for \( a \in [0, a^{(2)}] \) is also an equilibrium issuance policy induced by the alternative coupon policy. The fact that \( d\tilde{G}(a, C) \) is an equilibrium issuance policy on the rest of the state space, \( a \in [a^{(2)}, \infty) \) follows from the observation that \( \hat{e} \) is linear over \( a \in [a^{(1)}, \infty) \) and \( p \) is constant. This implies that jumping to any point in \( a \in [a^{(1)}, \infty) \) including \( a^* \) is an equilibrium issuance policy as we showed above. Our argument also implies that \( \hat{e}(a) = e(a) \) and \( \hat{p}(a) = p(a) \) for all \( a \in [0, a^*] \).

Third, we show that \( p(a) = 1 \) for \( a \in [a^{(1)}, a^{(2)}] \) in the equilibrium induced by the original policy, and thus \( \hat{p}(a) = 1 \) for all \( a \in [a^{(1)}, \infty) \). Equity value is linear over \([a^{(1)}, a^{(2)}]\) and the equilibrium issuance policy is to jump at \( a^* \in [a^{(1)}, a^{(2)}] \) when \( a \in [a^{(1)}, a^{(2)}] \). Hence, the price \( p(a) = p \) must be constant over \([a^{(1)}, a^{(2)}]\). Since \([0, a^*]\) is an absorbing subspace for the equilibrium induced by the original policy, it must be that \( p = 1 \). If not, investors never enjoy any liquidity benefit for \( a \in [0, a^*] \) and thus \( p(a) = e(a) = 0 \) for all \( a \in [0, a^*] \), which is a contradiction. To see this, suppose first \( p < 1 \). By monotonicity of \( p \), we have \( p(a) < 1 \) for all \( a \in [0, a^{(2)}] \) which implies investors never enjoy the liquidity benefit. Conversely, if \( p > 1 \) over \([a^{(1)}, a^{(2)}]\), we have \( p(a) = 1 \) for a unique \( a \in [0, a^{(1)}] \) because \( p \) is strictly increasing over \([0, a^{(1)}]\) since \( e \) is strictly convex (Step ??). With a smooth equilibrium issuance policy on \([0, a^{(1)}]\) this state is not visited with positive probability and thus investors enjoy liquidity benefit with zero probability, which again leads to a contradiction. Hence \( p(a) = 1 \) for \( a \in [a^{(1)}, a^{(2)}] \). This implies \( \hat{p}(a) = 1 \) for all \( a \in [a^{(1)}, \infty) \) in the equilibrium induced by the alternative policy.

Finally, we can show that the platform value at date 0 is higher under the alternative
policy than under the original policy. The platform’s value at date 0 is given by equation (10), which we rewrite here for convenience.

\[
E_0 = E \left[ \int_0^\infty e^{-rt} \ell(A_t, C_t) C_t 1_{p(A_t, C_t) = 1} + (\mu^k - r) \varphi C dt \right] | A_0, C_0 = 0
\]

In any equilibrium, liquidity benefits are only enjoyed when \( a \in [a^{(1)}, a^{(2)}] \) because \( p(a) = 1 \) for \( a \in [a^{(1)}, a^{(2)}] \). Under the alternative policy \( a^* \in [a^{(1)}, a^{(2)}] \) is reached immediately at date 0 by design because the equilibrium issuance policy is to jump to \( a^* \) when no stablecoins are outstanding (\( a = \infty \)). In the equilibrium induced by the original policy, the optimal choice at date 0 is some \( a^{**} > a^{(2)} \) by design of the original policy. Denote \( \tau_f \) the first (stochastic) time the platform enters the region \([a^{(1)}, a^{(2)}]\) under the original policy. We have

\[
E_0 = E[E^{-r\tau_f}] \hat{E}_0 + E \left[ \int_0^\infty e^{-rt} (\mu^k - r) \varphi C dt \right] < E_0
\]

because no liquidity benefit is enjoyed before the platform reaches \([a^{(1)}, a^{(2)}]\). The inequality follows from the fact that \( E[\tau_f] > 0 \) by design of the original policy and \( \mu^k < r \).

We showed that the original policy is strictly dominated. Hence, in an equilibrium induced by an optimal coupon policy, the issuance policy must belong to the class of targeted Markov policies.

\[\square\]

This concludes the proof of Proposition Proposition 6.

H Proof of Proposition 7

Points 1 and 2 The properties of \( e \) and \( p \) in Points 1 and 2 follow directly from the proof of Proposition 6 in Appendix G. The HJB equation (??) for the equity value follows from equations (??) in the proof of ?? . The HJB equation for the price is given by equation (56) in the proof of Proposition 6. For future reference, we also provide the smooth-pasting condition at the boundary \( \bar{a} \) between the smooth region and the target region. We have

\[
e'(\bar{a}) = \frac{e^* + p^*}{a^*}, \tag{64}
\]
with $e$ the value of equity in the region $[0, \bar{a}]$. The right-hand side is the derivative of the equity value in the target region. We showed in the proof of Proposition 6 that equation (??) must hold in this region. Computing the derivative with respect to $a$ yields the right-hand-side of (64).

**Point 3** Next, we derive the equilibrium stablecoin issuance rate in the smooth region. From HJB equation (52) and the optimality of a smooth debt policy, we obtained equation (53) in the proof of Lemma ?? . Taking the first-order derivative of $e$ in equation (52) at $g = 0$, we obtain

$$(r + \lambda)e'(a) = \mu e'(a) + \mu ae''(a) - \delta'(a)p(a) - p'(a)\delta(a) + \frac{\sigma^2}{2}a^2e''(a) + \sigma^2 ae'''(a) + \lambda \mathbb{E}[Se'(Sa)]$$

The HJB for the stablecoin price is given by

$$(r + \lambda)p(a) = \ell(a) + \delta p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda \mathbb{E}[p(Sa)].$$

with $\ell(a) = 0$ because the price $p$ is strictly below one by construction. We can then use (53) to obtain a condition on $g$. We have

$$0 = (r + \lambda)(p(a) + e(a) - e'(a)a)$$

$$= \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda \mathbb{E}[p(Sa)]$$

$$- \delta(a)p(a) + \mu ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda \mathbb{E}[e(Sa)]$$

$$+ \delta(a)p'(a)a + \delta'(a)p(a)a - \mu a^2e''(a) - \mu ae'(a) - \frac{\sigma^2}{2}a^3e'''(a) - \sigma^2 a^2e''(a) - \lambda \mathbb{E}[e'(Sa)Sa]$$

$$= -g(a)ap'(a) + \delta'(a)ap(a) + \mu \left( p'(a) - ae''(a) \right) + \frac{\sigma^2}{2}a^2 \left( p''(a) + e''(a) - ae'''(a) \right)$$

$$+ \lambda \mathbb{E}[p(Sa) + e(Sa) - e'(Sa)Sa]$$

$$= -g(a)ap'(a) + \delta'(a)ap(a).$$

The terms above can be set to 0 because equation (53) was shown to hold for all values of $a$ in the proof of Proposition 6. This implies directly that the last term is equal to 0.
Besides, taking derivatives of equation (53), we also have

\[ p'(a) = e''(a), \]
\[ p''(a) = e''(a) + e''(a), \]

which allow us to set other terms to 0. This proves our claim.

**Point 4.** We showed in ?? that a smooth issuance is strictly optimal when the equity value is convex. Hence, we are left to show that the jump to \( a^* \) is optimal when \( a \in [\overline{a}, \infty) \).

By Point 1 and ??, equity value is linear and price is constant. Hence, from any value of \( a \in [\overline{a}, \infty) \) the platform is ex-post indifferent about jumping to any value in \([\overline{a}, \infty)\). In particular, jumping to \( a^* \) is weakly optimal.

We are left to characterize conditions such that for any \( a \in [\overline{a}, \infty) \), deviating with a smooth issuance policy is suboptimal under condition (28). Given that the return to issuance is zero by construction, it is enough to check that equity owners prefer the equilibrium policy over inaction during time interval \( dt \). For state \((A, C)\) with \( A/C \geq \overline{a} \), the equilibrium value of equity is

\[ E(A, C) = E(A, C^*(A)) + (p(a^*) - \varphi)(C^*(A) - C) \]

If instead equity owners wait during \( dt \) before reverting to the equilibrium policy (one-step deviation), they enjoy

\[ \hat{E}(A, C) = (1 - rdt)(1 - \lambda dt)E \left[ E(A + dA, C + \delta C dt) + (\mu^k dt + C^*(A) - C + \delta C dt) \varphi \right] \\
+ (1 - rdt)\lambda dt E[S(A, C)\right] \]

(65)

When \( a \in [\overline{a}, \infty) \), the equilibrium equity value is given by

\[ E(A, C) = \frac{A}{\alpha^*} e(a^*) + (p(a^*) - \varphi)(C^*(A) - C) = \frac{e(a^*) + p(a^*) - \varphi}{\alpha^*} A - (p(a^*) - \varphi)C \]

(66)

Hence, we get

\[ E[A + dA, C + \delta C dt] = E(A, C) + \mu \frac{e(a^*) + p(a^*) - \varphi}{\alpha^*} \delta C dt - (p(a^*) - \varphi)\delta C dt \]
Plugging back into (65) and keeping only terms of order at least \( dt \), we obtain

\[
\hat{E}(A, C) = E(A, C) - (r + \lambda)E(A, C)dt + \mu [e(a^*) + p(a^*) - \varphi] C^*(A)dt - (p(a^*) - \varphi)Cdt + \mu^k \varphi Cdt + \lambda \mathbb{E}[E(S, A, C)]dt
\]

Equity owners do not deviate if and only if \( \hat{E}(A, C) < E(A, C) \), that is,

\[
(r + \lambda)E(A, C) \geq \mu [e(a^*) + p(a^*) - \varphi] C^*(A) - (p(a^*) - \varphi)C + \mu^k \varphi C + \lambda \mathbb{E}[E(S, A, C)]
\]

Rearranging the inequality above can be written

\[
(r + \lambda - \mu)e(a^*)C^*(A) \geq -(r + \lambda)(p(a^*) - \varphi)(C^*(A) - C) + \mu(p(a^*) - \varphi)C^*(A) - (p(a^*) - \varphi)C + \mu^k \varphi C + \lambda \mathbb{E}[E(S, A, C)]
\]

Using now equation (15) to substitute for \( e(a^*) \), we get

\[
\mu^k \varphi C^*(A) - (p(a^*) - \varphi)C^*(A) + \mu(p(a^*) - \varphi)C^*(A) + \lambda \mathbb{E}[E(S, A, C)]
\]

\[
\geq -(r + \lambda)(p(a^*) - \varphi)(C^*(A) - C) + \mu(p(a^*) - \varphi)C^*(A) - (p(a^*) - \varphi)C + \mu^k \varphi C + \lambda \mathbb{E}[E(S, A, C)]
\]

which we can finally rewrite as

\[
\left[ (r + \lambda)(p(a^*) - \varphi) + \mu^k \varphi \right] (C - C^*(A)) - (p(a^*))(\delta(a)C - p(a^*)C^*(A)) \leq \lambda \mathbb{E}[E(S, A, C)] - \lambda \mathbb{E}[E(S, A, C)]
\]

Let us derive the value \( \Delta E(A_t, C_t) \) of selling a quantity of stablecoins \( \Delta C_t dt \) instead of repurchasing \( C^*_t - C_t \) where \( C^*_t = A_t/a^* \) over the time interval \( dt \) for \( A_t/C_t \in [\bar{a}, \infty] \):

\[
\Delta E(A_t, C_t) = \Delta C_t p(A_t, C_t)dt + (1 - rdt)\mathbb{E}_t [E(A_{t+dt}, C_{t+dt})] - E(A_t, C_t).
\]

Using Ito’s lemma, we get

\[
\mathbb{E}_t [E(A_{t+dt}, C_{t+dt})] = E(A_t, C_t) + \mu A_t A_t E(A_t, C_t) dt + (\Delta C_t + \delta(A_t, C_t)) E(C(A_t, C_t) dt
\]

\[
+ \frac{\sigma^2}{2} E_{AA}(A_t, C_t) dt + \lambda dt(\mathbb{E}_t [E(SA, C_t)] - E(A_t, C_t)). \tag{67}
\]

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From equation (??), we get

\[ E(A_t, C_t) = \frac{e(a^*) + p(a^*)}{a^*}A_t - C_t p(a^*), \]

\[ E_A(A_t, C_t) = \frac{e(a^*) + p(a^*)}{a^*}, \]

\[ E_C(A_t, C_t) = -p(a^*), \]

\[ E_{AA}(A_t, C_t) = E_{CC}(A_t, C_t) = E_{AC}(A_t, C_t) = 0. \]

Substituting in (67), we get

\[
\mathbb{E}_t [E(A_{t+dt}, C_{t+dt})] = E(A_t, C_t) + \mu (E(A_t, C_t) + p(a^*)C_t)dt - (\Delta C_t + \delta(A_t, C_t)C_t)p(a^*)dt \\
+ \lambda dt \mathbb{E}_t [E(SA_t, C_t) - E(A_t, C_t)].
\]

We can follow the same steps as in Appendix ?? to obtain

\[
(r + \lambda)E(A_t, C_t^*) = -\delta(A_t, C_t^*)p(a^*)C_t^* + (p(a^*)C_t^* + E(A_t, C_t^*))\mu + \lambda \mathbb{E}[E(SA_t, C_t^*)].
\]

Consequently,

\[
(r + \lambda)E(A_t, C_t) = -\delta(a^*)p(a^*)C_t^* + (p(a^*)C_t^* + E(A_t, C_t^*))\mu + \lambda \mathbb{E}[E(SA_t, C_t^*)] + (r + \lambda)p(a^*)(C_t^* - C_t).
\]

Also note that \( p(A_t, C_t) = p(A_t, C_t^*) = p(a^*) \). Hence, the net benefit of a smooth deviation is

\[
\Delta E(A_t, C_t) = -\delta(a_t)p(a^*)C_t dt + \delta(a^*)p(a^*)C_t^* dt - (r + \lambda)p(a^*)(C_t^* - C_t)dt \\
+ \lambda [\mathbb{E}[E(SA_t, C_t^*)] - \mathbb{E}[E(SA_t, C_t^*)]] dt.
\]

Thus, as \( p(a^*) = 1 \), a deviation is optimal if

\[
-\delta(a_t) + \delta(a^*)a_t/a^* - (r + \lambda)p(a^*)(a_t/a^* - 1) + \lambda (\mathbb{E}[e(Sa_t)] - \mathbb{E}[e(Sa^*)]) > 0.
\]

In other words, to ensure time-consistency, it must be that

\[
\delta(a_t) \geq (r + \lambda)p(a^*) + (\delta(a^*) - (r + \lambda)p(a^*) - \lambda \mathbb{E}[e(Sa^*)]) \frac{a_t}{a^*} + \lambda \mathbb{E}[e(Sa_t)]
\]

for \( a_t \geq \bar{a} \). We have \( p(a^*) = 1 \) and by the HJB for \( p \) at \( a^* \), equation (??), we have

\[
(r + \lambda)p(a^*) = \ell(a^*)p(a^*) + \delta(a^*)p(a^*) + \lambda \mathbb{E}[p(Sa^*)].
\]
Thus we obtain the following condition to rule out a smooth deviation:

$$\forall a \in [\bar{a}, \infty), \quad \delta(a) \geq (r + \lambda) + \lambda \mathbb{E}[e(Sa)] - (\ell(a^*) + \lambda \mathbb{E}[e(Sa^*) + p(Sa^*)]) \frac{a}{a^*}$$

which is equivalent to (28) in the main text.

I Proof of ??

Maximization Program We first show that the solutions for $e$ and $p$ are as derived in the proof of Proposition ?? for the commitment case. The statement is obvious in the target region in which equity is linear and the price is constant and equal to 1. For the smooth region, observe that the stablecoin issuance rate is $g(a) = \delta'(a)p(a)/p'(a)$ by Proposition 7. By Assumption 3, $\delta'(a) = 0$, which implies $g(a) = 0$. This feature implies that HJB equations (??) and (??) for the equity value and the price in the smooth region are the same as in the commitment case, respectively (??) and (??).

Hence, given a policy set $\theta_0 = \{\delta, \delta^*, \pi, a^*\}$, the only difference when constructing the equity value and price function is the smooth pasting condition (64) at $\bar{a}$ derived in the proof of Proposition 7. Using the functional form for $e(a)$ in the smooth region given by (??), this condition becomes

$$-\frac{\gamma}{\pi} \left( [e(a^*) + p(a^*)] \frac{\pi}{a^*} - p(a^*) \right) = \frac{e(a^*) + p(a^*)}{a^*} \iff \frac{e(a^*) + p(a^*)}{a^*} = \frac{\gamma}{1 + \gamma} \frac{p(a^*)}{\pi} \quad (68)$$

We can now characterize the program of the platform at date 0. The platform maximizes its date-0 value subject to the limited liability constraint, the smooth-pasting condition (68), and the relevant conditions on parameter. We obtain
\[ E_0 = \max_{\theta_0} \frac{e(a^*) + p(a^*)}{a^*} A_0 \]  

subject to  

\[ \frac{e(a^*) + p(a^*)}{a^*} = \frac{\gamma}{1 + \gamma} \frac{p(a^*)}{a} \]  

\[ e(a) \geq 0, \quad \forall a \geq 0, \]  

\[ p(a^*) = 1, \]  

\[ \bar{a} \in [0, a^*], \quad \delta \geq 0, \quad \delta^* \geq 0. \]

Equations (71) and (73) show that limited liability constraint (71) is satisfied if and only if  

\[ [e(a^*) + p(a^*)] \bar{a} - p(a^*) a^* \geq 0. \]  

This condition is implied by equality (68) because \( \gamma < -1 \) which means (71) is redundant. 

Next, observe that peg constraint \( p(a^*) = 1 \) pins down \( \delta^* \). Equation (72) shows that to ensure \( p(a^*) = 1 \), we must have  

\[ \delta^* = r - \frac{\lambda \gamma}{\xi - \gamma} \left( \frac{a^*}{a} \right)^{-\xi} - \ell(a^*). \]  

Using then equation (72) to substitute for \( e(a^*) \), we obtain the following program  

\[ E_0/A_0 = \max_{\theta} \left\{ \frac{p(a^*)}{a^*} \left[ \frac{r - \delta^* - \frac{\lambda \gamma}{\xi - \gamma} \left( \frac{a^*}{a} \right)^{-\xi}}{r + \frac{\lambda}{\xi + 1} - \mu + \left( \frac{\lambda \xi}{\xi + 1} - \frac{\lambda \xi}{\xi - \gamma} \right) \left( \frac{a^*}{a} \right)^{-\xi}} \right] \right\}. \]  

The stablecoin price in the target region must be pegged because investors enjoy no liquidity benefit otherwise. Furthermore, because \( \gamma < -1 \), the sequential optimality of \( \bar{a} \) constraint implies that the limited liability constraint is always satisfied. Plugging (75) into (76) and
using \( p(a^*) = 1 \) we can rewrite the optimization problem as follows:

\[
E_0/A_0 = \max_{\{a,a^\star,\delta\}} \frac{\ell(a^*)/a^*}{c + b(\gamma) \left( \frac{a^*}{\theta} \right)^{-(\xi+1)}}
\]

subject to

\[
\frac{\gamma}{1 + \gamma \bar{a}} = \frac{\ell(a^*)/a^*}{c + b(\gamma) \left( \frac{a^*}{\theta} \right)^{-(\xi+1)}}
\]

(77)

\[\bar{a} \in [0,a^*],\]

\[\gamma \text{ is the lowest negative root of (32)}.\]

where \( c \) and \( b(\gamma) \) are the reduced-form parameters given in equation (??).

**Optimal Policies**

Building on our analysis in Section ??, constraint (77) can be written \( H(\bar{a},a^*) = \frac{\gamma}{1+\gamma} > 1 \) where \( H \) is defined in (??). We now call \( \bar{a}_{opt}(a^*,\gamma) \) the lowest value of \( \bar{a} \) if any that satisfies constraint (77). Following similar steps as in Section ??, this value exists only if

\[
\ell(a^*) \geq \frac{\gamma - c + b \min \left\{ 1, \frac{c}{\theta b(\gamma)} \right\}}{1 + \gamma \min \left\{ 1, \frac{c}{\theta b(\gamma)} \right\}^{1/\xi+1}}.
\]

(78)

This condition is more restrictive than condition (51) which obtains under full commitment. To see this, observe first that \( \gamma/(1 + \gamma) > 1 \) for all \( \gamma < -1 \). The second term on the right-hand-side of (78) is decreasing with \( \gamma \) because it is increasing with \( b \) and \( b'(\gamma) < 0 \). Hence, it is higher than the term on the right-hand-side of (51) because \( \gamma \) is equal to its maximum value under full commitment.

We showed in Section ?? that the variable \( \delta \) only affects \( \gamma \) and that \( \gamma \) strictly decreases with \( \delta \). Hence, we can write the problem as if the platform directly chose \( \gamma \).

\[
\max_{\{a^\star,\gamma\}} \frac{\gamma}{1 + \gamma \bar{a}_{opt}(a^\star,\gamma)},
\]

subject to \( \bar{a}_{opt}(a^\star,\gamma) \in [0,a^\star] \).

In what follows, we derive optimality constraints with respect to \( a^\star \) and \( \gamma \), assuming the
constraint does not bind. The optimality condition with respect to \( a^* \) writes

\[
0 = \frac{\partial \pi_{opt}(a^*, \gamma)}{\partial a^*}
\]

\[
\Leftrightarrow 0 = \ell'(a^*)\pi_{opt} - \frac{\gamma}{1 + \gamma} \left[ c - \xi b(\gamma) \left( \frac{\pi}{a^*} \right)^{\xi+1} \right]
\]

using the implicit characterization for \( \pi_{opt}(a^*, \gamma) \) and the steps for the derivation from Section ???. The optimality condition with respect to \( \gamma \) is

\[
0 = \frac{1}{(1 + \gamma)^2} \frac{1}{\pi_{opt}(a^*, \gamma)} - \frac{\gamma}{1 + \gamma} \frac{\partial \pi_{opt}(a^*, \gamma)}{\partial \gamma} \left( \frac{\pi_{opt}(a^*, \gamma)}{(1+\gamma)^2} \right)^2.
\] (79)

Applying the Implicit Function Theorem to the equation \( H(\pi, a^*) \), we obtain

\[
\frac{\partial \pi_{opt}(a^*, \gamma)}{\partial \gamma} = \frac{b'(\gamma) \left( \frac{\pi}{a^*} \right)^{\xi+1} H(\pi, a^*)}{c + b(\gamma) \left( \frac{\pi}{a^*} \right)^{\xi+1} + 1} + \frac{1}{(1+\gamma)^2}
\]

\[
= \frac{b'(\gamma) \left( \frac{\pi}{a^*} \right)^{\xi+1}}{c + b(\gamma) \left( \frac{\pi}{a^*} \right)^{\xi+1} + 1} + \frac{1}{(1+\gamma)^2}
\]

with \( H_x \) the derivative of \( H \) with respect to \( x = \frac{\pi}{a^*} \).

The first term on the right-hand-side of (79) is strictly positive. If the derivative of \( \pi_{opt}(a^*, \gamma) \) is negative, it is optimal to maximize \( \gamma \) subject to constraint (78) as in the full commitment case. If not, there exists a counteracting force implying that the highest possible \( \gamma \) and thus the lowest possible \( \delta \) are not optimal.

\section{Proof of ??}

The condition for \( \pi \) is given by

\[
-\frac{\gamma}{\bar{a}} \left( \frac{(e(a^*) + 1)\bar{a}}{a^*} - 1 \right) = \frac{e(a^*) + 1}{a^*}.
\]
Thus,

\[
\frac{\bar{a}}{a^*} = \frac{\gamma}{1 + \gamma e(a^*) + 1}.
\]

If \( \lambda = 0 \),

\[
\frac{\bar{a}}{a^*} = \frac{\gamma r - \mu}{1 + \gamma \ell(a^*)}.
\]

Since

\[
\lim_{\mu \to r} \gamma = -1,
\]

we can use the Hospital’s rule

\[
\lim_{\mu \to r} \frac{\bar{a}}{a^*} = \lim_{\mu \to r} \frac{1}{\gamma \ell(a^*)}
\]

where

\[
\gamma = \frac{1 - \frac{(\mu - \delta - \sigma^2/2)}{\sqrt{(\mu - \delta - \sigma^2/2)^2 + 2\sigma^2(r - \delta)}}}{\sigma^2}.
\]

As

\[
\gamma = \frac{\mu - \delta - \sigma^2/2 - \sqrt{(\mu - \delta - \sigma^2/2)^2 + 2\sigma^2(r - \delta)}}{\sigma^2},
\]

the implicit function theorem yields

\[
\lim_{\mu \to r} \gamma = \frac{1 - \frac{r - \delta - \sigma^2/2}{r - \delta + \sigma^2/2}}{\sigma^2} = \frac{1}{r - \delta + \sigma^2/2}.
\]

Thus,

\[
\lim_{\mu \to r} \frac{\bar{a}}{a^*} = \frac{r - \delta + \sigma^2/2}{\ell(a^*)}.
\]

For an equilibrium, we need \( \bar{a} \leq a^* \), which is satisfied if and only if \( \geq r - \ell(a^*) + \sigma^2/2 \).
K Solution for Centralized Equity Price with Collateral

We assume that $\varphi \leq 1$ so there is never residual value of collateral for equity holders after liquidation. We conjecture then verify that the equity value per unit of stablecoin is given by

$$e(a, k) = \begin{cases} 0 & \text{if } k < \varphi, \\ e(a) + k - k^*(a) & \text{if } k \geq \varphi, \end{cases}$$

and

$$e(a) = \begin{cases} 0 & \text{if } a < a, \\ c_0 + \sum_{k=1}^{3} c_k a^{-\gamma_k} & \text{if } a \leq a < a, \\ e(a^*)a/a^* + p(a^*)(a/a^* - 1) + k(a) - k^*a/a^* & \text{if } a \geq a. \end{cases}$$

where $\gamma_k$s are roots of the characteristic equation

$$r + \lambda = -\mu \gamma + \frac{\sigma^2}{2} (1 + \gamma) \gamma.$$

**Smooth Region** $a \in [a, a]$ Given the optimal collateral policy $k(a)$ and the result of Lemma 5, we can define $E(A, C) + K - K(A, C) \equiv E(A, C, K)$. In the smooth region, the HJB is given by

$$(r + \lambda)E(A_t, C_t) = p(A_t, C_t)G_t - M_t + \mu A_t E_A(A_t, C_t) + \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t)$$

$$+ (G_t + \delta_t C_t) E_C(A_t, C_t) + \lambda E[E(SA_t, C_t) + SK(A_t, C_t) - K(SA_t, C_t)].$$

where $E[dM_t/dN_t = 0] = M_t dt$ and $E[dM_t/dN_t = 1] = E[(k(Sa_t) - Sk(a))C_t]$. Given the definition of $e(a)C \equiv E(A, C)$, we can rewrite

$$(r + \lambda)e(a) = \max_g \left\{ gp(a) - m + \mu e'(a) + (g + \delta)(e(a) - e'(a)a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda E[e(Sa) + Sk(a) - k(Sa)] \right\}.$$ 

Thus, $m_t$ is such that

$$dK_t = C_t d(k(A_t/C_t) + k(A_t/C_t)dG_t + k(A_t/C_t)C_t \delta_t dt.$$
Since
\[ dK_t = \mu K_t \, dt + \sigma K_t \, dZ_t + dM_t, \]
we get
\[ M_t \, dt = k'(A_t/C_t)(\mu A_t dt - A_t(g_t + \delta)dt) + k''(A_t/C_t) \frac{\sigma^2}{2} A_t^2/C_t \, dt + k(A_t/C_t)(g_t + \delta)C_t dt - \mu k(A_t/C_t)C_t \, dt \]
and
\[ m_t = \mu a t k'(a_t) + \frac{\sigma^2}{2} a^2 t k''(a_t) + (g_t + \delta)(k(a_t) - k'(a_t)a_t) - \mu k(a_t). \]

Plugging it in the HJB yields
\[
(r + \lambda) e(a) = \max_g \left\{ \mu k(a) + gp(a) + (g + \delta)(e(a) - e'(a)a - k(a) + k'(a)a) + \mu a(e'(a) - k'(a)) \right. \\
+ \left. \frac{\sigma^2}{2} a^2(e''(a) - k''(a)) + \lambda E[e(Sa) + Sk(a) - k(Sa)] \right\}.
\]

The first order condition for \( g(a) \) becomes
\[
p(a) = -e(a) + e'(a)a + k(a) - k'(a)a. \tag{80}
\]

Thus, we get
\[
(r + \lambda - \delta)e(a) = (\mu - \delta)k(a) + (\mu - \delta)a(e'(a) - k'(a)) + \frac{\sigma^2}{2} a^2(e''(a) - k''(a)) + \lambda E[e(Sa) + Sk(a) - k(Sa)].
\]

Assume \( k(a) = \phi \). We get
\[
(r + \lambda - \delta)e(a) = (\mu - \delta)\phi + (\mu - \delta)ae'(a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda E[\max(0, S\phi - 1)].
\]
We can plug in the guess into the HJB and solve for the undetermined coefficients:

\[(r + \lambda - \delta) e(a) = (\mu - \delta) \varphi - (\mu - \delta) \sum_{k=1}^{3} \gamma_k c_k a^{-\gamma_k} \]

\[+ \frac{\sigma^2}{2} \sum_{k=1}^{3} (1 + \gamma_k) \gamma_k c_k a^{-\gamma_k} + \lambda \mathbb{E}[\max(0, S\varphi - 1)]. \tag{81}\]

Let us compute the expectation \(\mathbb{E}[e(Sa)]\) assuming that \(\varphi \geq 1\):

\[
\mathbb{E}[\max(0, S\varphi - 1)] = \int_{0}^{\ln(\varphi)} \left\{ (\varphi e^{-s} - 1) \xi e^{-\xi s} \right\} ds
= \frac{\xi \varphi}{\xi + 1} \left( 1 - \varphi^{-(\xi + 1)} \right) - \left( 1 - \varphi^{-\xi} \right)
= \frac{\xi \varphi}{\xi + 1} - 1 + \frac{\varphi^{-\xi}}{\xi + 1}.
\]

For the previous equation to hold, constant terms must be such that

\[(r + \lambda - \delta) c_0 = (\mu - \delta) \varphi + \lambda \mathbb{E}[\max(0, S\varphi - 1)]. \]

Additionally, terms in \(c_k a^{-\gamma_k}\) must be such that

\[r + \lambda - \delta = -(\mu - \delta) \gamma_k + \frac{\sigma^2}{2} (1 + \gamma_k) \gamma_k. \]

Thus, \(\gamma_k\)s must be the roots of that characteristic equation. The first boundary condition is given by

\[c_0 + \sum_{k=1}^{2} c_k a^{-\gamma_k} = 0. \tag{82}\]

The second boundary condition is given by

\[c_0 + \sum_{k=1}^{2} c_k \bar{a}^{-\gamma_k} = e(\bar{a}). \tag{83}\]

Thus, the two coefficients \(c_k\)s must satisfy conditions (82) and (83).
The value of equity at $a^*$ is equal to:

$$
E(a^*C_t, C_t) = p(a^*)E[dG_t] - E[dM_t] \\
+ (1 - rdt - \lambda dt)E[E(a^*C_t, C_t) | dN_t = 0] \\
+ (1 - rdt)\lambda dt E[E(a^*C_t, C_t) | dN_t = 1].
$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the equity holders issue/repurchase debt to compensate for all Brownian shocks and reissue maturing debt so that

$$
da_t/a_t = dA_t/A_t - dG_t/C_t - \delta^* dt = 0.
$$

Thus,

$$
E[dG_t | dN_t = 0] = E[C_t(dA_t/A_t - \delta^* dt)] = C_t(\mu - \delta^*)dt.
$$

Furthermore, they need to issue/repurchase collateral at market value such that

$$
dk_t/k_t = dK_t/K_t - dG_t/C_t - \delta^* dt = 0.
$$

Thus,

$$
E[dM_t | dN_t = 0] = E[dG_t K_t/C_t + \delta^* K_t dt - \mu K_t dt] = 0.
$$

The continuation value in this case is equal to

$$
E[E(a^*C_t, C_t)| dN_t = 0] = E[e(a^*)C_t | dN_t = 0] = e(a^*)C_t(1 + \mu dt).
$$

If there is a Poisson jump $dA_t/A_t = S_t - 1$ so that $k^*S_t > \varphi$, then the equity holders compensate this jump so that the state returns to $a^*$. Thus, they repurchase $C_t(1 - S_t)$ units of debt and $dG_t = C_t(1 - S_t - 1)$. In that case,

$$
E[E(a^*C_t, C_t)| dN_t = 1, S_t = \tilde{S}] = E[e(a^*)C_t | dN_t = 1, S_t = \tilde{S}] = e(a^*)C_t \tilde{S}.
$$
Therefore, we can write

\[
\mathbb{E}[E(A_t, C_t)| dN_t = 1] = e(a^*)C_t \int_0^{\ln(k^*/\phi)} e^{-s} \xi e^{-\xi s} ds.
\]

Also,

\[
\mathbb{E}[dG_t] = \mu C_t dt + \lambda dt \int_0^{\ln(k^*/\phi)} (e^{-s} - 1)C_t \xi e^{-\xi s} ds
\]

\[
= \mu C_t dt - \lambda C_t dt \left( \frac{\xi e^{-(\xi+1)s}}{\xi + 1} - e^{-\xi s} \right) \bigg|_0^{\ln(k^*/\phi)}
\]

\[
= \mu C_t dt + \lambda C_t dt \left( \frac{\xi}{\xi + 1} \left( 1 - \left( \frac{k^*}{\phi} \right)^{-\xi+1} \right) - \left( 1 - \left( \frac{k^*}{\phi} \right)^{-\xi} \right) \right).
\]

In the case of a Poisson jump, the collateral value jumps by the same amount as the crypto asset. Thus, there is no need to adjust the value of the collateral as the debt level is also adjusted for. Therefore,

\[
\mathbb{E}[dM_t] = 0.
\]

Regrouping all terms and scaling by \( C_t \), we get

\[
e(a^*) \equiv e(a^*) = p(a^*)(\mu - \delta(a^*))dt + p(a^*)\lambda dt \int_0^{\ln(k^*/\phi)} (e^{-s} - 1)\xi e^{-\xi s} ds
\]

\[
+ (1 - rd)\xi_0 dt \xi_0 (1 + \mu dt)
\]

\[
+ (1 - rd)\lambda dt \left( e(a^*) \int_0^{\ln(k^*/\phi)} e^{-s} \xi e^{-\xi s} ds \right).
\]

Removing terms in \( dt dt \) and scaling by \( dt \), we have

\[
(r + \lambda - \mu)e(a^*) = p(a^*)(\mu - \delta(a^*)) + \lambda \int_0^{\ln(k^*/\phi)} (p(a^*)(e^{-s} - 1) + e^{-s}e(a^*))\xi e^{-\xi s} ds.
\]

Another useful way to write that equation is

\[
(r - \mu)e(a^*) = p(a^*)(\mu - \delta(a^*)) + \lambda(\mathbb{E}[e(Sa^*)] - e(a^*)).
\]

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Solving for the integral, we get

\[(r + \lambda - \mu)E(a) = p(a)(\mu - \delta(a)) + \frac{\lambda \xi}{\xi + 1} \left( 1 - \left( \frac{k^*}{\varphi} \right)^{-\xi} \right) \left( p(a) + e(a) \right) - \lambda p(a) \left( 1 - \left( \frac{k^*}{\varphi} \right)^{-\xi} \right).\]

Putting all terms in \(e(a)\) on the left hand side gives

\[\left( r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda \xi}{\xi + 1} \left( \frac{k^*}{\varphi} \right)^{-\xi} \right) e(a) = p(a) \left( \mu - \delta(a) - \frac{\lambda \xi}{\xi + 1} \left( \frac{k^*}{\varphi} \right)^{-\xi} \right) + \lambda \left( \frac{k^*}{\varphi} \right)^{-\xi}.\]

L Solution for Centralized Stablecoin Price with Collateral

We conjecture then verify that the price of a stablecoin is given by

\[p(a) = \begin{cases} 
0 & \text{if } a < \underline{a}, \\
\sum_{k=1}^{2} b_k (a/\underline{a})^{-\gamma_k} & \text{if } \underline{a} \leq a < \overline{a}, \\
p(a^*) & \text{if } a \geq \overline{a}.
\end{cases}\]

where \(\gamma_k\)s are roots of the characteristic equation

\[r + \lambda = -\mu \gamma + \frac{\sigma^2}{2} (1 + \gamma) \gamma.\]

Smooth Region \(a \in [\underline{a}, \overline{a}]\) In the smooth region, the HJB is given by

\[(r + \lambda)p(a) = \ell(a) + \delta(a)p(a) - g(a)ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2} a^2 p''(a) + \lambda E[p(Sa)].\]

To solve for \(g(a)\), we need the first order derivative of \(e(a)\)

\[(r + \lambda)e'(a) = \mu k'(a) - \delta p'(a) + \mu a e''(a) + \mu e'(a) + \frac{\sigma^2}{2} a^2 (e'''(a) - k'''(a)) + \sigma^2 a(e''(a) - k''(a))
+ \lambda \text{E}[e'(Sa) + Sk'(Sa) - k'(Sa)S].\]
Assume \( k \)

Then, the HJB for \( g(a) \) from equation (??) and its derivatives:

\[
\begin{align*}
p(a) &= -e(a) + e'(a)a + k(a) - k'(a)a, \\
p'(a) &= e''(a)a - k''(a)a, \\
p''(a) &= e'''(a)a + e''(a) - k'''(a)a - k''(a).
\end{align*}
\]

Thus, we get

\[
0 = (r + \lambda)(p(a) + e(a)) - e'(a)a - k(a) + k'(a)a),
\]

\[
= \ell(a) + \delta p(a) - (g(a) + \delta)ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda E[p(Sa)]
\]

\[
+ \mu k(a) - \delta p(a) + \mu a e'(a) + \frac{\sigma^2}{2}a^2(e''(a) - k''(a)) + \lambda E[e(Sa) + Sk(a) - k(Sa)]
\]

\[
- \mu k'(a)a + \delta p'(a)a - \mu a^2 e''(a) - \mu e'(a)a - \frac{\sigma^2}{2}a^3(e'''(a) - k'''(a)) - \sigma^2 a^2(e''(a) - k''(a))
\]

\[
- \lambda E[e'(S)Sa + k'(S)Sa - k'(Sa)Sa]
\]

\[
- (r + \lambda)(k(a) - k'(a)a)
\]

\[
= (\mu - r - \lambda)(k(a) - k'(a)a) + \ell(a) - g(a)ap'(a) + \lambda E[S](k(a) - k'(a)a).
\]

That is,

\[
g(a) = \frac{(\mu - r - \lambda + \lambda E[S])(k(a) - k'(a)a) + \ell(a)}{ap'(a)}.
\]

Then, the HJB for \( p(a) \) becomes

\[
(r + \lambda)p(a) = (r + \lambda - \mu - \lambda E[S])(k(a) - k'(a)a) + \delta p(a) - \delta p'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda E[p(Sa)].
\]

Note that

\[
E[S] = \int_0^\infty e^{-s}\xi e^{-\xi}s ds = \frac{\xi}{\xi + 1}.
\]

Assume \( k(a) = \varphi \). Thus we get

\[
(r + \lambda)p(a) = (r + \lambda - \mu - \lambda E[S])\varphi + \delta p(a) - \delta p'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda E[S\varphi].
\]
We can plug in the guess into the HJB and solve for the undetermined coefficients:

\[
(r + \lambda - \delta)p(a) = (r + \lambda - \mu - \lambda E[S])\varphi - (\mu - \delta)\sum_{k=1}^{2} \gamma_k b_k a^{-\gamma_k}
\]

\[
+ \frac{\sigma^2}{2} \sum_{k=1}^{2} (1 + \gamma_k)\gamma_k b_k a^{-\gamma_k} + \lambda E[p(Sa)].
\]

(84)

For the previous equation to hold, the constant must be such that

\[
(r + \lambda - \delta)b_0 = (r + \lambda - \mu)\varphi.
\]

Furthermore, terms in \(b_k a^{-\gamma_k}\) must be such that

\[
r + \lambda - \delta = -(\mu - \delta)\gamma_k + \frac{\sigma^2}{2}(1 + \gamma_k)\gamma_k.
\]

The first boundary condition is given by

\[
b_0 + \sum_{k=1}^{3} b_k a^{-\gamma_k} = \varphi.
\]

(85)

The second boundary condition is given by

\[
b_0 + \sum_{k=1}^{3} b_k a^{-\gamma_k} = p(a^*).
\]

(86)

Thus, the two coefficients \(b_k\)s must satisfy conditions (85) and (86).

**Target Region** \(a \in [\bar{a}, \infty)\)  

At the target demand ratio \(a^*\), the HJB for the price of a stablecoin is given by

\[
p(a^*) \equiv p(a^*) = \ell(a^*)dt + \delta(a^*)p(a^*)dt + (1 - r dt - \lambda dt)E[p(a^*)|dN_t = 0]
\]

\[
+ (1 - r dt)\lambda dt E[p(a^*C_t/C_t)|dN_t = 1].
\]

If no jumps occur in the interval \([t, t + dt]\) (i.e., \(dN_t = 0\)), then the equity holders issue/repurchase debt to compensate for all Brownian shocks and reissue maturing debt so
that
\[ da_t = dA_t / A_t - dG_t / C_t - \delta^* dt = 0. \]

The continuation value in this case is equal to
\[ \mathbb{E} \left[ p(a^*) | dN_t = 0 \right] = p(a^*). \]

If there is a Poisson jump \( dA_t / A_t = S_t - 1 \), then there is immediate default the stablecoin holders get the collateral. Therefore, we can write
\[ \mathbb{E} \left[ p(a^*) | dN_t = 1 \right] = \int_0^\infty \varphi \xi e^{-\xi s} ds = \frac{\varphi \xi}{\xi + 1}. \]

Regrouping all terms, we get
\[ p(a^*) = \ell(a^*) dt + \delta^* p(a^*) dt + (1 - r dt - \lambda dt) p(a^*) + (1 - r dt) \lambda dt \frac{\varphi \xi}{\xi + 1}. \]

Removing terms in \( dtdt \) and scaling by \( dt \), we have
\[ (r + \lambda - \delta^*) p(a^*) = \ell(a^*) + \lambda \frac{\varphi \xi}{\xi + 1}. \]

### M Optimal \( a^* \) for Centralized Platform with Collateral

We have to find \( a^* \) defined as
\[ a^* = \arg \max_{a^*} \left\{ e(a^*; a^*) + p(a^*; a^*) \right\} = \arg \max_{a^*} \left\{ \ell(a^*) + \lambda \mathbb{E}[e(Sa^*; a^*) + p(Sa^*; a^*)] \right\} \]

where we write \( e(a) = e(a; a^*) \) and \( p(a) = p(a; a^*) \) to be explicit about the fact that \( a^* \) also enters as a parameter in the price functions. Note that because \( \pi \) is implicitly defined by
\[ -2 \sum_{k} \gamma_k c_k \bar{a}^{-\gamma_k - 1} = \frac{e(a^*) + p(a^*)}{a^*} \]

we get that
\[ \frac{\partial \pi}{\partial a^*} \approx \frac{\partial}{\partial a^*} \frac{e(a^*) + p(a^*)}{a^*} = 0. \]
That is, \( \bar{a} \) is already maximizing the value of \( (e(a^*) + p(a^*)) / a^* \). Assume that \( k(a) / \varphi \leq a^* / \bar{a} \). The expectation is given by

\[
\mathbb{E}[e(Sa^*; a^*) + p(Sa^*; a^*)] = \frac{\xi}{\xi + 1} \left( 1 - \left( \frac{k(a^*)}{\varphi} \right)^{-(\xi+1)} \right) (e(a^*) + p(a^*)) + \left( \frac{k(a^*)}{\varphi} \right)^{-(\xi+1)} \frac{\xi k(a^*)}{\xi + 1}.
\]

The partial derivative of that expectation is given by

\[
\frac{\partial \mathbb{E}[e(Sa^*; a^*) + p(Sa^*; a^*)]}{\partial a^*} \approx \frac{\partial k(a^*)}{\partial a^*} \approx \frac{\partial}{\partial a^*} \frac{e(a^*) + p(a^*)}{a^*} = 0.
\]

Thus, the first order condition for \( a^* \) is given by

\[
\ell'(a^*) a^* = \ell(a^*) + \lambda \mathbb{E}[e(Sa^*; a^*) + p(Sa^*; a^*)].
\]

N Proof of Lemma 5

Given any collateral level \( K \), governance token owners have the option to adjust collateral to \( K' \) by buying \( K - K' \) at cost of \( K - K' \). Therefore, the value of the governance token given \( K \) must be at least as high as the value that governance token owners would obtain by changing the collateral level to \( K' \)

\[
E(A, K, C) \geq E(A, K', C) + K - K'.
\]

Thus, 1 is the constant subgradient of \( E(A, K, C) \) and \( E(A, K, C) \) is linear and increasing in \( K \).

O No-Deviation in Target Region with Collateral

Consider \( (A, C, K) \) such that the platform is in the target region. Let \( \tilde{E}(A, C, K) \) be the value of deviating to a smooth issuanced \( \Delta C dt \) before reverting back to the conjectured equilibrium policy. For simplicity we denote \( K_t^* \) for \( K^*(A_t, C_t) \) as an argument of the value function. Both along the equilibrium path and in the deviation, it is assumed that the optimal collateral policy is \( K^*(A, C) = \varphi C \). Let \( p = p(a^*) \) be the price in the target region.
The equity value in the deviation is

\[ \tilde{E}(A_t, C_t, K_t^*) = p\Delta C_t dt - dM_t + (1 - rd)E[E(A_t+dt, C_{t+dt}, K_{t+dt}^*)] \]

\[ = (p - \varphi)\Delta C_t dt + (1 - rd)\left\{ E(A_t, C_t, K_t^*) + \mu AE_A dt + \Delta C_t \frac{E_C}{-p} dt + E_K \left( \mu^k \varphi C_t dt + \varphi \Delta C t dt \right) \right\} \]

\[ + \lambda dt \left[ E(SA_t, C_t, K_t^*) + K^*(A_t, C_t)(1 + \rho(S - 1)) - K^*(SA_t, C_t) - E(A_t, C_t, K_t^*) \right] \]

In the region where equity is flat, we have

\[ E(A_t, C_t, K_t) = E(A_t, C^*(A), K^*_t) + p(C^*(A) - C_t) - (K^*(A_t, C_t) - K_t) \]

\[ = A_t e(a^*) + \frac{A_t}{a^*} - pC_t - \varphi \frac{A_t}{a^*} + K_t \]

We have

\[ AE_A = C^*(A) [e(a^*) + p(a^*)] - \varphi C^*(A) \]

Substituting for \( AE_A \), the deviation is not profitable, that is, \( \tilde{E}(A_t, C_t, K_t^*) \leq E(A_t, C_t, K_t^*) \)
if and only if

\[ (r+\lambda)E(A_t, C_t, K_t^*) \geq \mu(e(a^*) + p(a^*))C^*(A) dt + (\mu^k - \mu) \varphi C_t dt + \lambda E[(SA_t, C_t, K_t^*) + \rho(S - 1) \varphi C_t] \]

We now replace the LHS of the inequality above. The conjectured equilibrium is to jump to \( C^*(A_t) \). Hence we have

\[ E(A_t, C_t, K_t^*) = E(A_t, C^*(A_t), K_t^*) + p(C^*(A_t) - C_t) + K^*(A_t, C_t) - K^*(A_t, C^*(A_t)) \]

\[ = E(A_t, C^*(A_t), K_t^*) + (p - \varphi)(C^*(A_t) - C_t) \]

To substitute for the value of equity at the target \( E(A_t, C^*(A_t), K_t^*) \), we use the derivations from the analysis without collateral. Adapting equation (43), we have

\[ (r+\lambda)E(A_t, C^*(A_t), K_t^*) = \mu(e(a^*) + p(a^*))C^*(A) + (\mu^k - \mu) \varphi C^*(A_t) + \lambda E[(SA_t, C^*(A_t), K_t) + \rho(S - 1) \varphi C^*(A_t)] \]
Replacing for \((r + \lambda)E(A_t, C_t, K_t^*)\), we get

\[
\mu(e(a^*) + p(a^*))C^*(A)dt + (\mu^k - \mu)\varphi C^*(A)dt + \lambda \mathbb{E}\left[ E(SA, C^*(A_t), K_t) + \rho(S - 1)\varphi C^*(A_t) \right]
\]

\[
+ (r + \lambda)(p - \varphi)(C^*(A_t) - C_t) \geq \mu(e(a^*) + p(a^*))C^*(A)dt + (\mu^k - \mu)\varphi C_t dt + \lambda \mathbb{E}\left[ E(SA_t, C_t, K_t^*) + \rho(S - 1)\varphi C^*(A_t) \right]
\]

Hence, overall the no-deviation condition becomes

\[
\left[ (r + \lambda)(p - \varphi) + (\mu^k - \mu)\varphi \right] (C - C^*(A)) \leq \lambda \mathbb{E}\left[ E(SA, C^*(A), K^*) - E[S, C, K^*] \right] + \lambda \rho \varphi [1 - S] (C - C^*(A))
\]

We can rewrite this expression as follows

\[
\left[ (r + \lambda)(p - \varphi) + (\mu^k - \mu)\varphi - \lambda \rho \varphi [1 - S] \right] (C - C^*(A)) \leq \lambda \mathbb{E}\left[ E(SA, C^*(A), K^*) - E[S, C, K^*] \right]
\]

When \(\varphi = 0\), we obtain the same expression as in the uncollateralized case for \(\delta = 0\). To get some intuition about the condition suppose \(C^*(A) < C\) so that the equilibrium policy is to repurchase \(C - C^*(A)\) units of stablecoins. The LHS is the sum of three terms. The first one is the net cost of the repurchase proportional to \(p - \varphi\). A repurchase frees up collateral. Hence, the net cost if equal to \(p - \varphi\). The second term corresponds to the net benefit from owning collateral. As collateral value grows at rate \(\mu^k\) but stablecoin issuance only grows at rate \(\mu\) together with stablecoin demand, there is a net windfall \((\mu^k - \mu)\varphi\) from each unit of stablecoin. If \(\mu^k - \mu > 0\), this force pushes against repurchasing stablecoins. The last term on the right-hand-side is negative. It says that when holding more stablecoins and thus more collateral, the effect of a Poisson shock is more severe.

\section*{P Proof of ??}

Let us derive the value \(\Delta E(A_t, C_t, \Delta K_t)\) of staying at a higher level of collateral \(K_t + \Delta K_t\) instead of jumping directly to \(K^*(A_t, C_t)\) when the issuance policy \(dG_t\) is smooth:

\[
\Delta E(A_t, C_t, K_t + \Delta K_t) = p_t G_t dt + (1 - r dt) \mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) \right] - E(A_t, C_t, K_t).
\]
Using Ito’s lemma, we get

\[
\mathbb{E}_t\left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) \right] = E(A_t, C_t, K_t + \Delta K_t) + \mu A_t E(A_t, C_t, K_t + \Delta K_t) dt \\
+ G_t E_C(A_t, C_t, K_t + \Delta K_t) dt \\
+ \mu (K_t + \Delta K_t) E_K(A_t, C_t, K_t + \Delta K_t) dt \\
+ \frac{\sigma^2}{2} E_{AA}(A_t, C_t, K_t + \Delta K_t) dt + \sigma^2 E_{KK}(A_t, C_t, K_t + \Delta K_t) dt \\
+ \lambda dt (\mathbb{E}_t[E(S A_t, C_t, S(K_t + \Delta K_t))] - E(A_t, C_t, K_t + \Delta K_t)).
\]

Following Lemma 5, we have that

\[
E(A_t, C_t, K_t + \Delta K_t) = E(A_t, C_t, K_t) + \Delta K.
\]

Thus,

\[
E_A(A_t, C_t, K_t + \Delta K_t) = E_A(A_t, C_t, K_t), \\
E_{AA}(A_t, C_t, K_t + \Delta K_t) = E_{AA}(A_t, C_t, K_t), \\
E_C(A_t, C_t, K_t + \Delta K_t) = E_C(A_t, C_t, K_t), \\
E_K(A_t, C_t, K_t + \Delta K_t) = 1, \\
E_{KK}(A_t, C_t, K_t + \Delta K_t) = 0,
\]

and

\[
\mathbb{E}_t[E(S A_t, C_t, S(K_t + \Delta K_t))] = \int_0^{\ln((K_t + \Delta K_t)/(\varphi C_t))} E(e^{-s} A_t, C_t, e^{-s}(K_t + \Delta K_t)) \xi e^{-\xi s} ds.
\]

Thus,

\[
\mathbb{E}_t[E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt})] = \mathbb{E}_t[E(A_{t+dt}, C_{t+dt}, C_{t+dt} + \Delta K_t) + \Delta K_t + \mu \Delta K_t dt \\
+ \lambda dt (\mathbb{E}_t[E(S A_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(S A_t, C_t, S K_t)] - \Delta K_t).
\]
Putting all of these together, we get

\[ \Delta E(A_t, C_t, K_t + \Delta K_t) = (1 - rdt)(\Delta K_t + \mu \Delta K_t dt) \]

\[ + (1 - rdt)\lambda dt[\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(SA_t, C_t, SK_t)] - \Delta K_t] - \Delta K_t \]

\[ = -(r + \lambda - \mu)dt\Delta K_t + \lambda dt[\mathbb{E}_t[E(SA_t, C_t, S\Delta K_t)] - \mathbb{E}_t[E(SA_t, C_t, SK_t)]]. \]

Furthermore,

\[
\lim_{\Delta K \to 0} \frac{\mathbb{E}_t[E(SA_t, C_t, S(K_t + \Delta K_t))] - \mathbb{E}_t[E(SA_t, C_t, SK_t)]}{\Delta K_t} = E\left( A_t, e \right) C_t/K_t, C_t - K(A_t e C_t/K_t, C_t) \xi \left( \frac{K_t}{\rho C_t} \right)^{-\xi} + \frac{\xi}{\xi + 1} \left( 1 - \left( \frac{K_t}{\varphi C_t} \right)^{-(\xi + 1)} \right) \]

\[ = \left( e(a_t e k_t) + \varphi - k(a_t e k_t) \right) \xi \left( \frac{K_t}{\varphi} \right)^{-\xi} + \frac{\xi}{\xi + 1} \left( 1 - \left( \frac{k_t}{\varphi} \right)^{-(\xi + 1)} \right). \]

Thus, no infinitesimal deviation from the level \( k(a) \) is optimal if and only if

\[ \lambda \left( e(a\varphi/k(a)) + \varphi - k(a\varphi/k(a)) \right) \xi \left( \frac{k(a)}{\varphi} \right)^{-\xi} + \frac{\lambda \xi}{\xi + 1} \left( 1 - \left( \frac{k(a)}{\varphi} \right)^{-(\xi + 1)} \right) = r + \lambda - \mu. \]

We can derive \( k'(a) \) as

\[
k'(a) = \frac{F_a}{F_k} = -\frac{e'(a\varphi/k) - k'(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right)^{-(\xi + 1)} - \frac{e'(a\varphi/k) - k'(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right)^{-(\xi + 1)} + \frac{e(a\varphi/k) + \varphi - k(a\varphi/k)}{k^2} \left( \frac{k}{\varphi} \right)^{-\xi}
\]

\[ = -\frac{e'(a\varphi/k) - k'(a\varphi/k)}{k} + \frac{e(a\varphi/k) + \varphi - k(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right) + \frac{e'(a\varphi/k) - k'(a\varphi/k)}{k^2} \left( \frac{k}{\varphi} \right)^{-\xi} + \frac{e(a\varphi/k) + \varphi - k(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right) - \frac{e(a\varphi/k) + \varphi - k(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right) \]

\[ = \frac{e'(a\varphi/k) - k'(a\varphi/k)}{k} + \frac{e(a\varphi/k) - k(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right) + \frac{e(a\varphi/k) - k(a\varphi/k)}{k} \left( \frac{k}{\varphi} \right). \]

Assume that the optimal policy is at the lower bound. That is, \( k(a) = \varphi \). The left-hand
side (marginal benefit) becomes
\[ \lambda \frac{e(a)\xi}{\varphi}. \]

Since \( e(a) = 0 \), it is always optimal to be at the lower bound at \( a = a \).

Assume that the optimal policy is at the upper bound. That is, \( k(a) = \varphi a / a \). The left-hand side becomes
\[ \frac{\lambda \xi}{\xi + 1} \left( 1 - \left( \frac{a}{a} \right)^{-\xi+1} \right) = \lambda \mathbb{E}[S 1 \{ S \geq a / a \}], \]

where \( \mathbb{E}[S 1 \{ S \geq a / a \}] \) is the expected residual value of the collateral after a Poisson shock. If the probability of hitting the default boundary is close to zero, that is, \( \left( \frac{a}{a} \right) - \xi \approx 0 \), then it is optimal to deviate from the upper bound to a lower collateral level as \( r > \mu - \frac{\lambda}{\xi+1} \).

As \( \lambda \mathbb{E}[S 1 \{ S \geq a / a \}] \leq \frac{\lambda \xi}{\xi+1} \), it is never optimal to be at the upper bound.

When the issuance policy \( dG_t \) is not smooth, the continuation value conditional on no Poisson shock is equal to
\[
\mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) | dN_t = 0 \right] = \mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt}) + \Delta K_{t+dt} | dN_t = 0 \right] = \mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt}) | dN_t = 0 \right] + \Delta K_t (1 + \mu dt). 
\]

If there is a Poisson shock, we get
\[
\mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) | dN_t = 1 \right] = \mathbb{E}_t \left[ E(SA_t, C_t, S(K_t + \Delta K_t)) \right].
\]

Thus,
\[
\mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt} + \Delta K_{t+dt}) \right] = \mathbb{E}_t \left[ E(A_{t+dt}, C_{t+dt}, K_{t+dt}) \right] + \Delta K_t (1 + \mu dt) \\
+ \lambda dt (\mathbb{E}_t \left[ E(SA_t, C_t, S(K_t + \Delta K_t)) \right] - \mathbb{E}_t \left[ E(SA_t, C_t, SK_t) \right] - \Delta K_t).
\]

Therefore, the condition is exactly the same as when the issuance policy \( dG_t \) is smooth.
Q  Proof of ??

The condition for \( \overline{a} \) is given by

\[
-\frac{\gamma}{\overline{a}} \left( \frac{e(a^*) + 1 - \varphi}{a^*} - (1 - \varphi) - \phi \varphi \right) = \frac{e(a^*) + 1 - \varphi}{a^*}.
\]

Thus,

\[
\frac{\overline{a}}{a^*} = \frac{\gamma}{1 + \gamma e(a^*) + 1 - \varphi} (1 - (1 - \phi) \varphi).
\]

If \( \lambda = 0 \),

\[
\frac{\overline{a}}{a^*} = \frac{\gamma (r - \mu)}{1 + \gamma \ell(a^*) - (r - \mu) \varphi} \varphi.
\]

\[
-\frac{\gamma (r - \mu)}{1 + \gamma \ell(a^*) - (r - \mu) \varphi} \frac{r - \mu - \varphi}{r - \mu}.
\]

Since

\[
\lim_{\mu \to r} \gamma = -1,
\]

we can use the Hospital’s rule

\[
\lim_{\mu \to r} \frac{\overline{a}}{a^*} = \lim_{\mu \to r} \frac{1}{\gamma \mu \ell(a^*)}
\]

and we get the same result as in Appendix J.

R  Proof of Lemma 5

Given any debt level \( \overline{C} \), vault owners have the option to adjust debt to \( \overline{C}' \) by buying \( \overline{C} - \overline{C}' \) at cost of \( p(A, C) \). Therefore, the value of the vault given \( \overline{C} \) must be at least as high as
the value that vault owners would obtain by changing the debt level to $\tilde{C}'$

$$E(A, C, \tilde{C}) \geq E(A, C, \tilde{C}') + (p(A, C) - \varphi)(\tilde{C}' - \tilde{C}).$$

Thus, $p(A, C) - \varphi$ is the constant subgradient of $E(A, C, \tilde{C})$ and $E(A, C, \tilde{C})$ is linear and decreasing in $\tilde{C}$.

### S Proof of Lemma 7

### T Proof of Proposition 8

In this section, we solve for the value of the vault given that $K^i(A_t, C_t, C^i_t) = \varphi C^i_t$. The value of a vault at $a^*$ is equal to:

$$V(a^* C_t, C_t, C^i_t) = p(a^*) E[dG^i_t] - E[dM^i_t] + (1 - rdt - \lambda dt) E[V(a^* C_{t+dt}, C_{t+dt}, C^i_{t+dt}) | dN_t = 0] + (1 - rdt) \lambda dt E[V(a^* C_{t+dt}, C_{t+dt}, C^i_{t+dt}) | dN_t = 1].$$

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the vault owners issue/repurchase collateral at market value such that

$$\frac{dk^i_t}{k^i_t} = \frac{dK^i_t}{K^i_t} = dG^i_t/C^i_t - s(a_t) = 0.$$

Thus,

$$E[dM_t | dN_t = 0] = dG^i_t K^i_t/C^i_t = s(a_t) K^i_t - \mu K^i_t dt.$$

The continuation value in this case is equal to

$$E[V(a^* C_t, C_t, C^i_t) | dN_t = 0] = E[v(a^*) C^i_t | dN_t = 0] = v(a^*) (C^i_t + s(a^*) C^i_t dt + dG^i_t).$$

If there is a Poisson jump, the vault is liquidated. In that case,

$$E[V(a^* C_t, C_t, C^i_t) | dN_t = 1] = E[\max(0, S\varphi - 1) C^i_t].$$
Regrouping all terms and scaling by $C_t^i$, we get

$$v(a^*) = p(a^*)dG_t^i / C_t^i - \varphi dG_t^i / C_t^i - s(a_t)\varphi dt + \mu \varphi dt$$
$$+ (1 - rdt - \lambda dt)v(a^*)(1 + s(a^*)dt + dG_t^i / C_t^i) + (1 - rdt)\lambda dt \mathbb{E}[\max\{0, S\varphi - 1\}]$$.

Removing terms in $dtdt$ and scaling by $dt$, we have

$$(r + \lambda - s(a^*))v(a^*) = (p(a^*) - \varphi + v(a^*))dG_t^i / C_t^i + \varphi(\mu - s(a^*)) + \lambda \mathbb{E}[\max(0, S\varphi - 1)]$$.

Given that $v(a^*) = \varphi - p(a^*)$, we get

$$(r + \lambda)(\varphi - p(a^*)) = \mu \varphi - s(a^*)p(a^*) + \lambda \mathbb{E}[\max(0, S\varphi - 1)]$$.

Thus, with $p(a^*) = 1$, we need

$$s(a^*) = \mu \varphi - (r + \lambda)(\varphi - 1) + \lambda \mathbb{E}[\max(0, S\varphi - 1)]$$.

Similarly, the value of an equity token at $a^*$ is equal to:

$$E(a^*C_t, C_t) = (s(a^*) - \delta(a^*))p(a^*)C_t^i dt$$
$$+ (1 - rdt - \lambda dt)\mathbb{E}[E(a^*C_t, C_t) | dN_t = 0]$$
$$+ (1 - rdt)\lambda dt \mathbb{E}[\max\{0, E(a^*C_t, C_t) + \min\{S\varphi - 1, 0\}C_t\} | dN_t = 1]$$.

If no jumps occur in the interval $[t, t + dt]$ (i.e., $dN_t = 0$), then the vault owners issue/repurchase stablecoins such that

$$da_t/a_t = dA_t/A_t - dG_t/C_t^i - s(a_t) = 0.$$ 

Thus,

$$E[dG_t | dN_t = 0] = (\mu - s(a_t))C_t^i dt.$$ 

The continuation value in this case is equal to

$$E[E(a^*C_t, C_t) | dN_t = 0] = E[e(a^*)C_t | dN_t = 0] = e(a^*)(1 + \mu dt)C_t^i.$$
If there is a Poisson jump, all vaults are liquidated before new ones are reopen and the equity token owners need to pay for the losses. In that case, equity after losses is equal to 0 when

\[ s = \log(\varphi/(1 - e(a^*))). \]

Thus, if \( e(a^*) < 1 \),

\[
E \left[ \max \{0, E(a^* C_t, C_t) + \min \{S\varphi - 1, 0\} C_t\} \right] dN_t = 1
\]

\[
= \int_0^{\log(\varphi/(1-e(a^*))} (e(a^*) + \min \{e^{-s}\varphi - 1, 0\}) C_t \xi e^{-\xi s} ds
\]

\[
= \int_0^{\log(\varphi)} e(a^*) C_t \xi e^{-\xi s} ds + \int_{\log(\varphi)}^{\log(\varphi/(1-e(a^*))} (e(a^*) + e^{-s}\varphi - 1) C_t \xi e^{-\xi s} ds
\]

\[
= e(a^*) C_t \left( 1 - \varphi^{-\xi} \right) + (e(a^*) - 1) C_t \left( \varphi^{-\xi} - \left( \frac{\varphi}{1 - e(a^*)} \right)^{-\xi} \right) + \frac{\varphi \xi}{\xi + 1} C_t \left( \varphi^{-\xi - 1} - \left( \frac{\varphi}{1 - e(a^*)} \right)^{-\xi - 1} \right)
\]

Otherwise, if \( e(a^*) \geq 1 \),

\[
E \left[ \max \{0, E(a^* C_t, C_t) + \min \{S\varphi - 1, 0\} C_t\} \right] dN_t = 1 = E(a^* C_t, C_t) + E \left[ \min \{S\varphi - 1, 0\} \right] C_t.
\]

Assuming that \( e(a^*) \geq 1 \), and regrouping all terms and scaling by \( C_t \), we get

\[
e(a^*) = (s(a^*) - \delta(a^*))p(a^*) dt + (1 - rt - \lambda dt)e(a^*) (1 + \mu dt) + (1 - rd t) \lambda dt \left( e(a^*) + E \left[ \min \{S\varphi - 1, 0\} \right] \right).
\]

Removing terms in \( dtdt \) and scaling by \( dt \), we have

\[
(r + \mu)e(a^*) = (s(a^*) - \delta(a^*))p(a^*) + \lambda E \left[ \min \{S\varphi - 1, 0\} \right].
\]

Since \( p(a^*) = 1 \) and

\[
s(a^*) = \mu \varphi - (r + \lambda)(\varphi - 1) + \lambda E[\max \{0, S\varphi - 1\}],
\]

we get

\[
(r + \mu)e(a^*) = \mu \varphi - \delta(a^*) - (r + \lambda)(\varphi - 1) + \lambda \left( \frac{\xi \varphi}{\xi + 1} - 1 \right).
\]

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Furthermore, as $\delta(a^*) = r - \ell(a^*)$, we get

$$(r - \mu)e(a^*) = \ell(a^*) - (r + \lambda - \mu)\varphi + \lambda \frac{\xi\varphi}{\xi + 1}$$

$$= \ell(a^*) - \left( r + \frac{\lambda}{\xi + 1} - \mu \right) \varphi.$$ 

\[ \text{U} \quad \text{No Loss of Generality for Policies without Brownian Component} \]

In this section, we show that considering a policy function $dG_t = g_tC_t dt$ instead of a more general functional form $dG_t = g_tC_t dt + \kappa_tC_t dZ_t$ is without loss of generality. We proof the case for the centralized uncollateralized protocol in the smooth region but the proof can be adapted to any case. The intuition of the results is straightforward: If fighting brownian shocks with $\kappa_t$ has any expected impact on the value of equity, it will also be taken into account in the smooth issuance decision $g_t$ and cancel out. With a stochastic term in $dG_t$ we can write the value of equity in the smooth region as

$$E(A_t, C_t) = E[p(A_t + dA_t, C_t + dG_t) dG_t] + (1 - rdt - \lambda dt) E[E(A_t + dA_t, C_t + dG_t)] + (1 - rdt) \lambda dt E[E(SA_t, C_t)].$$

Using Ito’s lemma and the fact that terms in $dt dt$ converge to 0 faster than terms in $dt$, we can get

$$E[p(A_t + dA_t, C_t + dG_t) dG_t] = E[p(A_t, C_t) g_tC_t dt + \sigma A p_A(A_t, C_t) \kappa_t C_t dt + \kappa_t^2 C_t^2 p_C(A_t, C_t) dt]$$

and

$$E[E(A_t + dA_t, C_t + dG_t)] = E[E(A_t, C_t) + \mu A E_A(A_t, C_t) dt + g_t C_t E_C(A_t, C_t) dt$$

$$+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t) dt + \frac{\kappa_t^2}{2} C_t^2 E_{CC}(A_t, C_t) dt + \sigma A_t \kappa_t C_t E_{AC}(A_t, C_t) dt].$$

The first order condition for $g_t$ is still given by

$$p(A, C) + E_C(A, C) = 0.$$
while the first order condition for $\kappa_t$ is given by

$$\sigma A p_A(A, C) + \kappa C p_C(A, C) + \kappa C E_{CC}(A, C) + \sigma A E_{AC}(A, C) = 0.$$  

As

$$p_A(A, C) + E_{AC}(A, C) = 0$$

and

$$p_C(A, C) + E_{CC}(A, C) = 0$$

the first order condition for $\kappa_t$ is satisfied if and only if the first order condition for $g_t$ is satisfied. The HJB for $p(A, C)$ becomes

$$(r + \lambda - \delta(A, C))p(A, C) = \mu A p_A(A, C) + (g(A, C) + \delta(A, C)) C p_C(A, C)$$

$$+ \frac{\sigma^2}{2} A^2 p_A A(A, C) + \frac{\kappa^2}{2} C^2 p_{CC}(A, C) + \sigma A \kappa C p_{AC}(A, C) + \lambda \mathbb{E}[p(SA, C)].$$

Given that $p(A/C) = p(A, C)$, we get

$$(r + \lambda - \delta(a))p(a) = \ell(a) + \mu a p'(a) - (g(a) + \delta(a)) a p'(a)$$

$$+ \frac{\sigma^2}{2} a^2 p''(a) + \frac{\kappa(a)^2}{2} (p''(a) a^2 + 2 p'(a) a) - \sigma \kappa(a) (p'(a) a^2 + p'(a) a) + \lambda \mathbb{E}[p(Sa)].$$

Similarly,

$$e(a) = -\delta(a) p(a) + \mu a e'(a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)]$$

and

$$e'(a) = -\delta'(a) p(a) - \delta(a) p'(a) + \mu a e''(a) + \mu e'(a) + \frac{\sigma^2}{2} a^2 e'''(a) + \sigma^2 a e''(a) + \lambda \mathbb{E}[e'(Sa)].$$
Using the first order condition for \( g(a) \) and its derivatives:

\[
\begin{align*}
    p(a) &= -e(a) + e'(a)a, \\
    p'(a) &= e''(a)a, \\
    p''(a) &= e'''(a)a + e''(a),
\end{align*}
\]

we get

\[
0 = (r + \lambda)(p(a) + e(a) - e'(a)a),
\]

\[
= \ell(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2} a^2 p''(a)
\]

\[
+ \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) - \sigma \kappa(p'(a)a^2 + p'(a)a) + \lambda \mathbb{E}[p(Sa)]
\]

\[
- \delta(a)p(a) + \mu ae'(a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)]
\]

\[
+ \delta'(a)ap(a) + \delta(a)p'(a)a - \mu a^2 e''(a) - \mu ae'(a) - \frac{\sigma^2}{2} a^3 e'''(a) - \sigma^2 a^2 e''(a) - \lambda \mathbb{E}[e'(Sa)a]
\]

\[
= \ell(a) + \delta'(a)ap(a) - g(a)ap'(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma \kappa(a)(p'(a)a^2 + p'(a)a).
\]

Thus, in the smooth part of the equilibrium, it must be that

\[
g(a) = \frac{\ell(a) + \delta'(a)ap(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma \kappa(a)(p'(a)a^2 + p'(a)a)}{ap'(a)}.
\]

Therefore, the HJB for \( p(a) \) is given by

\[
(r + \lambda)p(a) = \delta(a)p(a) - \delta'(a)ap(a) + \mu ap'(a) + \frac{\sigma^2}{2} a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]
\]

and none of the equilibrium price functions are affected by \( \kappa(a) \).

V No Commitment

In the main text, we assume that a centralized platform has some commitment power with respect to the coupon policy and the collateralization rule. As claimed in Section ??, we show that the platform has no value if it cannot commit at all.
Lemma 12. Without commitment, there is no MPE with strictly positive equity value $E(A, C, K) > 0$ and stablecoin price $p(A, C, K) > 0$.

The problem of a platform without any commitment to policies is similar to that of a firm that can choose whether or not to make coupon payments on perpetuity debt without defaulting. Once stablecoins/debt are issued, the firm strictly prefers not to make coupon payments because it already captured any benefits from issuance. As a result, the platform would always set the coupon payment to 0 ex-post, which means that stablecoin have no value ex-ante because the peg is not guaranteed. Lemma 12 thus shows that some commitment to a coupon policy is necessary; otherwise the platform and the stablecoin it issues have no value.

Proof of Lemma 12. Note that we have

$$dC_t = \delta_t C_t dt + G_t dt + (\mathcal{G}_t - \mathcal{G}_t^-)$$

and

$$dK_t = \mu K_t dt + \sigma K_t dZ_t + M_t dt + K_t^- (S_t - 1) dN_t + (\mathcal{M}_t - \mathcal{M}_t^-).$$

If $(\mathcal{G}_t - \mathcal{G}_t^-)$ and $(\mathcal{M}_t - \mathcal{M}_t^-)$, using Ito’s lemma we get

$$(r + \lambda)E(A_t, C_t, K_t) = p(A_t, C_t, K_t)G_t - M_t + \mu A_t E_A(A_t, C_t, K_t) + (G_t + \delta_t C_t) E_C(A_t, C_t, K_t) + (M_t + \mu K_t) E_K(A_t, C_t, K_t)$$

$$+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t, K_t) + \frac{\sigma^2}{2} K_t^2 E_{KK}(A_t, C_t, K_t) + \sigma^2 A_t K_t E_{AK}(A_t, C_t, K_t)$$

$$+ \lambda E\left[E(SA_t, C_t, SK_t)\right].$$

Therefore, if $E_C(A, C, K)$ is strictly negative, given a strategy $\delta(A, C)$, there is always an optimal deviation to a lower interest payment $\delta(A, C) - \Delta$ where $\delta > 0$ until $\Delta(A, C) = 0$. By Proposition I of DeMarzo and He (2021), $E(A, C, K)$ is strictly decreasing in $C$ when $p(A, C, K) > 0$.

Otherwise, we get

$$E(A_t, C_t, K_t) = E(A_t, C_t + \mathcal{G}_t - \mathcal{G}_t^-, K_t + \mathcal{M}_t - \mathcal{M}_t^-) + p(A_t, C_t + \mathcal{G}_t - \mathcal{G}_t^-, K_t + \mathcal{M}_t - \mathcal{M}_t^-)(\mathcal{G}_t - \mathcal{G}_t^-) - (\mathcal{M}_t - \mathcal{M}_t^-),$$

which is not impacted by $\delta_t$. 

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Similarly, without commitment to $K(A_t, C_t)$, it is always optimal to put no collateral in the platform as $r < \mu - \frac{\lambda}{\xi+1}$ and $K(A, C) = 0$. (See Appendix P.) If $\delta(A, C) = 0$, then $p(A, C, 0) < 1$ as $\ell(A, C) < r$.\[\Box\]