Can Stablecoins Be Stable? *

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Abstract

This paper provides a general model to assess the stability of stablecoins, cryptocurrencies pegged to a traditional currency. We study the problem of a monopolist platform that can earn seigniorage revenues from issuing stablecoins. We characterize stablecoin issuance-redemption and pegging dynamics under various degrees of commitment to policies. Even under full commitment, the stablecoin peg is vulnerable to large demand shocks. Backing stablecoins with collateral helps to stabilize the platform but does not provide commitment. Decentralization of issuance, combined with collateral, can act as a substitute for commitment.

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1 Introduction

A stablecoin is a cryptocurrency designed to maintain a peg with an official currency. Stablecoins can purportedly cater to investors' demand for alternative means of payment by combining the benefits of blockchain technology with the stability of traditional currencies. These cryptocurrencies have recently gained in popularity, with the market value of stablecoins growing from $3 billion in 2019 to $130 billion in June 2023 (CoinMarketCap, 2023). Confronted with this rapid development, along with multiple depegging events and crashes,[1] policymakers have started to introduce initiatives to regulate stablecoins.[2]

We propose a model to study the stability of various stablecoin protocols and the optimal design of stablecoins. In our model, a monopolistic stablecoin platform faces a demand for money-like assets. Like any money issuer, the platform has strong ex post incentives to overissue and devaluate stablecoins, which limits its ability to maintain the peg. A stablecoin issuer, however, can rely on new tools, such as smart contracts and decentralization, to potentially address this time-consistency problem inherent to monetary institutions. Our model encompasses an array of protocol designs that have emerged in practice, such as algorithmic supply adjustments, collateralization, and the decentralization of the issuance process.[3] Our main result is that the delegation of issuance to users can substitute for commitment when smart contracts are imperfect.

In our dynamic model, a monopolist platform caters to a time-varying demand for stablecoins, which depends on the platform's ability to maintain the peg. A sufficient statistic for demand is the marginal utility flow of users from holding stablecoins. This liquidity benefit varies with an exogenous demand shock and with the total stock of stablecoins, which may reflect network effects or demand satiation. To reflect a preference for stable money-like assets, we assume that users enjoy these liquidity benefits only if the stablecoin price is pegged to the unit of account. This liquidity premium proxies for various benefits users can enjoy in practice; for instance, they may use stablecoins as a means of payment on the blockchain, a safe and tax-efficient "parking space" for crypto volatility, or as a cheap vehicle for international remittances.[4]

The monopolistic platform chooses its policy to extract seigniorage revenues from these

[1]For example, the collapse of the Terra-Luna platform in May 2022.

[2]The US Congress introduced the Stablecoin Tethering and Bank Licensing Enforcement (STABLE) Act; the EU introduced the Markets in Crypto Assets (MiCA) Regulations; and the UK Treasury has launched a "UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence."

[3]Examples include Terra and Nubits for algorithmic stablecoins; FRAX, USDT, UDC, and DAI for collateralized stablecoins; and DAI for decentralized stablecoins.

[4]See European Central Bank (2021); Federal Deposit Insurance Corporation and Office of the Comptroller of the Currency (2021); and Gorton, Klee, Ross, Ross, and Vardoulakis (2022a).

liquidity benefits, taking into account how its decisions affect stablecoin demand and pricing. At any point in time, the platform can adjust its issuance-repurchase policy for stablecoins—similar to a central bank's open market operations—and its interest-rate policy paid in stablecoins to users, similar to interest on reserves. The platform may also collateralize stablecoin issuance with a safe asset. In this case, the platform must hold as collateral a fraction of the par value of outstanding stablecoins. To finance these policies, the platform can freely issue equity (tokens) that represents claims to the platform's future seigniorage revenues. On the demand side, users price the stablecoin competitively, based on their expectations of future platform policies.

As a benchmark, we first study a stablecoin platform that can fully commit to all of its policies thanks to immutable smart contracts. Even in this case, an uncollateralized (also called "algorithmic") stablecoin admits an equilibrium in which its price is zero. This classic monetarist result arises because no anchor ties the stablecoin to the unit of account, as both liquidity benefits and interest payments depend on the value of stablecoins. In this full-commitment benchmark, a second equilibrium exists in which the peg is robust to small negative demand shocks. In this equilibrium, the platform mints stablecoins when demand increases and issues equity tokens to buy back stablecoins when demand drops.[5] These policies correspond to the adjustment mechanisms of the Terra-Luna algorithmic protocol in practice. Finally, via its interest-rate policy, the platform can simultaneously maintain the peg and implement the supply that maximizes seigniorage revenues. Doing so, it implements a profit-maximizing version of the Friedman (1960) rule.

Although uncollateralized platforms can defend the peg against small negative demand shocks, they are vulnerable to large demand drops. After such shocks, the present value of seigniorage revenues may fall short of the cost of repurchasing stablecoins to maintain the peg. In this case, the platform's equity holders are neither willing to inject cash nor able to issue new equity tokens to finance the desired stablecoin buyback. As a result, the stablecoin loses its peg, and the value of equity tokens drops to zero. The prediction that equity tokens trade at zero when the stablecoin depegs is consistent with the collapse of Terra-Luna in 2022.[6] Overall, we stress that programming monetary policy via smart contracts does not allow the platform to avoid the consequences of limited liability.

Our analysis under full commitment further highlights the role of collateral for stability.

[5] This ability of stablecoin platforms to issue equity shares or "tokens" at no substantial cost is key to allowing pure algorithmic stablecoins to perform the equivalent of open market operations without holding any tangible assets on their balance sheets.

[6] Following a depegging event, the protocol reacted by minting increasing quantities of Luna equity tokens to burn/buy back Terra stablecoins. In Appendix A, we provide a descriptive analysis of the May 2022 stress for the five largest stablecoin platforms. See also Liu, Makarov, and Schoar (2023) for a more detailed account of the Terra-Luna downfall.

Repurchasing stablecoins frees up collateral held against these stablecoins. Hence, collateralization reduces the amount of equity tokens the platform must issue to finance stablecoin buybacks, thereby relaxing its limited liability constraint. In our model, holding collateral is costly, which induces a trade-off between the collateral cost and the stability it confers to the stablecoin. Specifically, for a fully collateralized stablecoin to be profitable, the liquidity benefits the platform captures must exceed the collateral holding cost. The existence condition for uncollateralized stablecoins emphasizes instead the need for demand growth, as the growth rate of issuance must exceed the interest payments to users necessary to maintain the peg.

In the second part of our paper, we analyze the stability of a stablecoin platform under a weaker form of commitment. In practice, not all contingencies can be programmed via smart contracts, and the platform has discretion over many policies.[7] We thus relax our assumption that all policies are programmed via smart contracts. While the platform can still commit to a collateral and interest-rate policy, it now chooses its issuance-repurchase policy sequentially. The platform then suffers from a durable-good monopolist problem (Coase, 1972). Issuance of new stablecoins affects the liquidity benefit enjoyed by past stablecoin buyers because these benefits depend on the total stock of outstanding stablecoins. Hence, new issuance dilutes legacy stablecoins, which the platform considers as a sunk cost when it lacks commitment. The platform's time-consistency problem evokes central banks' incentives to inflate away the stock of money, which is a government liability (Kydland and Prescott, 1977; Persson, Persson, and Svensson, 1987).

The time-consistency problem is so severe in our model that the stablecoin platform fails to earn any seigniorage revenues when it lacks commitment. This result is related to the leverage ratchet effect in trade-off models of debt. DeMarzo and He (2021) show that without commitment, a firm cannot capture any tax benefit of debt due to ex post incentives to overissue. In our model, stablecoins represent the platform's debt, and liquidity benefits correspond to the tax advantage of debt for a firm. A key difference, however, is that collateralization does not restore commitment in our model. In the trade-off model, fully securing debt prevents equity holders from diluting legacy debt via an increase in the likelihood of default. In our model, dilution operates instead via the liquidity benefit that depends on the total stablecoin stock, so collateralization does not neutralize dilution incentives and thus cannot restore commitment. Any policy aiming to restore commitment should instead neutralize the platform's price impact on

[7] In our model, the smart contract requires the exogenous demand shock as an input. In cryptocurrency jargon, such information qualifies as "off-chain" because it is not generated by the blockchain on which the stablecoin protocol runs. Incorporating off-chain information via "oracles" comes with implementation costs and vulnerabilities relative to on-chain information. See Czsun (2023) for details.

the stablecoin. We show how a programmable interest-rate rule can achieve this objective.

Finally, we consider the decentralization of issuance as a potential solution to the time-consistency problem of the platform. With DAI, for instance, anyone can mint stablecoins freely. So-called vault owners issue stablecoins by locking the required fraction of collateral in their vault. The platform charges these vault owners a seigniorage fee proportional to their outstanding stablecoin balance. We incorporate these elements in our framework and show that decentralization of issuance affects the platform's problem in two key ways. First, competitive vault owners arbitrage away price deviations from the peg. Second, the platform earns a rental-based income on the total stock of stablecoins rather than an issuance-based income on new stablecoins. Under full collateralization, these two features restore time consistency: the platform then sets fees charged to vault owners and interest rates paid to users that implement the full-commitment solution. The fact that a rental-based income for the platform restores commitment highlights the connection between our result and the leasing solution to the Coase (1972) problem.

Our main finding is that decentralization of issuance by the stablecoin platform can restore commitment. While we study optimal stablecoin design, this insight can apply more broadly to any intermediary with market power and money-like liabilities.

Related literature. Our paper contributes to an emerging literature on stablecoins. Several works have studied stablecoin protocols and their pegging mechanisms empirically (Arner, Auer, and Frost, 2020; Berentsen and Schär, 2019; Bullmann, Klemm, and Pinna, 2019; ECB, 2019; Eichengreen, 2019; G30, 2020; Ho, Darbha, Gorelkina, and Garcia, 2022). Gorton, Ross, and Ross (2022b) estimate the convenience yield on stablecoins and argue that further technological advances and reputation formation are required to make stablecoins money-like. Liu, Makarov, and Schoar (2023) study the crash of the Terra-Luna algorithmic stablecoin and attribute it to growing concerns about sustainability rather than manipulation. Kereiakes, Kwon, DiMaggio, and Platias (2019) and DiMaggio and Platias (2019) propose partial equilibrium models specific to the Terra-Luna stablecoin. Like them, we find that a stablecoin peg may be robust to small shocks, but we stress that uncollateralized stablecoins always risk losing their pegs after large shocks. Uhlig (2022) proposes a model of exchange dynamics to study the crash of Terra-Luna. As in our model, a positive probability of re-pegging prevents the price of the stablecoin from instantaneously collapsing to zero. Our model encompasses several other stablecoin designs that rely on collateralization and/or decentralization of issuance.

Closely related to our work, Li and Mayer (2022) propose a dynamic model of stablecoin in which platforms devalue after a negative shock to their reserves in order to avoid costly liquidation. Their analysis is similar to the full-commitment case of our model. However, we further show that the platform suffers from a time-consistency problem when managing issuance, which jeopardizes the stablecoin peg. Jermann and Xiang (2022) also focus on commitment issues attached to cryptocurrency issuance. We differ from their approach as we incorporate users' preference for a stable means of payment and study decentralization of issuance as a way to restore commitment. Similar to Lyons and Viswanath-Natraj (2020), we emphasize the role of arbitrage by vault owners for decentralized stablecoins' stability, but we also stress the importance of seigniorage fees.

Several works highlight the similarities between stablecoin platforms and banks as providers of means of payment. Routledge and Zetlin-Jones (2021) relate stablecoin platforms to central banks under a fixed exchange rate regime (Krugman, 1991; Obstfeld, 1996). They show that policies akin to suspension of deposit convertibility (Diamond and Dybvig, 1983) preempt self-fulfilling runs. Bertsch (2023) also studies stablecoin runs and highlights the link between stablecoin adoption and fragility. Zhang, Ma, and Zeng (2023) link stablecoins to ETFs and study the trade-off between stablecoin peg stability and financial fragility. Despite some analogies between these approaches, stablecoins are not redeemable in our model; instead, users may only convert stablecoins by selling them at the market price. This modeling difference allows us to focus on the incentive problem of the stablecoin issuer in a rich dynamic environment.

A recent literature on Central Bank Digital Currencies (CBDCs) has emerged (e.g., Ahnert, Hoffmann, and Monnet, 2022; Benigno, Schilling, and Uhlig, 2022; Brunnermeier, James, and Landau, 2021; Fernandez-Villaverde, Sanches, Schilling, and Uhlig, 2021). As electronic money issued by central banks, CBDCs can be seen as public stablecoins, while our model studies private stablecoins issued by profit-maximizing institutions. Furthermore, to the best of our knowledge, decentralizing issuance to solve the commitment problem has not been proposed in the context of CBDCs. Also related, a series of works has investigated the pricing of nonstable cryptocurrencies and also highlighted the role of transactional value in this respect (Biais, Bisière, Bouvard, Casamatta, and Menkveld, 2023; Garratt and Wallace, 2018; Schilling and Uhlig, 2019).

The time consistency problem we analyze affects monetary issuance beyond stablecoins. Early works in monetary economics (Calvo, 1978; Kydland and Prescott, 1977; Persson, Persson, and Svensson, 1987) show that a central bank has ex post incentives to engineer surprise inflation to dilute the legacy stock of money, which represents nominal liabilities of the government. In corporate finance, Admati, DeMarzo, Hellwig, and Pfleiderer (2018) identify the *leverage ratchet effect*, building on Black and Scholes (1973) and Myers (1977). Under discretion, a firm overissues debt relative to the commitment solution. These time-consistency problems can be traced back to Coase's (1972) conjecture that a durable good monopolist competes against its future self and does not capture any markup. In financial applications, the durable good is an issuer's liability that commands a tax advantage or a convenience yield. We show that delegating issuance to competitive agents can solve the platform's time-consistency problem, and we highlight the connection between decentralization and the leasing solution to Coase's problem.[8]

Finally, recent works have discussed the implications of crypto innovations in contexts where the Coase conjecture applies. Goldstein, Gupta, and Sverchkov (2022) show that issuing utility tokens dilutes the market power of a monopolist platform by transforming a flow of services into a durable good. Similarly, in the model developed by Cong, Li, and Wang (2020a), blockchain technology mitigates underinvestment by addressing a time consistency problem.[9] Brzustowski, Georgiadis-Harris, and Szentes (2023) show that "smart" contracts whereby a mediator imperfectly transmits information to the durable good seller can solve Coase's problem. Their focus on smart contracts as communication devices differs from ours: We instead view smart contracts as a set of immutable rules.

2 General Environment

This section lays out our model of stablecoins. Section 2.1 describes stablecoin demand from users who enjoy money-in-the-utility benefits from holding stablecoins. Section 2.2 and Section 2.3 present the tools available to a monopolistic platform that caters to such demand and its optimization problem, respectively. Finally, Section 2.4 discusses the mapping between our model and stablecoin designs in practice.

2.1 Stablecoin Demand

Time is continuous with $t \in [0, \infty)$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. There is a monopolistic stablecoin platform and a unit mass of homogeneous users. All agents consume a numeraire good—the unit of account—and discount the future at a rate of r. A stablecoin is an infinite-maturity asset issued by the platform, in supply C_t, that pays an interest rate δ_t to users at date t in stablecoins.

Stablecoin users enjoy direct utility from holding stablecoins and can trade those at

[8] In a trade-off model of debt, Hu, Veras, and Ying (2022) point out that rolling over short-term debt solves the commitment problem. In practice, however, stablecoins are long-lived assets, so we do not consider instantaneous maturity as a design choice.

[9] Other works that study token adoption and valuation include Cong, Li, and Wang (2020b); Hinzen, John, and Saleh (2022).

price p_t. A user with c_t stablecoins chooses his path of consumption $\{x_s\}_{s\geq t}$ of the numeraire good and stablecoin holdings $\{c_s\}_{s\geq t}$ to solve

$$\max_{\{x_s,c_s\}_{s\geq t}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \big\{ x_s + u(A_s, p_s c_s) \big\} ds \right], \tag{1}$$

$$\text{subject to} \quad x_s + p_s \dot{c}_s = e_s + \delta_s p_s c_s, \tag{2}$$

where $u(A_s, p_s c_s)$ is users' utility for real balances of stablecoins, $p_s c_s$. This money-in-the-utility specification captures in reduced form the use of stablecoins as a store of value or as means of payment. Variable A_s is an exogenous demand shock for which we provide dynamics below. In budget constraint (2), e_s represents a user's endowment of the numeraire good at date s. Users also receive δ_s stablecoins as interest payment for each stablecoin held, which corresponds to the second term on the right-hand side of (2).

We derive the stablecoin demand equation by solving users' problem (1). User optimization over consumption and stablecoin holdings implies the following Euler equation:

$$rp_t = \delta p_t + p_t u_c(A_t, p_t c_t) + \mathbb{E}_t \left[\frac{dp_t}{dt} \right]. \tag{3}$$

Users require a return r, equal to their discount rate, to hold stablecoins. The right-hand side of (3) features all three sources of return from holding stablecoins: interest payment δ_t, marginal utility for real balances $u_c(A_t, p_t c_t)$, and expected relative price appreciation of the asset, $\frac{1}{p_t}\mathbb{E}_t\left[\frac{dp_t}{dt}\right]$. The entire unit mass of homogeneous users chooses the same holdings, c_t, equal to total supply C_t by market clearing. By setting $c_t = C_t$ in equation (3), we obtain the dynamic demand equation for stablecoins:

$$rp_t = \delta p_t + p_t u_c(A_t, p_t C_t) + \mathbb{E}_t \left[\frac{dp_t}{dt} \right]. \tag{4}$$

We refer to (4) as the dynamic demand equation for stablecoins, because it maps the (real) stablecoin supply $p_t C_t$ to the stablecoin price dynamics.

Equation (4) highlights users' liquidity benefit $\ell_t := u_c(A_t, p_t C_t)$ as a sufficient statistic for stablecoin demand. This liquidity benefit is akin to a convenience yield, which captures indirect returns from holding an asset.[10] From here onward, we treat the liquidity benefit

[10] We rely on an ad hoc demand for stablecoins via money-in-the-utility function for simplicity, but a similar liquidity benefit emerges from microfounded theories of money demand. For instance, the new monetarist model of Choi and Rocheteau (2020) yields a similar dynamic demand equation as equation (4). In their version—without interest payment or demand shock—the liquidity benefit is given by $\ell_t = \alpha(v'(p_t C_t) - 1)$ when the bargaining power is given to sellers. In this formulation, the parameter α measures the frequency of decentralized meetings when coins are used, and v corresponds to the utility of buyers from consuming decentralized goods.

as a primitive and impose the following assumptions:

Assumption 1. *The liquidity benefit for stablecoins* $\ell(A, pC)$ *is*

(i) *continuously differentiable in both arguments;*

(ii) *homogeneous of degree* 0*;*

(iii) *such that* $\ell(A, x)x$ *has a unique interior maximum with* $\max_x \ell(A, x)x > 0$*;*

(iv) *equal to* 0 *if* $p \neq 1$.

To obtain Property (i), one needs only differentiability of the money-in-the-utility function u. Property (ii) is a technical assumption made to economize on one state variable. Consequently, we can define the liquidity benefit function as a function of the ratio $a = A/C$: $\ell(a) \equiv \ell(A/C, 1) = \ell(A, C)$. As we will show, Property (iii) implies that the platform's problem is well behaved because $\ell(A, pC)pC$ is a measure of seigniorage revenue flows for the platform. To satisfy this property, the liquidity benefit $\ell(A, pC)$ must decrease for large values of pC at a faster rate than pC or become negative.[11] Finally, Property (iv) captures users' preference for stable stablecoins. For simplicity, we assume that the liquidity benefit vanishes if the current price deviates from a reference peg price. This feature implies that the platform must defend the peg to capture seigniorage revenues. The peg is set to one without loss of generality to reflect market practice.[12].

Stablecoin demand is subject to exogenous shocks through the demand index A_t, which follows the law of motion:

$$dA_t = \mu A_t dt + \sigma A_t dZ_t + A_{t^-}(S_t - 1)dN_t, \tag{5}$$

where dZ_t is the increment of a standard Brownian motion and dN_t is a Poisson process with constant intensity $\lambda > 0$ adapted to $\mathcal{F}$. The size of a downward jump, $-\ln(S)$, is exponentially distributed with parameter $\xi > 0$ and the expected jump size is $\mathbb{E}[S - 1] = -1/(\xi + 1)$. The expected growth rate of stablecoin demand is thus given by $\mu - \lambda/(\xi + 1) < r$. The Poisson process generates large negative shocks to stablecoin demand. We denote A_{t^-} (resp. A_t) for the value of the demand shock just before (after)

[11] That the liquidity benefit $\ell(A, pC)$ should be decreasing in pC follows naturally from its definition as a marginal utility. Given that we consider ℓ_t to be a primitive, however, we do not impose that the liquidity benefit decreases with pC everywhere. For low values of pC, network effects in stablecoin adoption could increase the marginal utility for stablecoins as the supply increases.

[12] Without this "extreme-peg" assumption, a stablecoin could have value, even though there is no active management of the supply of stablecoins to stabilize its price (as for standard cryptocurrencies). For example, this preference for stable means of transaction could arise in models in which they benefit from being information insensitive (Dang, Gorton, and Holmstrom, 2019).

the jump. We will use similar notations for policy variables chosen by the platform.[13] The demand shock A_t could represent the exogenous price process for non-stablecoin cryptocurrencies such as Bitcoin. Given that investors use stablecoin as a store of value in the crypto world, the value of volatile cryptocurrencies could drive stablecoin demand.

2.2 Platform Policies

In this section, we describe the policy decisions of the stablecoin platform. In practice, platforms either directly control the issuance of stablecoins (e.g., USDT, USDC) or decentralize the issuance process (e.g., DAI). For clarity, we postpone the description of a decentralized issuance protocol to Section 5.

A centralized platform chooses the stablecoin supply C_t and the interest rate δ_t paid to users. Moreover, the platform may collateralize the issuance of stablecoins. The collateral asset is safe and delivers a rate of return $\mu^k < r$.[14] The difference $r - \mu^k$ can be interpreted as an unmodeled convenience yield that is lost to the platform when encumbering the asset as collateral. This assumption generates a collateral holding cost.[15] Finally, the platform can enter liquidation and distribute the collateral uniformly between users.

Definition 1 (Centralized Platform Policies). *The platform chooses a collateral ratio $\varphi \in [0, 1]$ such that φ worth of collateral backs each stablecoin; a sequence of stablecoin issuance and repurchase $\{d\mathcal{G}_t\}_{t\geq 0}$; interest rates $\{\delta_t\}_{t\geq 0}$ paid in stablecoins; and a stochastic liquidation time τ, at which the platform shuts down and distributes collateral to users uniformly.*

The platform controls supply via issuance to increase the stock of stablecoins or repurchases to retire them. Hence, the process $\{d\mathcal{G}_t\}_{t\geq 0}$ can take both positive and negative values. The stablecoin interest policy, whereby every user receives $\delta_t \geq 0$ extra stablecoins per stablecoin owned, is similar to interest payment on reserves by a central bank. These two policies combined imply the following law of motion for the stablecoin supply C_t:

$$dC_t = d\mathcal{G}_t + \delta_t C_t dt. \tag{6}$$

Both the active issuance $d\mathcal{G}_t$ and interest payments $\delta_t C_t dt$ contribute to supply changes.

[13] For a variable X, X_{t^-} denotes the left limit $X_{t^-} = \lim_{h\to 0} X_{t-h}$.

[14] Our assumption of a safe collateral asset comes with some loss of generality because some stablecoin platforms are often backed by risky cryptoassets. In this case, the collateral price would likely be correlated with the demand process A_t. It is intuitive, however, that such a correlation would reduce the usefulness of collateral as a hedge against demand fluctuations.

[15] Because it pays a return μ^k strictly lower than their discount rate, users consider collateral to be a dominated asset. Hence, their optimization problem (1) is unaffected by the availability of this asset.

The platform can hold collateral, which plays a role similar to central banks' reserves. To reflect stablecoin designs in practice, we assume that the platform must maintain a constant ratio φ between the value of its collateral holdings and the stock of stablecoins. This feature simplifies our analysis because the platform's collateral holding policy mimics its supply policy. The uncollateralized case, $\varphi = 0$ corresponds to a "pure algorithmic stablecoin" such as Terra, whereas the fully collateralized case, $\varphi = 1$ is sometimes referred to as a "narrow stablecoin" in reference to narrow banks (e.g. USDT, USDC). Our specification also encompasses partially collateralized designs such as FRAX.[16]

Finally, the platform can shut down, which transfers the collateral value φ backing each stablecoin to stablecoin users. We refer to this event as the liquidation of the platform.

2.3 The Platform's Problem

We solve for the policy sequence that maximizes the platform's date-0 value. As for any sequential plan, the platform's ability to implement ex post the optimal date-0 policy depends on its ability to commit. A central technological proposition of stablecoins is that rules can be programmed in advance via algorithms—i.e., so-called smart contracts. We thus first characterize the platform's problem under full commitment to policies chosen at date 0 and defer the analysis of commitment problems to Section 4.

Problem 1 (Full Commitment Problem). *Under full commitment, the stablecoin platform maximizes its date-*0 *value*

$$E_0 := \max_{\varphi,\tau,\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0\left[\int_0^\tau e^{-rt}\big(p_t d\mathcal{G}_t + \varphi\mu^k C_t dt - \varphi dC_t\big)\Big| A_0, C_{0^-} = 0\right], \tag{7}$$

subject to the law of motion (6)*, stablecoin pricing equation* (4) *with* $p_\tau = \varphi$*, and*

$$\forall t, \qquad \lim_{T\to\infty} \mathbb{E}_t[e^{-rT} p_T C_T] = 0, \tag{8}$$

$$\forall t, \qquad E_t := \mathbb{E}_t\left[\int_t^\tau e^{-r(s-t)}\big(p_s d\mathcal{G}_s + \varphi\mu^k C_s ds - \varphi dC_s\big)\Big| A_t, C_{t^-}\right] \geq 0, \tag{9}$$

the No-Ponzi-Game condition and limited liability constraint, respectively.

The platform maximizes the present value of its profit flow, which is equal to issuance gains $p_t d\mathcal{G}_t$ and the return on collateral $\mu^k \varphi C_t dt$ less new collateral purchase costs φdC_t.

[16] While some stablecoins collateralized by risky assets feature overcollateralization in practice ($\varphi > 1$), this would be suboptimal in our model because the platform uses a safe and liquid asset as collateral. Also, note that $\varphi = 1$ corresponds to full collateralization in our model because the implicit peg price is set at 1. If it were set at $\bar{p}$, full collateralization would mean $\varphi = \bar{p}$.

By definition, increasing (decreasing) the stablecoin supply by one unit requires (frees up) φ worth of collateral, which explains the last term of the profit flow in (7).

As a monopolist, the platform internalizes the pricing effect of supplying stablecoins. Hence, it treats the demand equation (4) as an optimization constraint. Furthermore, at liquidation date τ (if any), the stablecoin price must be equal to collateral ratio φ. Next, condition (8) follows from the transversality condition in users' optimization problem.[17] This constraint rules out bubbles in which the stablecoin (real) supply would grow at the discount rate of r. The No-Ponzi-Game condition (8) implies that the value of stablecoins must hinge on the creation of liquidity benefits for users and cannot be a pure bubble.

The platform is also subject to a limited liability constraint, given by equation (9). At any date t, the present value of expected profit flows (the platform equity) must be positive given current demand A_t and cumulative supply up to date t, C_{t^-}. When a policy generates a negative cash flow—for instance, to repurchase stablecoins ($d\mathcal{G}_t < 0$)—constraint (9) ensures that equity holders of the platform are willing to inject cash or able to issue equity against future profits to meet this outflow.

Equilibrium Selection In our model, there exists an equilibrium in which the stablecoin price does not exceed the collateralization ratio φ. In particular, an uncollateralized stablecoin ($\varphi = 0$) may trade at a zero price, similar to fiat money. Without collateral, there exists no anchor between the stablecoin and the unit of account, since dividends are paid in stablecoins. As a result, the zero-price equilibrium is self-fulfilling, as can be seen from the pricing equation (4). For any collateralization ratio $\varphi < 1$, the platform has no value in this equilibrium because it captures liquidity benefits only when the price is pegged to one. In what follows, we abstract from equilibrium multiplicity and study instead whether the platform can sustain another equilibrium with a positive value.[18]

[17]This condition states that a stablecoin holding plan is admissible if, for all t,

$$\lim_{T\to\infty} e^{-r(T-t)}\mathbb{E}_t[p_T c_T] = 0,$$

where c_t is a (representative) user holdings of stablecoins at date t. Condition (8) thus follows from the above equation and market clearing, $C_t = c_t$.

[18]Equilibrium multiplicity with fiat money has been thoroughly studied by the new monetarist literature (Williamson and Wright, 2011). We note that with full collateralization—a central part of our analysis—a platform could eliminate this equilibrium by offering 1:1 redemption rights to users. See also our discussion on this point in Section 3.3.

2.4 Mapping to Stablecoins in Practice

Redemption Rights In our model, users cannot directly redeem stablecoins with the platform. Instead, they must trade in the market to convert their stablecoins back to the unit of account. Since the monopolistic platform controls the market price, however, it implicitly sets the conversion rate for users. We later discuss the link between our supply-based model and redeemable stablecoins in Section 3.3 and footnote 32.

In practice, the largest stablecoin issuers (e.g., Tether and Circle) grant redemption rights to a restricted set of authorized participants in order to manage changes in demand, similar to exchange-traded funds. Zhang, Ma, and Zeng (2023) document frictions in this redemption process. Besides, most users can only exchange stablecoins in a secondary market. Griffin and Shams (2020) document outright stablecoin issuance from the Tether platform, similar to the open market operations of our model.

Interest Rates The platform pays an interest rate to stablecoin users, similar to a central bank's interest on reserves. In practice, Terra paid an interest rate of 20% before it collapsed; DAI's interest rate fluctuates between 1% and 7%. However, several large stablecoins, such as USDT or USDC, do not pay interest. This restriction constrains the platform's choices in our model, but our main insights remain (see later Remark 1).[19]

Equity Tokens Our model implicitly allows for the costless issuance of equity to finance stablecoin repurchases.[20] Whereas equity issuance costs may be significant for traditional firms, stablecoin platforms can easily issue blockchain-based equity tokens—also often referred to as "governance" tokens. In practice, this technological innovation has been instrumental in the emergence of fully uncollateralized—so-called algorithmic stablecoins—whereby stablecoin repurchases can be financed instantaneously with equity tokens issuance. Moreover, equity tokens are often touted as a way to foster diversity in ownership. In our model, however, ownership is irrelevant because all equity holders share the same profit-maximizing objective. Hence, we only need to keep track of the total value of equity, and governance decisions are taken by a representative equity holder.

Platform Competition Our model features a single stablecoin platform. In practice, several stablecoin platforms compete to cater to users' demand for alternative means

[19]It is often claimed that some stablecoins do not pay interest to avoid the regulatory burden of qualification as securities by the US Securities and Exchange Commission.

[20]Fully collateralized stablecoins ($\varphi = 1$) need not issue equity because repurchases can be financed entirely out of collateral holdings.

of payment. If multiple platforms coexisted, the liquidity benefit in our model would represent investors' residual demand for one platform's stablecoins after accounting for supply from other platforms. What matters is that the platform benefits from some market power that could arise, for example, because of payment network effects, as in Cong, Li, and Wang (2020a).

3 Full Commitment

In this section, we characterize optimal policies under full commitment—an environment with complete and immutable smart contracts that govern all policies in all contingencies. This benchmark provides a minimal set of necessary conditions for a stablecoin platform to have value and to be able to maintain parity. Since limited liability plays an important role in this analysis, we first consider a benchmark with unlimited liability in Section 3.1 and with limited liability in Section 3.2.

3.1 Unlimited Liability Benchmark

First, we assume that the platform is not bound by limited liability and ignore constraint (9) from Problem 1; that is, the platform's equity value may become negative. Equity holders can thus commit at date 0 to meet any future outflow, even if it exceeds the expected continuation profit from operating the platform. In this case, the platform never liquidates—that is, $\tau = \infty$.[21]

To derive the platform's optimal policies, rewrite the date-0 equity value of the platform in (7) as follows:[22]

$$E_0 = \mathbb{E}\left[\int_0^\infty e^{-rt}\Big(\ell(A_t, C_t)C_t\mathbb{1}\{p_t = 1\} + (\mu^k - r)\varphi C_t\Big)dt\,\middle|\, A_0, C_{0\text{-}} = 0\right], \qquad (10)$$

Equation (10) expresses the platform's equity value at date 0 as the expected present value of its profit flow. The first component of this flow is the seigniorage revenue, which is equal to the stablecoin supply C_t multiplied by investors' flow demand for stablecoins, $\ell(A_t, C_t)$. To obtain the profit flow, the collateral flow costs $(r - \mu^k)\varphi$ must be subtracted

[21] Unlimited liability means that equity holders have access to a large source of pledgeable income generated outside the platform to meet outflows. To draw a parallel in a macroeconomic context, a central bank with unconditional fiscal backing from the sovereign could also be considered as having unlimited liability. (Reis, 2015)

[22] Detailed computations to obtain (10) from (7) are reported in the proof of Proposition 1 in Appendix A.

from the seigniorage revenues. As the collateral asset returns less than the discount rate r, the platform faces an opportunity cost from holding collateral. Recall that users enjoy liquidity benefits only if the price is pegged to 1 (Assumption 1). Hence, the platform earns seigniorage revenues at date t only if the peg currently holds—that is, $p_t = 1$. From pricing equation (4), the peg holds if and only if

$$r = \delta_t + \ell(A_t, C_t) \tag{11}$$

The discussion above suggests that the platform should maximize its profit flow at any date t while maintaining the peg via its interest rate policy. To formally characterize optimal policies under commitment, we introduce the following definition.

Definition 2. *Given collateralization ratio φ and demand shock A_t, the stablecoin supply that maximizes the platform's profit flow at date t in (10) is given by $C^\star(A_t, \varphi)$ with*

$$C^\star(A, \varphi) \equiv \arg\max_C \left\{\ell(A, C)C - \varphi(\mu^k - r)C\right\}. \tag{12}$$

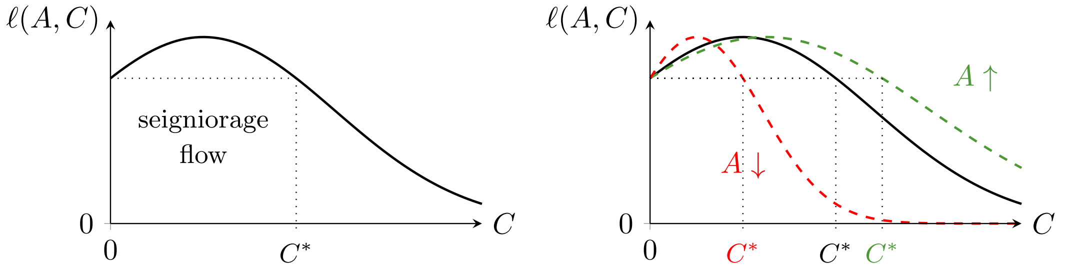
We define $a^\star(\varphi) \equiv A_t/C^\star(A_t, \varphi)$ as the constant "demand ratio" at this optimal supply.

When the platform implements the profit-maximizing supply, $C_t = C^\star(A_t, \varphi)$, the demand ratio $a_t = A_t/C_t$ is constant due to the homogeneity of liquidity benefit ℓ (Assumption 1). The following proposition describes the platform's optimal policies.

Proposition 1 (Full Commitment with Unlimited Liability). *The platform holds no collateral, $\varphi^\star = 0$, sets supply to $C_t = C^\star(A_t, 0)$ and pays interest rate $\delta^\star = r - \ell(a^\star(0))$ to peg the stablecoin price to 1.*

The platform chooses collateralization ratio φ and supply sequence $\{C_t\}_{t\geq 0}$ that maximize its equity value at date 0, given by (10). Under unlimited liability, this problem boils down to a static profit maximization at every date t since the platform faces no adjustment cost. Given φ, the platform thus sets date-t supply C_t to $C^\star(A_t, \varphi)$ as this maximizes its profit flow. The platform sets $\varphi^* = 0$ because there is no collateralization benefit that could justify bearing the collateral cost under unlimited liability: The platform can tap into unlimited resources outside the platform at no cost to meet outflows. The platform's optimal supply policy consists in maintaining a target demand ratio $a^\star(0)$ between demand for stablecoins A_t and its supply. To implement this target, the platform issues (buys back) stablecoins when demand A_t increases (decreases), in line with the stabilization mechanisms of algorithmic stablecoins in practice.

The argument above assumes that the platform can maintain the peg at all dates, as otherwise, it captures no seigniorage revenues. Proposition 1 shows how the platform



(a) Monopolist Profit Maximization

(b) Adjustments Following Demand Shocks

Figure 1: Demand for Stablecoins and Platform Profit Maximization. This figure displays examples of liquidity benefits along with the associated maximized stablecoin supply C^*. The functional form for the liquidity benefit is given by $\ell(a) = \exp(a/2(1 - a/2))$.

achieves this outcome by paying an interest rate $\delta^\star$ to stablecoin users to satisfy pricing equation (4). The platform can maintain the peg if the sum of the interest it pays and the equilibrium liquidity benefit $\ell(a^\star(0))$ matches the user's discount rate r. This relationship also highlights how the platform makes money from stablecoins: It pays on its liabilities a lower rate than the economy's discount rate—that is, $\delta^\star < r$—because users value stablecoins for liquidity benefits that are costless to the platform.

As shown in the proof of Proposition 1, the platform's total value at date 0 with unlimited liability is given by the net present value of seigniorage flows, that is,

$$E_0^\star = \frac{\ell(a^*(0))}{r - \mu + \frac{\lambda}{\xi+1}} \frac{A_0}{a^\star(0)}, \tag{13}$$

where the discount rate is adjusted for the growth rate of stablecoin demand $\mu - \lambda/(\xi+1)$.

Figure 1 illustrates the platform's optimal supply decision and its adjustments to demand shocks. In the unlimited liability benchmark, the platform maximizes the static monopolist revenue at each point in time, which corresponds to the surface of the dotted rectangle in Panel (a). Supply adjustments are depicted in Panel (b).

Remark 1. *Some stablecoins (e.g., USDT, USDC) do not pay interest. Equation (11) shows that with $\delta_t = 0$, supply C_t is pinned down by $r = \ell(A_t, C_t)$ for any date t, provided a solution exists. Hence, the platform cannot earn the monopolistic profit if it misses an interest rate policy as part of its toolkit and therefore relies on quantities alone to target the peg. Our analysis below would be otherwise unchanged.*

Remark 2. *The policy in Proposition 1 remains optimal without Property iv from Assumption 1, which states that users enjoy liquidity benefit only when the peg holds. In that case, only the real supply—rather than the nominal quantity—of stablecoin matters. Thus, the platform chooses the real supply to satisfy $p_t C_t = C^\star(A_t, 0)$, but the price level*

p_t is undetermined. Equation (11) becomes

$$r = \delta + \ell(A, C^\star(A, 0)) + \pi, \tag{14}$$

with π the deflation rate—the growth rate of the stablecoin price. Equation (14) then shows that deflation and interest payments are substitutable tools to sustain the optimal policy. This observation echoes a familiar result regarding the dual implementation of the Friedman rule in monetary economics. Unlike a benevolent government, however, our issuer maximizes profit and does not seek to equate the marginal benefit from money, ℓ_t, with its marginal cost of zero.

3.2 Limited Liability

We now consider Problem 1 in full, including limited liability constraint (9). The platform's equity value can no longer become negative, that is, equity holders' contribution to buy back stablecoins cannot exceed the present discounted value of future profits.

As a first step, we derive the platform's equity value at any date $t > 0$ for a general policy, following the same steps as when we derived equation (10). We obtain

$$E_t = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Big(\ell(A_s, C_s)C_s\mathbb{1}\{p_t = 1\} + (\mu^k - r)\varphi C_s\Big)ds\bigg| A_t, C_{t^-} = 0\right] - (p_t - \varphi)C_{t^-}. \tag{15}$$

The first term of (15) is the present discounted value of seigniorage revenues net of collateral costs. It represents the platform's total value at date t, similar to equation (10) for the date-0 equity value. At date t, however, the platform has C_{t^-} stablecoins outstanding. The second term, $(p_t - \varphi)C_{t^-}$, that enters E_t negatively, represents the net value of the platform's debt equal to the market value of stablecoins outstanding $p_t C_{t^-}$ minus the collateral backing, φC_{t^-}. Figure 2 provides a balance-sheet representation of equation (15), where the present discounted value of seigniorage revenues is represented as an (intangible) asset.

To see why the unlimited liability policy from Proposition 1 may lead to a negative equity value, compute (15) when the platform applies this policy to obtain[23]

$$E_t = \frac{\ell(a^\star(\varphi)) - (r - \mu^k)\varphi}{r - \mu + \frac{\lambda}{\xi+1}}\frac{A_t}{a^\star(\varphi)} - (p_t - \varphi)C_{t^-}. \tag{16}$$

[23]Steps for the derivation are in the proof of Proposition 1

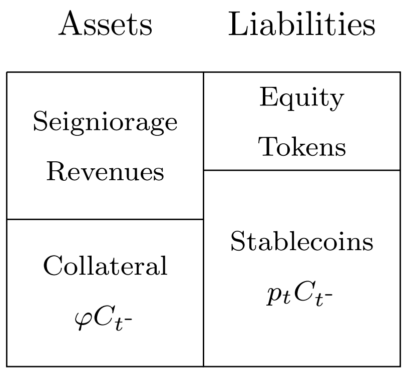


Figure 2: Balance Sheet of a Stablecoin Platform

For any outstanding stock of stablecoins C_{t^-} there exists a low enough demand shock A_t such that $E_t \geq 0$ cannot hold at the peg $p_t = 1$ unless $\varphi = 1$. Intuitively, if demand A_t jumps to a very low value, the total platform's value—the first term of (16)—falls below the net value of outstanding debt, $(1 - \varphi)C_{t^-}$, and equity value becomes negative.

Lemma 1. *The full-commitment policy from Proposition 1 violates limited liability constraint (9) if and only if $\varphi < 1$. A fully collateralized stablecoin platform ($\varphi = 1$) is profitable if and only if $\ell(a^\star(1)) \geq r - \mu^k$.*

The first result follows from the discussion above. For any level of collateralization $\varphi < 1$, the unlimited liability policy cannot be implemented. For fully collateralized (or narrow) stablecoins, however, limited liability is inconsequential. As stablecoin repurchases can then be fully financed by selling the platform's collateral, equity holders never need to inject cash. The second part of Lemma 1 follows from inspection of equation (16). A narrow stablecoin platform makes a profit if the liquidity benefit it captures from users, $\ell(a^\star(1))$, exceeds the collateral holding cost $r - \mu^k$. If this spread measures the collateral's own convenience yield, this condition implies that the stablecoin convenience yield must exceed that of the collateral at the equilibrium demand ratio.

Next, we analyze optimal policies and price dynamics for partially collateralized stablecoins. To do so, we consider a simplified version of Problem 1 by restricting the set of feasible policies. We first define this set and then explain the restriction. These policies are functions of state variable $a_t \equiv A_t/C_{t^-}$, called the demand ratio—the ratio of the current demand A_t to the outstanding stock of stablecoins C_{t^-} at date t.

Definition 3. *A target Markov policy (TMP) is given by collateralization ratio φ; liquidation threshold $\underline{a}$; peg threshold $\bar{a}$; target ratio $a^\star$; interest rate policy $\delta_t = \delta(a_t)$; and* *issuance policy*

$$d\mathcal{G}_t = \begin{cases} g(a_t)C_{t^-}dt & \text{if } \underline{a} \leq a_t < \overline{a}, \\ (a_t/a^\star - 1)\, C_{t^-} & \text{if } a_t \geq \overline{a}, \end{cases} . \tag{17}$$

with $\underline{a} < \overline{a} < a^\star$*. The issuance policy is said to be smooth over* $[\underline{a}, \overline{a}]$ *(of order dt). The policy supports a continuous stablecoin price function* $p_t = p(a_t)$ *with*

$$p(a) = \begin{cases} \varphi & \text{if } a \leq \underline{a}, \\ p(a) & \text{if } a \in [\underline{a}, \overline{a}], \\ 1 & \text{if } a \geq \overline{a} \end{cases} . \tag{18}$$

The Markov label for TMPs refers to the fact that policies depend only on two state variables: current demand shock A_t and outstanding stock of stablecoins C_{t^-}. Furthermore, once the issuance policy in (17) is normalized by the outstanding stablecoin stock C_{t^-}, it depends only on the demand ratio a_t. Restricting policies to be functions of state variables A_t and C_{t^-} comes with some loss of generality. However, the general problem is hard to solve because limited liability constraint (9) is a forward-looking constraint, which depends on the entire future sequence of actions by the platform. While focusing on TMPs only provides a partial solution to Problem 1, it greatly increases tractability and still allows us to highlight the main effect of limited liability on platform stability.[24]

A TMP is characterized by three regions for the demand ratio a: a liquidation region $[0, \underline{a}]$, a smooth-issuance region $[\underline{a}, \overline{a}]$, and a peg region $[\overline{a}, \infty]$. In the peg region, the platform implements a constant demand ratio $a^\star$, similar to the policy described in Proposition 1.[25] Unlike a strict target policy, however, a TMP allows the platform to abandon the target to accommodate the limited liability constraint. In the smooth-issuance region, the platform switches to a smooth-issuance policy and lets the price fall below 1. This region would be reached from the target $a^\star$ following a negative shock to demand. If this shock brings the demand ratio below the liquidation threshold $\underline{a}$, the

[24]The full solution to Problem 1 is not time consistent, which implies that optimal policies do not solve a standard recursive program with A_t and C_{t^-} as state variables. Marcet and Marimon (2019) develop a method to analyze programs similar to Problem 1 with forward-looking constraints. They show that recursive methods can still be used if one enriches the state space. Precisely, the recursive formulation should feature co-state variables that sum Lagrange multipliers associated with past forward-looking constraints. Note that the problem with unlimited liability is already time inconsistent, as we will discuss later, but without the limited liability constraint, the solution proves easy to characterize without such techniques (see Proposition 1).

[25]The optimal policy under unlimited liability from Proposition 1 is a TMP with $\underline{a} = \overline{a} = 0$. Note that the notation $a^\star$ refers to a generic target for a TMP, while $a^\star(\varphi)$ refers to the mapping introduced in Definition 2, which is also the target of the unlimited liability policy.

platform liquidates; that is, it distributes collateral to stablecoin holders. Definition 3 also includes the price function given by equation (18). While the stablecoin price is an equilibrium object, the monopolistic platform effectively sets the price subject to the dynamic demand equation (4).

The remainder of this section characterizes the TMP that maximizes platform's date-0 value E_0 under limited liability. To express E_0, let $E(A, C_-)$ be the platform's equity value as a function of state variables A and C_-. Guessing that E inherits the homogeneity property with respect to C_- from the TMP, we define $e(a) \equiv E(a, 1)$, the equity value per stablecoin outstanding. By definition of a TMP, at date 0, the platform issues $C^*(A_0) = A_0/a^\star$ at price $p(a^\star) = 1$. Its date-0 value is thus

$$E_0 = E(A_0, 0) = E(A_0, C^\star(A_0)) + (1-\varphi)C^\star(A_0) = A_0 \frac{e(a^\star) + 1 - \varphi}{a^\star}, \tag{19}$$

with $e(a^\star)$ the (endogenous) equity value at the target demand ratio.

To solve for the optimal TMP, we proceed in three steps. First, we characterize the equity value at target $e(a^\star)$. Second, we derive optimal issuance policy g in the smooth-issuance region $[\underline{a}, \overline{a}]$ and the interest rate policy for given TMP parameters $\{\underline{a}, \bar{a}, a^\star\}$ and collateralization ratio φ. Finally, we derive the optimal TMP—that is, the value of these parameters that maximize the date-0 value given in (19).

Step 1: Equity Value. To derive the platform's value at date 0, we must characterize the equity value dynamics over the state space for demand ratio a. In peg region $[\overline{a}, \infty)$, the issuance policy in (17) features a jump to target demand ratio $a^\star$. Hence, the equity value prior to the jump equals the post-jump equity value at the target, $E(A, C^\star(A))$, plus the issuance gains net of collateral costs:

$$E(A, C_-) = E(A, C^\star(A)) + (1-\varphi)(C^\star(A) - C_-). \tag{20}$$

Normalized equity value $e(a) = E(A, C_-)/C_-$ thus satisfies:

$$\forall a \in [\underline{a}, \bar{a}], \quad e(a) = e(a^\star)\frac{a}{a^\star} + (1-\varphi)\left(\frac{a}{a^\star} - 1\right). \tag{21}$$

In the smooth issuance region $[\underline{a}, \overline{a}]$, the dynamic equation for the equity value is:

$$e(a) = (p(a) - \varphi)g(a)dt + \mu^k \varphi dt - \varphi\delta(a)dt + (1 - rdt)\Big\{(1-\lambda dt)\mathbb{E}[e(a+da)] + \lambda dt \mathbb{E}[e(Sa)]\Big\}, \tag{22}$$

with da the change in the demand ratio over a period dt and Sa the new value of the demand ratio if a jump occurs during aperiod dt. The platform's profit flow during a period dt consists of three terms. The first term, $(p(a) - \varphi)g(a)$, corresponds to the platform's active issuance gains net of collateral purchases. The second term $\mu^k \varphi$ is the return on collateral. The third term, $-\varphi\delta(a)$, reflects the collateral cost of the interest rate policy: The interest rate payment increases the stablecoin supply which must be matched by collateral. Equation (22) can be used also to derive the equity value at the target $e(a^\star)$ by setting $a = a^\star$ and $g = 0$ (no issuance) in (22).

For a given TMP policy, equations (21) and (22) fully describe the dynamics of the platform's equity value together with boundary condition $e(\underline{a}) = 0$, whereby the equity value is zero when the platform liquidates.

Step 2: Optimal Issuance and Interest Rate. Lemma 2 characterizes the interest rate policy and the issuance policy in an optimal TMP.

Lemma 2. *In an optimal TMP, the interest rate in the peg region is*

$$\delta^\star = r - \ell(a^\star) + \lambda\left(1 - \mathbb{E}[p(Sa^\star)]\right). \tag{23}$$

In the smooth-issuance region $[\underline{a}, \overline{a}]$, the platform's equity value is zero, $e(a) = 0$. The platform pays no interest ($\delta(a) = 0$) and buys back stablecoins at rate

$$g(a) = -\frac{\mu^k \varphi}{p(a) - \varphi}. \tag{24}$$

Similar to equation (11) with unlimited liability, equation (23) determines the interest rate payment necessary to maintain the stablecoin price at 1 in the peg region. The new last term on the right-hand side of (23) reflects the extra interest rate the platform must pay to compensate users for the expected price devaluation. Indeed, any negative demand shock driving the demand ratio below $\overline{a}$ induces a loss of the peg. Intuitively, this interest rate premium is proportional to the probability λ of a negative jump in demand.[26]

Now, consider optimal policies in the smooth-issuance region $[\underline{a}, \overline{a}]$. The platform's value rests on its ability to capture investors' liquidity benefits as seigniorage revenues. Hence, it minimizes the time spent in region $[\underline{a}, \overline{a}]$ where the peg is lost ($p(a) < 1$) and investors enjoy no such benefit. To reach the peg region, the platform maximizes stablecoin repurchases and pays no interest to decrease the supply of coins at the fastest feasible rate. The platform must spend $p_t - \varphi$ to buy back one stablecoin as the operation

[26]High interest rates offered by the platform can indicate a larger probability of a price crash. Before it collapsed, the Terra-Luna platform was offering interest rates above 20%.

frees up φ worth of collateral. Hence, given that it earns collateral return $\mu^k\varphi$ for each stablecoin outstanding, equation (24) defines the maximum repurchase rate compatible with limited liability.

Step 3: Optimal TMP. Lemma 2 characterizes a TMP for given values of the threshold $\{\underline{a}, \overline{a}, a^\star\}$ and collateralization ratio φ. To solve for the optimal TMP, we must express the platform's date-0 value, given in (19), as a function of $\{\underline{a}, \overline{a}, a^\star\}$ and φ. This requires solving for $e(a^\star)$, the equity value following date-0 issuance.

As a first step, we derive the Hamilton-Jacobi-Bellman (HJB) equation for the equity value at the target demand ratio from equation (22) (see derivations in Appendix A.5):

$$(r + \lambda - \mu)e(a^\star) = \mu^k\varphi + \mu(1 - \varphi) - \delta(a^\star) + \lambda\mathbb{E}[e(Sa^\star)]. \tag{25}$$

Equity holders receive interest on collateral $\mu^k\varphi$, issue new stablecoins and purchase collateral at the (expected) rate μ for a gain $1-\varphi$, and pay interest $\delta(a^\star)$. The expected equity value following a negative jump in demand, $\mathbb{E}[e(Sa^\star)]$, can be expressed as a function of $e(a^\star)$ since $e(a) = 0$ for $a \le \overline{a}$ and $e(a)$ is given by (21) for $a \ge \overline{a}$. The other endogenous term on the right-hand side of (25) is $\delta(a^\star)$, which depends on expected price devaluation $\lambda(1 - \mathbb{E}[p(Sa^\star)])$, as shown by (23). No closed-form solution exists for the price over the smooth-issuance region $[\underline{a}, \overline{a}]$ and thus for $e(a^\star)$ unless $\varphi = 0$ or $\varphi = 1$.[27] As we already characterized the fully-collateralized case ($\varphi = 1$) in Lemma 1, we focus on the uncollateralized case ($\varphi = 0$) in the remainder of this section. While special from a theoretical standpoint, this case corresponds, for instance, to the design of Terra-Luna, an algorithmic stablecoin that notoriously crashed in May 2022. For completeness, we report numerical results for partially collateralized stablecoins in Appendix B.

Considering now exclusively an uncollateralized stablecoin ($\varphi = 0$), we can solve for the price function in the smooth-issuance region.

Lemma 3. *In region $[\underline{a}, \overline{a}]$, the equilibrium price for an uncollateralized stablecoin is*

$$p(a) = \left(\frac{a}{\underline{a}}\right)^{-\gamma} \tag{26}$$

where $\gamma < -1$ is the unique negative root of the following characteristic equation:

$$r + \lambda = -\mu\gamma + \frac{\sigma^2}{2}(1 + \gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \tag{27}$$

[27] The difficulty when $\varphi \in (0, 1)$ is that the law of motion for demand ratio a_t depends nonlinearly on the price via the optimal issuance policy (24). Hence, the HJB for the price is nonlinear.

The key insight from Lemma 3 is that the stablecoin price remains strictly positive when the peg is lost, although investors enjoy no liquidity benefit. The stablecoin value is then driven entirely by the probability that demand ratio a_t eventually reaches the peg threshold $\overline{a}$ following positive demand shocks. The speed of this process depends on the value of the root γ. Combining equation (25) and Lemma 3, we may finally characterize the optimal TMP choice for an uncollateralized stablecoin platform in Proposition 2.

Proposition 2 (Optimal Uncollateralized Stablecoin). *The optimal stablecoin policy for an uncollateralized platform features no liquidation,* $\underline{a} = 0$*. Optimal lower bound* $\overline{a} > 0$ *for the peg region and target ratio* $a^\star$ *solve*

$$\frac{e(a^\star) + 1}{a^\star} = \max_{\overline{a}, a^\star} \left\{ \frac{\ell(a^\star)/a^\star}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}\right)\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}} \right\} \tag{28}$$

$$\textit{subject to} \qquad e(\overline{a}) = \left[e(a^\star) + 1\right]\frac{\overline{a}}{a^\star} - 1 = 0. \tag{29}$$

First, Proposition 2 shows that an uncollateralized platform never commits to liquidate itself ($\underline{a} = 0$). Liquidation can only increase its date-0 value via an ex-post transfer of collateral to stablecoin users. No such transfer can take place without collateral, so the platform always continues to operate. In the smooth-issuance region $[0, \overline{a}]$, the platform's equity value is $e(a) = 0$, which implies that the platform is indifferent ex post between shutting down and continuing to operate. Staying in operation, however, maintains a positive price for the stablecoin (Lemma 3). The platform ultimately captures this benefit via a lower interest payment $\delta^\star$ in the peg region, and thus a higher value of $e(a^\star)$.

The second key result from Lemma 3 is that the smooth-issuance region is nonempty ($\overline{a} > 0$). This means that an uncollateralized stablecoin will lose its peg after a large enough negative demand shock. Even if it can fully commit to all policies, the platform cannot escape limited liability. After a large negative demand shock, maintaining the target proves too costly for equity holders to recapitalize relative to the platform's future expected profits. As a result, the peg is then lost. This result is reminiscent of Del Negro and Sims (2015) and Reis (2015), who show that an insolvent central bank without fiscal support cannot control inflation.

The platform's optimization problem, in equation (28), then amounts to the optimal choice of the lower bound of peg region $\overline{a}$ and target demand ratio $a^\star$. Given that $e(a) = 0$ for all $a \leq \overline{a}$ and $e(a)$ is linear and increasing for $a \in [\overline{a}, \infty)$, the limited liability constraint binds for all a if $e(\overline{a}) = 0$, which is constraint (29). Similar to equation (13) with unlimited liability, equation (28) shows that the platform's date-0 value is the present discounted value of seigniorage revenues. As a key difference, however, the effective discount rate is higher under limited liability to account for the risk that the platform loses the peg.

A direct corollary of Proposition 2 is that the optimal target demand ratio with limited liability, denoted $a^\star_{ll}(0)$, is higher than the target with unlimited liability, $a^\star(0)$: Reducing stablecoin issuance from $C^\star(A)$ to $C^\star_{ll}(A) < C^\star(A)$ protects the platform against large negative demand shocks. Next, we show in Corollary 1 an uncollateralized stablecoin platform is not always profitable, even though the cost of minting stablecoins is zero.

Corollary 1. *An uncollateralized platform exists only if*

$$\max_a \ell(a) \geq r - \mu + \frac{\lambda}{\xi + 1}. \tag{30}$$

Condition (30) is a sufficient condition for an uncollateralizated platform to exist.[28] It states that the growth rate of stablecoin demand, $\mu - \lambda/(\xi + 1)$ must exceed the interest paid by the platform, $\delta^\star \geq r - \ell(a^\star)$ in equation (23). Paying interest to users entails buying back stablecoins to maintain the demand ratio at the target. Hence, the difference between the growth rate of demand and the interest rate is the net issuance rate of stablecoins, which must be positive for equity tokens to have any value. Ultimately, Corollary 1 shows that a platform must face a steady growth in stablecoin demand to maintain the value of its equity tokens. In other words, an uncollateralized platform can emerge only if stablecoin demand is expected to keep growing over time.

To illustrate our results, Figure 3 contrasts the solutions under limited and unlimited liability for uncollateralized stablecoins. The left panel shows that limited liability protects equity holders, since their equity value is always positive after large negative shocks. From an ex ante perspective, however, the inability to conduct large repurchases lowers the total platform value, as can be observed in the right-most panel. The center panel shows the stablecoin price as a function of the demand ratio. With limited liability, the stablecoin trades below par in the smooth-issuance region, that is, when $a \leq \overline{a}$.

Figure 3 also illustrates our model prediction that the platform's equity value falls to zero when the stablecoin depegs even if it maintains a positive price. This prediction can rationalize the events observed during the crash of the two algorithmic stablecoins, Terra and NuBits. Faced with a demand crisis, both platforms issued equity tokens at an exponentially increasing rate to repurchase and reduce the supply of stablecoins. Once the price of equity tokens reached zero, it was not possible to proceed to further buybacks,

[28] We derive the necessary and sufficient condition for existence in the proof, and report a more intuitive sufficient condition in the main text.

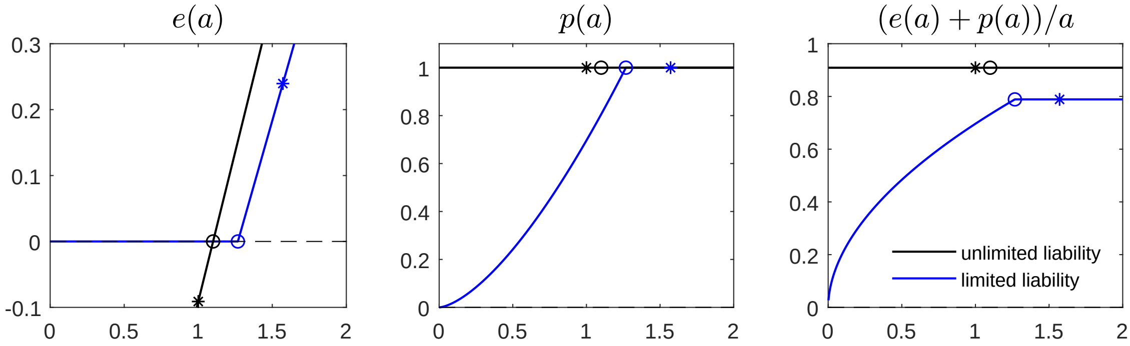


Figure 3: Full-commitment solution with limited liability without collateral (blue) and unlimited liability without collateral (black). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(a) = r\exp(-1/a)$, $\xi = 6$, $\lambda = 0.10$. Asterisks represent target demand ratio $a^{\star}$ and circles indicate $\bar{a}$, the point at which $e(a)$ reaches zero.

the supply of stablecoin plateaued, and the stablecoin lost its peg (see Appendix A).

3.3 Tradable vs. Redeemable Stablecoins

To conclude this section, we discuss the relationship between our supply-based model of stablecoins and one in which stablecoins are redeemable. In our model, the monopolistic platform controls the stablecoin supply and anticipates the effect of its issuance on the stablecoin price. In a redemption-based model, the platform would instead satisfy users' requests to convert stablecoins in the unit of account at an advertised conversion rate.

Despite these differences, one may interpret the market price in our model as the conversion rate in the redemption-based model, since the monopolistic platform effectively controls the former. To see this, consider a state a_t in the peg region $[\overline{a}, \infty)$ of a TMP. In the redemption-based model, the platform would offer conversion at rate $e_t = 1$. Then, the following would be an equilibrium: users redeem or buy new stablecoins until the secondary market price p_t adjusts to 1, and the stablecoin stock adjusts to $C^{\star}(A_t)$. Now, consider a state $a_t \leq \overline{a}$ for which the peg is lost in our model. The platform could achieve the same outcome in the redemption-based model by lowering its conversion rate from 1 to $e_t = p^{\star}(a_t)$, where $p^{\star}(a_t)$ denotes the equilibrium price of our model. Overall, this argument suggests that the equilibrium of our supply-based model could be implemented as an equilibrium of the redemption-based model.

As an important difference, however, runs could arise in a redemption-based model unless the platform is fully collateralized.[29] Although our model does not capture the strategic complementarities at play in bank runs, it still highlights the greater fragility

[29] See He and Xiong (2009) for a dynamic model of bank runs. As is well known, partial suspensions of stablecoin convertibility could kill nonfundamental runs.

of unbacked stablecoins. Moreover, in what follows, we focus on fully collateralized stablecoins that would be immune to runs because the collateral is liquid in our model.

4 Non-programmable Issuance

So far, we have assumed that the platform could fully commit to its future policies. In practice, many stablecoin protocols retain discretion over the repurchase and issuance of stablecoins. In this section, we let the platform choose policies sequentially to analyze optimal time-consistent policies.[30]

First, we present a heuristic argument to show that the full-commitment policy derived in Section 3 may not be time consistent. For this argument, we consider a fully collateralized platform ($\varphi = 1$) for two reasons. First, the commitment policy takes a simple form in this case because limited liability has no bite. Second, it allows us to show that the commitment problem arises even when stablecoin issuance is fully backed by collateral.

A fully collateralized platform that reoptimizes at date t chooses a policy sequence $\{C_\tau, \delta_\tau\}_{\tau \geq t}$ to maximize its equity value E_t, given by (15), subject to the pricing equation (4). To convey the intuition, we discretize the problem in this heuristic argument and let Δt be the time interval between two consecutive dates. Suppose that the platform implements the commitment policy for all future dates, that is, $(C_{t+n\Delta t}, \delta_{t+n\Delta t}) = (C^\star(A_{t+n\Delta t}), \delta^\star)$ for $n \geq 1$.[31] Abstracting from future terms $s > t$ in (15), the stablecoin supply C_t and interest rate δ_t that maximize the platform's date-t equity value solve

$$\max_{\delta_t, C_t} \quad \Big(\ell(A_t, C_t)\mathbb{1}\{p_t = 1\} - (r - \mu^k)\Big) C_t \Delta t - (p_t - 1) C_{t-\Delta t} \tag{31}$$

$$\text{subject to} \quad p_t = \ell(A_t, C_t)\mathbb{1}\{p_t = 1\}\Delta t + (1 - r\Delta t)\mathbb{E}_t\big[(1 + \delta_t \Delta t)\underbrace{p_{t+\Delta t}}_{=1}\big], \tag{32}$$

where the equality $p_{t+\Delta t} = 1$ obtains if the platform implements the commitment policy from date $t + \Delta t$ onward. The first term in (31) corresponds to the liquidity benefit net of collateral costs enjoyed by the platform for its date-t supply while the second term is the market value of outstanding stablecoins net of the collateral value backing them.

Absent the second term in (31), the platform would choose $(C_t, \delta_t) = (C^\star(A_t), \delta^\star)$ as in the commitment case. However, since it has price impact via its choice of (C_t, δ_t),

[30] Imposing time-consistency can only make the platform worse in our model because we allow commitment policies to be fully contingent. Our analysis thus highlights the value of such commitment. In practice, discretion may add value if some contingencies cannot be included in smart contracts.

[31] It can be shown that this is in fact optimal also from the point of view of date t. Hence, as done here, we only need to analyze the deviation with respect to date-t variables.

the platform is tempted to dilute previously issued stablecoins C_{t^-} by lowering the price p_t, thereby increasing the second term in (31). Under commitment, the platform would take into account the negative effect of dilution at date t on the price of stablecoins issued at date $t - \Delta t$. At date t, however, these past issuance costs are sunk. When it lacks commitment, the platform thus finds optimal ex post to dilute previously issued stablecoins. These dynamics are similar to a durable monopolist's temptation to lower its price over time, which erodes its market power (see Coase, 1972).[32]

In the rest of this section, we analyze the platform's problem when it lacks commitment and chooses its issuance policy sequentially. We maintain commitment, however, to the collateralization ratio and to the interest rate policy. If it chooses collateralization ratio φ at date 0, the platform cannot lower φ at any future date. Similarly, the platform can program an interest rate policy at date 0 and cannot renege ex post.

Definition 4. *A programmable interest rate rule is a mapping from the current state variables (A_t, C_{t^-}) to the interest rate $\delta_t = \delta(a_t)$.*

In line with our previous analysis, the interest rate may depend on the pre-issuance stablecoin stock only via the demand ratio a_t. Maintaining commitment to the interest policy and the collateralization ratio allows us to show that incentives to dilute stablecoin holders via issuance are pervasive even when the platform can commit to other policies.

4.1 Equilibrium Concept under Partial Commitment

We consider Markov perfect equilibria (MPE), defined with respect to the state variables of our economy (A_t, C_{t^-}). In an MPE, the platform's issuance policy and the stablecoin pricing function can depend only on (A_t, C_{t^-}), as opposed to the entire history of shocks.

Definition 5. *Given a programmable interest rule $\delta(a)$ and a collateralization ratio $\varphi \in [0, 1]$, an MPE is given by an equity token value function $E(A, C_-)$; a stablecoin pricing function $p(A, C_-)$; an issuance policy $d\mathcal{G}(A, C_-)$; and a default policy $\tau(A, C_-)$ such that the issuance policy $d\mathcal{G}$ and default policy τ maximize the platform's equity value sequentially*

$$E(A, C_-) = \max_{\tau, d\mathcal{G}} \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} \left(p_s d\mathcal{G}_s + \mu^k \varphi C_s - \varphi dC_s\right) \middle| A_t = A, C_{t^-} = C\right], \tag{33}$$

[32] Our discussion shows that the platform cannot commit to maintaining the price of its stablecoins after they are issued. The analogy with a redemption-based model discussed in Section 3.3 suggests that the platform would renege on a commitment to maintain a 1:1 conversion rate. Hence, the commitment problem applies independently of our modeling choice for a supply-based stablecoin platform.

and the pricing function satisfies

$$p(A, C_{-}) = \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}(\ell_s + \delta_s)p_s ds + e^{-r(\tau-t)}\varphi \middle| A_t = A, C_{t^-} = C\right], \tag{34}$$

where the expectation is under the evolution of C implied by the platform's policies.

The platform's objective is to maximize its equity value at date 0, which corresponds to $E(A, 0)$, similar to the commitment case analyzed in Section 3. Without commitment to future policies, however, the platform's policies must also be time consistent, that is, policies must also be optimal at any future date. In an MPE, time only matters via the value of state variables (A_t, C_{t^-}). Hence, optimization condition (33) formalizes the requirement that policies are sequentially optimal. The Markov property for policies, however, does not further constrain the optimization problem relative to the commitment case, as we focused on TMPs in that analysis.

More importantly, Markov perfection implies that the stablecoin price (34) may only depend on history via current state variables (A, C_{-}). This feature rules out collective punishment of the platform by stablecoin users if it deviates from some reference policy. Focusing on MPE disciplines the analysis in that the stablecoin price may only depend on fundamentals and users' expectations about the platform's future policies.[33]

First, we characterize the platform's policies in an MPE. As in Section 3, we let $e(a) = E(A/C^{-}, 1)$ be the equity value per coin outstanding. While we assumed TMPs under commitment, we can show that an MPE policy must be a TMP.

Proposition 3 (Equilibrium Policy). *For an optimal programmable interest rate rule, the equilibrium issuance policy $d\mathcal{G}$ in an MPE belongs to the class of TMP introduced in Definition 3.*

As observed above, policies satisfy the Markov property by definition of an MPE. Proposition 3 shows that equilibrium policies must belong to the class of TMPs with a default region $[0, \underline{a}]$, a smooth-issuance region $[\underline{a}, \overline{a}]$ and a target region $[\overline{a}, \infty)$. The ability to characterize equilibrium policies hinges on the time-consistency requirement that constrains the set of policies in the absence of commitment. In the proof of Proposition 3, we first establish that the equilibrium equity function is weakly convex and the stablecoin

[33] Enforcing collective punishments may prove challenging with dispersed and anonymous investors, as in our model. If investors could use "grim-trigger" strategies to punish the platform, additional equilibria could be supported. See Malenko and Tsoy (2020), who consider punishments in a related dynamic leverage choice problem for firms. Following a deviation from the equilibrium policy, the firm and investors play the MPE of DeMarzo and He (2021), which gives the lowest possible equilibrium payoff to the firm.

price is weakly increasing as a function of the demand ratio a. Following arguments by DeMarzo and He (2021), we then show that the equilibrium issuance policy is smooth (features jumps) on intervals for which the equity value is strictly convex (linear). The existence of a default threshold $\underline{a}$ follows from the fact that the equity value is increasing in a. Next, we show that if interest policy $\delta(a)$ is chosen optimally at date 0, the issuance policy is smooth over the first part of the no-default region, $[\underline{a}, \overline{a}]$ for some $\overline{a} \geq \underline{a}$. For values of $a \in [\overline{a}, \infty)$, it features a jump to some target demand ratio $a^\star$. By definition, these results imply that the equilibrium policy belongs to the class of TMP.

4.2 Markov Perfect Equilibrium

In the absence of commitment, the platform's policy must solve its sequential optimization problem. In particular, in a TMP, the platform must find it optimal ex post to implement the target from any demand ratio a in the target region $[\overline{a}, \infty)$. The following statement provides conditions under which implementing the target is indeed ex-post optimal.

Lemma 4. *Under limited commitment, the platform implements the target of a TMP if, for any $a, a' \geq \overline{a}$ in the target region, the following inequality holds*

$$\begin{aligned}&\Big[\ell(a^\star) - \varphi(r - \mu^k)\Big]\frac{a'}{a^\star} + \varphi(r - \mu^k) - (r - \delta(a))\frac{a'}{a} \\ &\geq \lambda\Big[\mathbb{E}[e(Sa')] + 1 - \varphi\Big] - \lambda\Big[\mathbb{E}[e(Sa^\star)] + \mathbb{E}[p(Sa^\star)] - \varphi\Big]\frac{a'}{a^\star},\end{aligned} \tag{35}$$

where e and p are the equilibrium equity value and pricing function, respectively.

Condition (35) ensures that implementing demand ratio $a^\star$ is ex post optimal when the demand ratio a lies in the target region $[\overline{a}, \infty)$. This implementation condition rules out a "one-step" deviation whereby, starting from some demand ratio $a \in [\overline{a}, \infty)$, the platform would choose demand ratio $a' \neq a^\star$, stay at a' during a period of length dt before reverting to the equilibrium policy. To get some intuition, it is useful to consider the limit case $\lambda \to 0$ in which the right-hand side of (35) disappears. Then, in state (A, C_-), the platform prefers implementing the target $C^\star(A)$ to deviating to C if and only if[34]

$$\Big[\ell(a^\star) - \varphi(r - \mu^k)\Big]C^\star(A) + \varphi(r - \mu^k)C \geq (r - \delta(a))C_- \tag{36}$$

The right-hand side of (36) reflects the difference in liquidity benefit (net of collateral costs) at the target relative to the deviation. In the latter case, the platform enjoys no

[34]This condition applies also with $\varphi = 1$ when $\lambda > 0$ because then $\mathbb{E}[p(Sa^\star)] = 1$ and $\mathbb{E}[e(Sa)]/a$ is constant in the candidate equilibrium induced by the TMP with $\overline{a} = \underline{a} = 0$ (see Section 3).

liquidity benefit because the peg is lost. By definition of $a^\star$, this term is positive. The left-hand side of (36) captures the ex-post benefits from diluting previously issued stablecoin in a deviation. To see this, consider equation (32) with $p_t \neq 1$, which characterizes the price in the deviation. Then, the stablecoin price would fall from 1 to $p_t = 1 - (r - \delta_t)$, which implies that the deviation dilutes the market value of outstanding stablecoins by $r - \delta_t$.

Next, we show that the platform's incentives to dilute past stablecoin holders are so strong that it cannot earn seigniorage when it lacks commitment to the issuance policy.

Proposition 4. *There is no programmable interest rate rule in the sense of Definition 4 such that an MPE exists in which the platform earns seigniorage revenues.*

In an MPE with a TMP, for any a in the target region, it must be that $\delta(a) = \delta^\star$ where $\delta^\star$ is defined in (23) because the equilibrium policy features a jump to $a^\star$. As the interest rate rule can depend only on the state a, it means that the same interest rate $\delta^\star$ would apply if the platform were instead to deviate to $a' \neq a^\star$. To see why a deviation cannot be avoided, consider again condition (36) (the limit case with $\lambda \to 0$) and set $C = C_{-}$. Then, the no deviation condition can hold for any C_{-} if and only if

$$r - \ell(a^\star) = \delta^\star = \varphi(r - \mu^k),$$

which means that the net revenue flow from the platform is equal to zero. Importantly, the result applies even with full collateralization ($\varphi = 1$)—that is, collateral does not solve the platform's commitment problem.

Proposition 4 implies that without commitment, a platform cannot earn revenues. Incentives to dilute past stablecoin holders lead the platform to deviate from the peg ex post. This behavior neutralizes its ability to earn seigniorage because users enjoy the liquidity benefit only when the price is pegged. Our result evokes the leverage ratchet effect in DeMarzo and He (2021), who show that a firm never enjoys any tax benefit of debt due to incentives to dilute debtholders. In both cases, the commitment problem resembles that of the durable good monopolist in Coase (1972). A key difference between their model and ours, however, is the role of collateral. In their framework, the firm cannot dilute past debtholders via an increase in the probability of default when debt is (fully) collateralized. In our model, the dilution operates via the liquidity benefit ℓ, which depends on the total stock of stablecoins outstanding. Hence, dilution incentives exist even with full collateralization.

We conclude this section by highlighting an interest rate rule that can restore commitment but does not satisfy Definition 4. To achieve this objective, the interest rate rule should neutralize the platform's incentive to dilute stablecoin users ex post via the stablecoin price. As we show below, such a rule must react not only to the state $(A_t, C_{t\text{-}})$ but also to the current supply choice C_t of the platform.

Proposition 5. *Let $a = A/C_{\text{-}}$ ($a' = A/C$) be the pre- (post-)issuance demand ratio. Under full collateralization, the interest rate rule*

$$\delta(a') = \begin{cases} r - \ell(a^\star(1)) & \textit{if } a' = a^\star \\ r & \textit{otherwise} \end{cases} \tag{37}$$

implements the full-commitment outcome under discretionary issuance.

Interest rate rule (37) reacts to the new supply chosen by the platform, which provides incentives not to deviate. Intuitively, the platform is punished with a higher interest rate if it deviates from the equilibrium policy $a' = a^\star$. To see why interest rate rule (37) neutralizes dilution incentives, consider again equation (32), which gives the price in a deviation, $p_t = 1 - (r - \delta_t)$. Then setting $\delta_t = \delta(a'_t)$ for all $a'_t \neq a^\star$ implies that the stablecoin price is equal to 1, independently of the supply choice by the platform. Given that the platform cannot benefit from diluting past stablecoin holders by lowering the price, choosing the target demand ratio $a^\star$ maximizes its date-t equity value, as can be seen from (31). Ultimately, commitment to issuance is restored indirectly via the interest rate rule that penalizes deviations from the target demand ratio.[35]

5 Decentralized Protocols

So far, our analysis focused on centralized protocols where the platform directly supplies stablecoins to the market. In this section, we consider instead *decentralized* protocols—with DAI as the most prominent example. Decentralized protocols delegate the issuance of stablecoins to any users holding eligible collateral. To issue stablecoins, users must lock assets in a smart contract generated by the protocol, called a "vault." Vault owners can unlock their collateral by repurchasing and "burning" stablecoins. In this scheme, the platform's equity holders charge a seigniorage fee to vault owners paid in stablecoins. This fee, net of the interest paid to stablecoin users, is the source of platform profit.[36]

[35] In a partially collateralized case ($\varphi < 1$), an interest rate rule that neutralizes dilution incentives in the peg region is given by $r + \lambda - \ell(a^\star) - \lambda\mathbb{E}[p(Sa^\star)]$ if $a' = a^\star$ and $r + \lambda - \lambda\mathbb{E}[p(Sa')]$ otherwise, where $a^\star$ is the new optimal target demand ratio with limited liability and no commitment to the issuance policy.

[36] Decentralized stablecoin protocols also feature decentralized decision-making whereby equity/governance token holders can vote on platform policies. To the extent that all token holders share a common profit-maximization objective, ownership decentralization does not play a role in our model. We thus focus on the decentralization of issuance, but still consider a representative equity token holder.

We showed in Section 4 that a platform suffers from a commitment problem under centralized issuance. Decentralizing issuance can thus add value if it mitigates this commitment problem. For a fair comparison with the centralized case, we assume that the decentralized platform re-optimizes continuously the interest rate it pays to users, δ_t, seigniorage fee, denoted s_t, that it charges to vault owners. As in Section 4, however, the collateralization ratio φ is set at date 0 and cannot be altered subsequently.

In what follows, we first present the vault owners' problem. We then present the platform's optimization problem and show that the full-commitment policy is time consistent with decentralized issuance—that is, decentralizing issuance can solve the commitment problem described in Section 4. Finally, we draw a parallel between our results and the rental solution to the durable good monopolist problem introduced by Coase (1972).

5.1 Vault Owners

Any agent can open a vault and issue stablecoins. A vault is indexed by i with $C^i_{t\text{-}}$ the amount of stablecoins outstanding for vault i at date t. Every period, a vault owner chooses to default or to keep the vault open. In the latter case, the vault owner chooses new issuance $d\mathcal{G}^i_t$ subject to collateralization ratio φ, its expectation about the platform's policies, as well as the price dynamics implied by dynamic demand equation (4), which she takes as given. Denoting τ^i the default time for vault owner i, its problem writes

$$V^i_t(C^i_{t\text{-}}) = \max_{\tau^i, d\mathcal{G}^i_t} \mathbb{E}_t\left[\int_t^{\tau^i\wedge\tau} e^{-r(s-t)}\left(p_s d\mathcal{G}^i_s + \varphi\mu^k C^i_s ds - \varphi dC^i_s\right)\right], \tag{38}$$

$$\text{subject to} \qquad dC^i_t = s_t C^i_t dt + d\mathcal{G}^i_s. \tag{39}$$

Vault owner's optimization problem (38) resembles that of a centralized platform, given by equation 7, with the vault fee s_t replacing the interest rate δ_t. A vault owner enjoys the return on collateral held in the vault and gains from issuance proceeds $p_s d\mathcal{G}^i_s$ net of collateral purchases φdC^i_s. As a key difference with a centralized platform, however, competitive vault owners do not internalize their impact on the stablecoin price p_t.

Now, we characterize equilibrium restrictions on the vault owners' value function, the stablecoin price, and the platform's policies implied by vault owners' optimization. Since vault owners have no price impact, their issuance gains are linear in the quantity issued. In any equilibrium with positive and finite stablecoin stock $C_t = \int_i C^i_t di$, free entry implies that issuance gains must be equal to zero. Hence, the equilibrium value of a vault is

$$V^i_t(C^i_{t\text{-}}) = (\varphi - p_t)C^i_{t\text{-}}. \tag{40}$$

That is, we can compute the vault value as if the vault owner never issues stablecoins ($d\mathcal{G}_t^i = 0$). Intuitively, the vault value equals the collateral held net of the value of outstanding stablecoins. This characterization of the vault owners' value function allows us to derive arbitrage restrictions that competitive vault owners impose on the platform.

Proposition 6. *In any equilibrium with positive and finite stablecoin stock C_t at date t, the stablecoin price cannot exceed the collateralization ratio—that is,*

$$p_t \leq \varphi, \tag{41}$$

and the stablecoin price and the vault fee satisfy

$$r(\varphi - p_t) = \varphi\mu^k - s_t p_t - \mathbb{E}_t\left[\frac{dp_t}{dt}\right]. \tag{42}$$

The first result follows from an arbitrage argument for vault owners. If the stablecoin price exceeds the collateralization ratio, vault owners could achieve an unbounded profit from issuing stablecoins at date t and defaulting next period. Equation (42) obtains by solving problem (38) for an interior optimum for supply $d\mathcal{G}_t^i$. From equation (40), $\varphi - p_t$ is a vault's value per stablecoin outstanding. Hence, the right-hand side of (42) is the opportunity cost from owning a vault, which must be equal to the flow return from owning a vault. A vault owner enjoys the return on collateral $\varphi\mu^k$ and pays the seigniorage fee s_t in stablecoins. In addition, the vault value decreases when the stablecoin appreciates because a vault amounts to a short position in stablecoins. Hence, when the stablecoin appreciates, a vault owner must pay more to release the vault's collateral.

Corollary 2. *The platform can earn seigniorage revenues in equilibrium only if $\varphi \geq 1$.*

This result shows that a decentralized platform must be fully collateralized. It follows directly from the no-arbitrage relationship (41) in Proposition 6 and the observation that users enjoy liquidity benefits only if the stablecoin is pegged. Hence, with any collateralization ratio $\varphi < 1$, the stablecoin price must satisfy $p_t < 1$, and the platform earns no seigniorage revenue. The platform thus optimally sets $\varphi = 1$ at date 0 because it cannot generate profit otherwise. In the next section, we characterize the problem of a fully collateralized decentralized platform.

5.2 Decentralized Platform Problem and Solution

This section describes the optimization problem of a decentralized platform with full collateralization ($\varphi = 1$). With decentralized issuance, the platform sets the interest rate

δ_t paid to users and the seigniorage fee s_t it charges to vault owners, to whom issuance is delegated. As we do not assume commitment, the platform reoptimizes at every date t.

Problem 2 (Decentralized Platform Problem Without Commitment). *A decentralized platform chooses its interest rate policy $\{\delta_t\}_{t\geq 0}$ and its seigniorage fee policy $\{s_t\}_{t\geq 0}$ sequentially to maximize its equity value at every date t,*

$$E_t = \max_{\tau,\delta,s} \mathbb{E}_t \left[\int_t^{\tau} e^{-r(s-t)} \big(s_s - \delta_s\big) p_s C_s ds \right], \tag{43}$$

subject to stablecoin pricing equation (4), the no-Ponzi Game condition (8), and the vault owner's arbitrage conditions (41) and (42).

Similar to the centralized case, the monopolistic platform effectively chooses both policy variables δ and s as well as the stablecoin price p and the supply C subject to the pricing constraints imposed by the competitive behavior of other agents. As before, equation (4) follows from competitive stablecoin pricing by users. Specific to the decentralized case, constraints (41) and (42) follow from vault owner's competitive supply decisions.

The decentralized issuance model changes the platform's sequential optimization problem in a fundamental way. As can be seen from (43), the platform's profit flow is now proportional to the stablecoin stock C_t as opposed to new issuance $d\mathcal{G}_t$. The latter feature generated the commitment problem with centralized issuance analyzed in Section 4 as it gave the platform's incentives to dilute previously issued stablecoins. With decentralized issuance, the platform's model changes from an issuance-based revenue flow to a rental based-revenue flow. As our next result shows, such incentives disappear with the rental-based revenue model under decentralized issuance.

Proposition 7 (Decentralized Protocol Equilibrium). *A fully-collateralized decentralized platform without commitment implements the full-commitment outcome of Proposition 1. The equilibrium stock of stablecoins thus satisfies*

$$C_t = \arg\max_C \big\{\ell(A_t, C)C + (\mu^k - r)C\big\} \equiv C^\star(A_t, 1). \tag{44}$$

To implement $C^\star(A)$, the platform sets a vault fee schedule of the form

$$s(a) - \delta(a) = \begin{cases} \ell(a) + (\mu^k - r)/p(a) + \varepsilon & \text{if} \quad C_t > C^\star(A_t, 1), \\ \ell(a^\star(1)) + \mu^k - r & \text{if} \quad C_t = C^\star(A_t, 1), \\ \ell(a) + (\mu^k - r)/p(a) - \varepsilon & \text{if} \quad C_t < C^\star(A_t, 1), \end{cases} \tag{45}$$

where ε is strictly positive. The equilibrium interest rate is $\delta^\star = r - \ell(a^\star(1))$.

Proposition 7 shows that the full-commitment outcome of Section 3 is time consistent when the platform delegates issuance to vault owners, in contrast with our result for centralized issuance in Section 4. Hence, delegated issuance dominates centralized issuance when the platform lacks commitment. As mentioned above, a decentralized platform earns a rental income on the total stock of stablecoins, as opposed to an issuance profit from new stablecoins issued. This feature kills incentives to dilute past stablecoins, which generates the commitment problem in the centralized case.

With decentralized issuance, the platform implements the full-commitment solution even as it reoptimizes sequentially. Similar to Proposition 1, the platform implements the stablecoin stock $C^\star(A_t)$ that maximizes the liquidity benefit flow net of collateral costs, given by (44). Unlike in the centralized case, however, the platform does not directly control supply. The second part of Proposition 7 thus shows how the platform can use the seigniorage fee to steer issuance by vault owners. To get some intuition, suppose that the stock of stablecoins lies below the platform's target—that is, $C_t < C^\star(A_t)$. The platform would then lower the fee to stimulate issuance until C_t reaches $C^\star(A_t)$. At this point, vault owners become indifferent about issuance. These adjustments occur instantaneously in equilibrium, so that equation (45) describes an off-equilibrium fee schedule.

To explain why decentralized issuance neutralizes the platform's incentives to manipulate the stablecoin price, consider the discretized version of Problem 2, as in the argument of Section 4. As noted before, the platform implicitly chooses all endogenous variables under constraints imposed by the competitive behavior of other agents. Denoting E the platform's equity value function, the discretized problem is

$$\max_{\delta_t, s_t, p_t, C_t} (s_{t-\Delta t} - \delta_{t-\Delta t}) p_t C_{t-\Delta t} \Delta t + (1 - r\Delta t) \mathbb{E}_t[E(A_{t+\Delta t}, C_t, \delta_t, s_t)], \tag{46}$$

subject to pricing equation (4) and the arbitrage constraints, (41) and (42), respectively

$$p_t = \ell(A_t, C_t) \mathbf{1}_{\{p_t=1\}} \Delta t + (1 - r\Delta t) \mathbb{E}_t[(1 + \delta_t \Delta t) p_{t+\Delta t}], \tag{47}$$

$$1 - p_t = (1 - r\Delta t) \mathbb{E}_t\big[(1 + \mu^k \Delta t) - (1 + s_t \Delta t) p_{t+\Delta t}\big], \tag{48}$$

$$p_t \leq 1. \tag{49}$$

At date $t - \Delta t$, the platform chose the interest rate $\delta_{t-\Delta t}$ and the vault fee $s_{t-\Delta t}$ that applies to the stock $C_{t-\Delta t}$. These interest payments occur at date t, which explains the expression for the platform's rental profit flow—the first term of (46). The stablecoin price enters the platform's rental flow because its net seigniorage fee $s - \delta$ is paid in stablecoins.

To see why the full-commitment solution is time consistent, we follow the same one-step deviation argument as in Section 4. Suppose that the platform implements the full-commitment solution at all dates but t and consider its date-t optimization problem under these premises. Then, given date-t preset variables $C_{t-\Delta t} = C^\star(A_{t-\Delta t})$, $\delta_{t-\Delta t} = r - \ell(a^\star)$ and $s_{t-\Delta t} = \mu^k$, the platform maximizes

$$(\ell(a^\star) + \mu^k - r)p_t C^\star(A_{t-\Delta t})\Delta t + (1 - r\Delta t)\mathbb{E}_t[(s_t - \delta_t)p_{t+\Delta t}C_t\Delta t]. \tag{50}$$

The expression above does not include the terms for periods $t + n\Delta t$ for $n \geq 2$ because the platform implements the commitment policy after date t. Using pricing equations (47) and (48) to substitute for the second term in (50), the platform maximizes:[37]

$$\Pi_t = (\ell(a^\star) + \mu^k - r)p_t C^\star(A_{t-\Delta t})\Delta t + \left[\ell(A_t, C_t)\mathbf{1}_{\{p_t=1\}} - \varphi(r - \mu^k)\right]C_t\Delta t. \tag{51}$$

The second term of (51) is the liquidity benefit net of the collateral flow cost, which is maximized at the full-commitment solution with $C^\star(A_t) = A_t/a^\star(1)$ and $p_t = 1$. A deviation is profitable only if it increases the first term of (51) relative to its value under the full-commitment policy. This requires increasing the stablecoin price p_t above 1, which arbitrage condition (49) rules out because competitive vault owners maintain the price below the collateralization ratio $\varphi = 1$ (Proposition 6). Hence, the full commitment policy is time-consistent for a decentralized stablecoin protocol. That is, decentralizing issuance solves the commitment problem that plagues a centralized platform.[38]

5.3 Decentralizing Issuance vs. Renting Stablecoins?

Decentralizing issuance solves the platform's commitment problem because its profit flow, which is proportional to new issuance $d\mathcal{G}_t$ in the centralized case, becomes rental-based since it is now proportional to the stablecoin stock C_t. In this respect, decentralized issuance is similar to the rental solution to the durable good monopolist problem analyzed by Coase (1972) and Bulow (1982), among others. Renting a durable good solves the monopolist's commitment problem because the full stock of goods is repriced every period. Instead, in an outright sale market, a seller without commitment fails to internalize the

[37] In equation (50) we ignored the term $-r\mu^k(\Delta t)^2$ which the platform cannot influence via its policy.

[38] The reader may have noticed that any deviation gain from altering the first term of (51) relative to the commitment solution is of order $(\Delta t)^2$ because the price deviation $p_t - 1$ must be of order Δt if $p_{t+\Delta t} = 1$. Hence as $\Delta t \to 0$, any deviation gain becomes negligible relative to the order-Δt deviation loss generated by the second term of (51). Our argument based on arbitrage constraint (49), however, shows that the result is not an artifact of the continuous time limit.

effect of a drop in the price of previously sold goods caused by future issuance. Similarly, in our model, decentralized issuance and its rental-based revenue model neutralizes the platform's incentives to dilute previously issued stablecoins with new issuance.

While the analogy with the rental solution of Coase (1972)'s problem helps explain our result, nontrivial differences remain with our model. First, in the rental solution to the durable good monopolist problem, the rental market between the producer and users replaces the primary market. Every period, users return the good to the producer and can rent it again at a newly set rate. Instead, in our model, a market for buying and selling stablecoins between vault owners and users still exists alongside the decentralized protocol. Indeed, the liquidity benefit depends directly on the stablecoin price, which implicitly requires the presence of such a market. While the latter feature is an assumption of our model, it captures the fact that, unlike for durable goods, direct benefits from holding money-like assets derive from users' ability to exchange those stablecoins.

The role of collateral is a second difference from the original rental solution. Suppose that users could rent stablecoins without locking collateral in a vault. Anonymous users would then enter a rental contract and immediately sell the stablecoin in the market without paying rental fees to the platform. Collateral helps prevent this behavior. Users thus deposit collateral in a vault to draw stablecoins, which ensures that the platform can collect fees. In our model, we introduce a distinct group of users who play this role—vault owners—to better connect the model with the empirical design of decentralized stablecoin protocols, but this distinction is unimportant for the result.

6 Conclusion

This paper proposes a model to study the (merits and) vulnerabilities of various stablecoin designs when the issuer faces a time-consistency problem. Our analysis shows that partially collateralized platforms are always vulnerable to large demand shocks, even under full commitment. The optimal collateralization level under commitment thus trades off resilience against these shocks against collateral holding costs. Collateral alone, however, does not solve the platform's time-consistency problem, whereby the issuer tends to inflate previously issued stablecoins via peg deviations. When combined with full collateralization, decentralization of issuance to competitive vault owners—a design similar to DAI—can restore commitment. To focus on our main research question, we assumed a reduced-form liquidity benefit for stablecoins and considered the problem of a single stablecoin issuer using riskless collateral. We leave the analysis of these microfoundations and extensions to future research.

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Appendices

A Proofs

A.1 Proof of Proposition 1

Substituting for $d\mathcal{G}_t = dC_t - \delta_t C_t dt$, the objective function can be written as

$$E_0 = \max_{\varphi,\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \Big(p_t dC_t - \delta_t p_t C_t dt + \mu^k \varphi C_t dt - \varphi dC_t \Big) \right]. \tag{A.52}$$

Integrating the terms in dC_t by parts, we obtain

$$\begin{aligned} E_0 &= \max_{\varphi,\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\left[(p_t - \varphi) C_t e^{-rt} \right]_0^\infty - \int_0^\infty e^{-rt} C_t \left(dp_t - r(p_t - \varphi)dt + \delta_t p_t dt - \mu^k \varphi \right) \right] \\ &= \max_{\varphi,\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \Big(\ell(A_t, C_t) \mathbb{1}\{p_t = 1\} + (\mu^k - r)\varphi \Big) C_t dt \right]. \end{aligned} \tag{A.53}$$

To obtain the second line, we guess and verify that $\lim_{t\to\infty} \mathbb{E}_0[(p_t - \varphi)C_t e^{-rt}] = 0$. We use pricing equation (4) to substitute for $dp_t - (r - \delta)p_t dt$ within the expectation.

Equation (A.53) shows that setting $\varphi = 0$ is optimal. Next, δ_t is only determined to the extent that it maintains the price peg, and we can rewrite equation (A.53) as

$$E_0 = \max_{\{\delta_t, d\mathcal{G}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \ell(A_t, C_t) C_t \mathbb{1}\{p_t = 1\} dt \right]. \tag{A.54}$$

Assuming that such interest rate policy can be chosen, the platform's problem is static, and the optimal issuance rule is such that C_t maximizes $\ell(A_t, C_t)C_t$. By Property iii in Assumption 1, this maximizer exists, is unique, and is given by (12). The fact that $C^\star(A, 0) = A/a^\star(0)$ is linear in A follows from Property ii in Assumption 1. Moreover, our conjecture $\lim_{t\to\infty} \mathbb{E}_0[(p_t - \varphi)C_t e^{-rt}] = 0$ and the fact that the objective function is bounded follows from the fact that A_t grows at a rate inferior to r. Finally, the interest rate policy must be such that $p_t = 1$ for all t, which holds with $\delta(a^\star(0)) = r - \ell(a^\star(0))$.

The optimal issuance-repurchase policy $\{d\mathcal{G}_t\}_{t\geq 0}$ features a jump from 0 to $C^*(A_0, 0)$ at date 0 and is such that $d\mathcal{G}_t + \delta_t C_t dt = dA_t$ for $t > 0$. This concludes the proof.

Now, we derive the expression for the date-0 equity value, equation (13), thanks to Proposition 1. For future reference, we compute the equity value E_t at any date, and for all values of $\varphi \in [0, 1]$ under the optimal full-commitment policy. Starting from equation

(A.53), we have

$$E_t = -(p_t - \varphi)C_{t\text{-}} + \mathbb{E}_0\left[\int_t^\infty e^{-r(s-t)}\Big(\ell(A_s, C_s)\mathbb{1}\{p_s = 1\} + (\mu^k - r)\varphi\Big)C_s ds\right],$$
$$= -(p_t - \varphi)C_{t\text{-}} + \frac{\ell(a^\star(\varphi)) + \mu^k - r}{a^\star(\varphi)}\mathbb{E}_0\left[\int_t^\infty e^{-r(s-t)}A_s dt\right]. \tag{A.55}$$

Remember that $C_{t\text{-}}$ is the left limit of the supply process C_s at $s = t$. To obtain the second line, we substituted for the optimal supply policy, $C_s = C^\star(A_s, \varphi) = A_s/a^\star(\varphi)$. Using then equation (5) that describes the law of motion for A_t, we obtain

$$E_t = -(p_t - \varphi)C_{t\text{-}} + \frac{\ell(a^\star(\varphi)) + \mu^k - r}{a^\star(\varphi)\left(r - \mu + \frac{\lambda}{\xi+1}\right)}A_t. \tag{A.56}$$

Setting $t = 0$ and $C_{0\text{-}} = 0$, we obtain equation (13).

A.2 Proof of Lemma 1

The fact that the optimal full-commitment policy violates limited liability (9) if and only if $\varphi < 1$ obtains directly from equation (16).

For the second part of Lemma 1, set $p_t = \varphi = 1$ in equation (16). The equity value is positive if and only if the numerator of the first term is positive, that is, if and only if $\ell(a^\star(1)) \geq r - \mu^k$. This concludes the proof.

A.3 Proof of Lemma 2

First, we derive the value of the interest rate $\delta^\star$ paid by the platform at the target ratio $a^\star$ in order to maintain the peg. To do so, we derive the dynamic equation for the price at the target. Given that the price at the target ratio is 1, we obtain

$$1 = \big(\delta^\star + \ell(a^\star)\big)dt + (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(a^\star + da)] + (1 - rdt)\lambda dt\mathbb{E}[p(Sa^\star)]. \tag{A.57}$$

The third (last) term in equation (A.57) corresponds to smooth changes (jumps) in demand A_t. After a smooth change in demand, the peg still holds given that the platform maintains the target, that is, $p(a^\star + da) = 1$ in this case. After a jump, however, the price may fall below 1 if $Sa^\star \leq \overline{a}$. Plugging $p(a^\star + da) = 1$ into equation (A.57), we obtain equation (23).

Now, we prove the second part of Lemma 2. The main step is to show that $e(a) = 0$ is optimal for all $a \leq [\underline{a}, \overline{a}]$. Note that $e(a) = 0$ for $a \leq \underline{a}$ holds by definition of a TMP. We then derive the optimal issuance policy in the smooth region.

To show that $e(a) = 0$, for all $a \leq [\underline{a}, \overline{a}]$, consider the total platform value at date t, denoted F_t. The total platform value includes both the equity value, E_t and the net value of stablecoins outstanding, $(p_t - \varphi)C_{t\text{-}}$. At date 0, equity value and total platform value are equal, so the platform's objective is to maximize the date-0 total platform value.

Consider a demand ratio $a = A/C > \overline{a}$. In this, case F only depends on A—not on the outstanding stock of stablecoins C—and we denote $\bar{F}(A)$ to avoid confusion. Let τ_S denote the first (stochastic) time when a shock $S \leq \underline{a}/a^\star$ hits. We have

$$\bar{F}(A_0) = E_{\tau_S}\left[\int_0^{\tau_S} e^{-rt}\Big(\ell(A_t, C^\star(A_t))C^\star(A_t) + \varphi(\mu^k - r)C^\star(A_t)\Big)dt \right. \\ \left. + e^{-r\tau_S}\mathbb{E}\left[F\left(SA_{\tau_S}, C^*(A_{\tau_S})\right)\Big|Sa^* \leq \overline{a}\right]\right]. \tag{A.58}$$

Given values for $(a^\star, \overline{a})$, maximizing value $\bar{F}(A_0)$ consists in maximizing the second term of the above equation. We thus explicit the dynamic equation for $F(A, C)$ in the region where $a = A/C \in [\underline{a}, \overline{a}]$. For a given $a \in [\underline{a}, \overline{a}]$, denote $\tau^+(a)$ the first stochastic time when $a_t = \overline{a}$ and $\tau^-(a)$ the first stochastic time when $a_t = \underline{a}$. Let $\tau(a_0) = \min\{\tau^+(a_0), \tau^-(a_0)\}$. We have

$$F(A_0, C_0) = \mathbb{E}_{\tau(a_0)}\left[\int_0^{\tau(a_0)} e^{-rt}(\mu^k - r)\varphi C_t dt \right. \\ \left. + \quad e^{-r\tau(a_0)}\Big(\mathbb{1}\{\tau(a_0) = \tau^+(a_0)\}\bar{F}(A_{\tau(a_0)}) + \mathbb{1}\{\tau(a_0) = \tau^-(a_0)\}\varphi C_{\tau(a_0)}\Big)\right], \tag{A.59}$$

where the law of motion for C_t is given by (6). The dividend flow for the total platform is negative in the region $[\underline{a}, \overline{a}]$. Hence, maximizing $F(A, C)$ in region $[\underline{a}, \overline{a}]$ and thus $\bar{F}(A)$ amounts to minimizing the expected time $\tau^+(a)$ from any given point a. Given the policies in $[\underline{a}, \overline{a}]$ in (17), we have

$$\mathbb{E}\left[\frac{da_t}{a_t}\right] = \left(\mu - \frac{\lambda}{\xi + 1}\right)dt - (\delta_t + G_t/C_t)dt. \tag{A.60}$$

Hence the platform seek to minimize δ_t and G_t subject to the constraint that equity value $E(A, C)$ remains positive for $A/C \in [\underline{a}, \overline{a}]$.

In the next step, we derive the recursive equation for the equity value in order to pin down the minimum value of g and δ such that limited liability holds in region $[\underline{a}, \overline{a}]$. In doing so, we guess and verify that it holds for $[\overline{a}, \infty)$. Adapting Equation (7), we have

$$E(A, C) = p(A, C)G(A, C)dt + \mu^k\varphi C dt - \varphi dC \\ + (1 - rdt)(1 - \lambda dt)\mathbb{E}[E(A + dA, C + dC)] + (1 - rdt)\lambda dt\mathbb{E}[E(SA, C)]. \tag{A.61}$$

Using Ito's Lemma for the term $E(A + dA, C + dC)$ above and keeping only terms of order dt, we obtain the following HJB:

$$(r + \lambda)E(A, C) = (p(A, C) - \varphi)G(A, C) + (\mu^k - \delta(A, C))\varphi C + \mu A E_A(A, C) + \frac{\sigma^2}{2}E_{AA}(A, C) \\ + (\delta(A, C)C + G(A, C))E_C(A, C) + \lambda\mathbb{E}[E(SA, C)]. \tag{A.62}$$

Using the normalized equity value, $e(a) = E(A,C)/C$, we obtain $E_A(A,C) = e'(a)$, $E_{AA}(A,C) = e''(a)$, $E_C(A,C) = e(a) - ae'(a)$. Introducing the normalized issuance rate, $g(a) \equiv G(A,C)/C$, we get

$$\begin{aligned}(r+\lambda)e(a) =&(p(a)-\varphi)g(a) + \mu a e'(a) + \frac{\sigma^2}{2}e''(a) + (\delta(a)+g(a))(e(a)-ae'(a)) \\ &+ (\mu^k - \delta(a))\varphi + \lambda E[e(Sa)]. \end{aligned} \tag{A.63}$$

It follows from the equation above that the minimum value of $g(a)$ such that $e(a) \geq 0$ for all $a \in [\underline{a}, \overline{a}]$ is given by

$$g(a) = -\frac{\mu^k - \delta(a)}{p(a)-\varphi}\varphi. \tag{A.64}$$

Given policy $g(a)$ above and $e(a) = e'(a) = 0$, the impact of $\delta(a)$ is offset in HJB equation (A.63) and we can set $\delta(a)$ to its minimum at 0 for $a \in [\underline{a}, \overline{a}]$. This concludes the proof.

A.4 Proof of Lemma 3

First, we characterize the price dynamics in region $[\underline{a}, \overline{a}]$. The price equation can be written as

$$p(A,C) = (1-rdt)(1-\lambda dt)\mathbb{E}[p(A+dA, C+dC)] + (1-rdt)\lambda dt\mathbb{E}[p(SA,C)]. \tag{A.65}$$

When $a \in [\underline{a}, \overline{a}]$, stablecoin owners enjoy no cash flow because the platform optimally sets $\delta(a) = 0$ and liquidity benefits are equal to 0 since $p(a) < 1$. Using $dC = g(a)Cdt$, the first term on the right-hand side can be expanded using Ito's Lemma:

$$\begin{aligned}\mathbb{E}[p(A+dA, C+dC)] &= p(A,C) + p_A(A,C)\mu A dt + \frac{\sigma^2}{2}A^2 p_{AA}(A,C)dt + p_C(A,C)g(a)Cdt \\ &= p(a) + (\mu - g(a))ap'(a)dt + \frac{\sigma^2}{2}a^2p''(a)dt. \end{aligned} \tag{A.66}$$

To obtain the second line, we use the homogeneity of degree 0 of the price function, that is, $p(A/C) \equiv p(A,C)$, to replace $p_A(A,C) = p'(a)/C$, $p_{AA}(A,C) = p''(a)/C^2$ and $p_C(A,C) = -p'(a)A/C^2$. Substituting (A.66) into (A.65) and keeping only terms of order dt, we obtain

$$0 = -(r+\lambda)p(a) + (\mu - g(a))ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)]. \tag{A.67}$$

Equation (A.67) characterizes the price dynamics in region $[\underline{a}, \overline{a}]$ together with the boundary conditions $p(\overline{a}) = 1$ and $p(\underline{a}) = \varphi$.

Next, we solve for the price function for $\varphi = 0$. Anticipating on the result from

Proposition 2 that $\underline{a} = 0$ when $\varphi = 0$, we conjecture the following pricing function:

$$p(a) = \begin{cases} \sum_{k=1}^{3} b_k a^{-\gamma_k} & \text{if } \ 0 \le a < \overline{a}, \\ 1 & \text{if } \ a \ge \overline{a}. \end{cases} \tag{A.68}$$

The following computations will be useful to solve for equation (A.67). We have

$$p'(a) = -\sum_{k=1}^{3} b_k \gamma_k a^{-(\gamma_k+1)}, \tag{A.69}$$

$$p''(a) = \sum_{k=1}^{3} b_k \gamma_k (\gamma_k + 1) a^{-(\gamma_k+2)}, \tag{A.70}$$

$$\mathbb{E}[p(Sa)] = \int_0^\infty p(e^{-s}a)\xi e^{-\xi s} ds = \int_0^\infty \sum_{k=1}^{3} b_k e^{s\gamma_k} a^{-\gamma_k} \xi e^{-\xi s} ds = \sum_{k=1}^{3} \frac{b_k \xi a^{-\gamma_k}}{\xi - \gamma_k}. \tag{A.71}$$

Substituting into (A.67) and setting $g = 0$ (Lemma 2), we derive conditions on $\{\gamma_k\}_{k=1,2,3}$. Equalizing terms proportional to $a^{-\gamma_k}$, we obtain that for each $k \in \{1,2,3\}$, γ_k must be a root of characteristic equation (27). The roots of this third-order polynomial are

$$\gamma_k = -\frac{1}{2t_1}\left(t_2 + \zeta^\nu R + \frac{\Delta_0}{\zeta^\nu R}\right) \tag{A.72}$$

where

$$\Delta_0 = t_2^2 - 3t_1t_3, \quad \Delta_1 = 2t_2^3 - 9t_1t_2t_3 + 27t_1^2t_4,$$
$$R = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1+\sqrt{-3}}{2}, \quad \nu = \{0,1,2\},$$
$$t_1 = -\frac{\sigma^2}{2}, \quad t_2 = \mu + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -\mu\xi + \frac{\sigma^2}{2}\xi + r + \lambda, \quad t_4 = -r\xi.$$

According to Descartes' rule of sign, this polynomial has two positive roots and one negative root. Furthermore, using Budan-Fourier theorem, we can show that the negative root is strictly lower than -1. Because the price is bounded below by 0, the coefficients b_k, which correspond to positive roots must be 0. We now call γ the negative root of this polynomial and b the corresponding coefficient.

The price function is thus given by $p(a) = ba^{-\gamma}$ for a $\in [0, \overline{a}]$. To determine b, we use boundary condition $p(\overline{a}) = 1$ to get $b = \overline{a}^\gamma$. This concludes the proof.

A.5 Proof of Proposition 2

We proceed in three steps. First, we derive $e(a^\star)$ as a function of $\delta^\star$ for any $\varphi \in (0,1)$ to derive equation (25). Then, we show that $\overline{a} = 0$ when $\varphi = 0$. Finally, we solve explicitly for $e(a^\star)$ when $\varphi = 0$.

Step 1. The equity value for $a \in [\overline{a}, \infty)$ is given by (21). We are thus left to derive the HJB for the equity value at demand ratio $e(a^\star)$. The recursive equation is given by

$$E(a^\star C_{\text{-}}, C_{\text{-}}) = \mu^k \varphi C_{\text{-}} dt - \varphi dC + (1 - rdt)(1 - \lambda dt)\mathbb{E}\left[E(a^\star C_{\text{-}} + dA, C_{\text{-}} + dC) | \, dN_t = 0\right] \\ + (1 - rdt)\lambda dt \mathbb{E}\left[E(Sa^\star C_{\text{-}}, C_{\text{-}}) | \, dN_t = 1\right], \tag{A.73}$$

where the term on the first line corresponds to the case in which the adjustment in demand A_t is smooth ($dN_t = 0$), while the second term corresponds to the case in which demand jumps ($dN_t = 1$).

To express the term corresponding to Brownian shocks, use equation (21) and $dC = \delta(a^\star)Cdt$ to obtain the following relationship by Ito's Lemma:

$$\mathbb{E}\left[E(a^\star C_{\text{-}} + dA, C_{\text{-}} + dC) | \, dN_t = 0\right] = E(a^\star C_{\text{-}}, C_{\text{-}}) + \mu\big[e(a^\star) + 1 - \varphi\big]C^\star(A)dt \\ - (1 - \varphi)\delta^\star C^\star(A)dt. \tag{A.74}$$

Keeping only terms of order at least dt and dividing by $C^\star(A)$ in equation (A.73), we obtain

$$e(a^\star) = e(a^\star) + \big(-(r + \lambda)e(a^\star) + \mu\big[e(a^\star) + p(a^\star) - \varphi\big] - \delta^\star + \mu^k \varphi + \lambda \mathbb{E}[e(Sa^\star)]\big)\, dt, \tag{A.75}$$

which simplifies to equation (25).

Next, we compute the term $\mathbb{E}[e(Sa^\star)]$ in equation (25). Using (21), we get

$$\mathbb{E}[e(Sa^\star)] = \int_0^{ln(a^\star/\overline{a})} e(e^{-s}a^\star)\xi e^{-\xi s} ds \tag{A.76}$$

$$= \int_0^{ln(a^\star/\overline{a})} \Big[(e(a^\star) + 1 - \varphi)e^{-s} - 1 + \varphi\Big]\xi e^{-\xi s} ds \tag{A.77}$$

$$= \frac{\xi}{\xi + 1}\left(1 - \left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)(e(a^\star) + 1 - \varphi) - \left(1 - \left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)(1 - \varphi). \tag{A.78}$$

Plugging this equation in (A.75), we get

$$\left(r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda\xi}{\xi + 1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)e(a^\star) \\ = \left(\mu^k - \mu + \frac{\lambda}{\xi + 1} - \lambda\left(\frac{a^\star}{\overline{a}}\right)^{-\xi} + \frac{\lambda\xi}{\xi + 1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}\right)\varphi \\ + \left(\mu - \delta^\star - \frac{\lambda}{\xi + 1} - \frac{\lambda\xi}{\xi + 1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)} + \lambda\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right)p(a^\star). \tag{A.79}$$

After some manipulations, we can rewrite the objective function as

$$\frac{e(a^\star)+1-\varphi}{a^\star}=\frac{(\mu^k-r)\varphi+r-\delta^\star+\lambda(1-\varphi)\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{r-\mu+\frac{\lambda}{\xi+1}+\frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}. \tag{A.80}$$

Step 2. Now, we prove that $\underline{a}=0$ when $\varphi=0$. Equation (A.80) shows that for given $\{\overline{a},a^\star\}$, the objective function may depend on $\underline{a}$ only via the term $-\delta^\star$. From equation (23), this term itself may depend on $\underline{a}$ only via $\mathbb{E}[p(Sa^\star)]$ and is increasing with $\mathbb{E}[p(Sa^\star)]$. For given $(\underline{a},\overline{a})$, however, we showed in the proof of Lemma 3 that the price when $a\leq\overline{a}$ is fully pinned down by the boundary condition $p(\overline{a})=1$ and that $p(a)$ is positive and strictly increasing for all $a\in[0,\overline{a}]$ with $p(0)=0$. Hence, it must be that $\underline{a}=0$.

Step 3. Finally, we show that the maximization problem of the platform at date 0 is given by (28). Rewriting equation (A.80) for $\varphi=0$, we obtain

$$\frac{e(a^\star)+1}{a^\star}=\frac{r-\delta^\star+\lambda\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{r+\frac{\lambda}{\xi+1}-\mu+\frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}. \tag{A.81}$$

We derive $\delta(a^\star)$ as a function of the TMP parameters below. From equation (23), we have

$$\begin{aligned}
\delta^\star &= r-\ell(a^\star)+\lambda\left(1-\mathbb{E}[p(a^\star S)]\right),\\
&= r-\ell(a^\star)+\lambda-\lambda\left[\int_0^{\ln(a^\star/\overline{a})}\xi e^{-\xi s}ds+\int_{\ln(a^\star/\overline{a})}^{\infty}\left(\frac{a^\star}{\overline{a}}\right)^{-\gamma}e^{s\gamma}\xi e^{-\xi s}ds\right],\\
&= r-\ell(a^\star)+\lambda-\lambda\left[1-\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}\right]-\lambda\frac{\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}.
\end{aligned} \tag{A.82}$$

Substituting for $\delta^\star$ into (A.81), we obtain

$$e(a^\star)+1=\frac{\ell(a^\star)+\frac{\lambda\xi}{\xi-\gamma}\left(\frac{a^\star}{\overline{a}}\right)^{-\xi}}{r+\frac{\lambda}{\xi+1}-\mu+\frac{\lambda\xi}{\xi+1}\left(\frac{a^\star}{\overline{a}}\right)^{-(\xi+1)}}. \tag{A.83}$$

Simple computations show that this equation is equivalent to (28) if the liability constraint (29) holds. From Lemma 2, we have $e(a)=0$ for all $a\in[0,\overline{a}]$ and, from equation (21), $e(a)$ strictly increases with a for $a\in[\overline{a},\infty)$. Hence, limited liability holds for all a if $e(\overline{a})=0$. Using equation (20) with $\varphi=0$ and $p(a^\star)=1$, condition $e(\overline{a})=0$ is equivalent to equation (29). This concludes the proof.

A.6 Proof of Corollary 1

Proposition 2 shows that an equilibrium with positive equity value exists if there exist $(\overline{a},a^\star)$ with $\overline{a}\leq a^\star$ such that condition (29) holds. Using equation (28) to substitute for

$e(a^\star) + 1$, this condition holds if there exists $a^\star$ and $x \in [0,1]$ such that

$$\ell(a^\star)x - u - v(\gamma)x^{\xi+1} \geq 0, \quad \text{with} \quad u \equiv r + \frac{\lambda}{\xi+1} - \mu, \quad v(\gamma) \equiv \frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}. \quad \text{(A.84)}$$

To derive implications from this condition, define $H : x \mapsto \frac{x}{u+v(\gamma)x^{\xi+1}}$ and let x_{max} be the value of x where H attains its global maximum on $[0,1]$. We have

$$H'(x) \propto u - v(\gamma)\xi x^{\xi+1}, \tag{A.85}$$

which is strictly decreasing with x because $v(\gamma) > 0$ since $\gamma < -1$ (Lemma 3). Two cases are then possible. Either $H'(1) = u - \xi v(\gamma) \geq 0$ and $x_{max} = 1$ or $H'(1) < 0$ and $x_{max} = \left(\frac{u}{v(\gamma)\xi}\right)^{\frac{1}{\xi+1}}$ so that overall $x_{max} = \min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}$ and, for a given $a^\star$, a necessary and sufficient condition for the desired equilibrium to exist is

$$\ell(a^\star) \geq \frac{u + v(\gamma)x_{max}^{\xi+1}}{x_{max}} = \frac{u + v(\gamma)\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}}{\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}}. \tag{A.86}$$

A necessary condition for (A.86) to hold is $\ell(a^\star) \geq u$, which is equivalent to (30). This concludes the proof.

A.7 Proof of Proposition 3

We first state a series of Lemmas and prove them at the end of this section.

Lemma 5. *The equity value $e(a)$ is weakly convex and continuously differentiable, and stablecoin price function $p(a)$ is continuous and increasing.*

Lemma 6. *If equity value $e(a)$ is linear over some interval $[a_L, a_U]$, the equilibrium issuance policy features a target demand ratio $a^{jump} \in [a_L, a_U]$ such that the issuance policy for any $a \in [a_L, a_U]$ is to jump at a^{jump}.*

Lemma 7. *If equity value $e(a)$ is strictly convex over some interval $[a_L, a_U]$, the equilibrium debt policy is smooth in that region. Furthermore, there is no MPE with strictly positive equity value if the equilibrium issuance policy is smooth everywhere.*

Proposition 3 is then a corollary of the next result.

Lemma 8. *If the programmable interest rate rule is optimally chosen at date 0, there exist $(\overline{a}, a^\star)$ such that the equilibrium issuance policy is smooth over $[0, \overline{a}]$ and features a jump at some $a^\star \in [\overline{a}, \infty)$ when $a \in [\overline{a}, \infty)$.*

We now provide a proof for these lemmas.

Proof of Lemma 5. These properties follow from Lemma A.1 in DeMarzo and He (2021). □

Proof of Lemma 6. We first show that if equity value $e(a)$ is linearly increasing in a over some segment $[a_L, a_U]$ (with strictly positive slope), the equilibrium issuance policy is not smooth over this interval. We then show that there is a single jump point in this segment.

The proof is by contradiction. Suppose that $d\mathcal{G}_t = G(a)dt$ over $[a_L, a_U]$ with $g(a) \equiv G(a)/C$, the stablecoin issuance rate per unit of stablecoins. With a smooth debt policy, use equation (A.62) to rewrite the HJB equation for the equity value as follows:

$$(r+\lambda)e(a) = \max_g \Bigg\{ g(p(a)-\varphi) + \mu a e'(a) + (\mu^k - \delta(a))\varphi \\ + (g+\delta(a))(e(a)-e'(a)a) + \frac{\sigma^2}{2}a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \Bigg\}. \tag{A.87}$$

A smooth debt policy is optimal if the first-order condition with respect to g is satisfied; that is, if

$$p(a) - \varphi = e'(a)a - e(a). \tag{A.88}$$

The assumption that $e(a)$ is linear in a further implies that $p'(a) = e''(a)a = 0$, so we write $p(a) = p$ in what follows. Hence, equation (A.87) simplifies to

$$(r+\lambda)e(a) = \mu^k\varphi - \delta(a)p + \mu a e'(a) + \lambda\mathbb{E}[e(Sa)]. \tag{A.89}$$

We now establish a contradiction between equations (A.88) and (A.89) when $e(a)$ is linear. Taking the first-order-derivative with respect to a of the terms in (A.89), we obtain

$$(r+\lambda)e'(a) = -\delta'(a)p + \mu e'(a) + \lambda\mathbb{E}[e'(Sa)S]. \tag{A.90}$$

Adapting equation (A.67) to the general case with δ and ℓ different from 0, the HJB equation for the stablecoin price is given by

$$(r+\lambda)p(a) = \ell(a)p(a) + \delta(a)p(a) - (g(a)+\delta(a))ap'(a) + \mu a p'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda\mathbb{E}[p(Sa)], \tag{A.91}$$

which, for a constant $p(a) = p$, simplifies to

$$(r+\lambda)p = \ell(a)p + \delta(a)p + \lambda\mathbb{E}[p(Sa)]. \tag{A.92}$$

Combining equations (A.89), (A.90), and (A.92), we obtain

$$0 = (r+\lambda)(p(a) - \varphi + e(a) - e'(a)a) = (\mu^k - r)\varphi + \ell(a)p + \delta'(a)ap(a). \tag{A.93}$$

The last equality follows from equation (A.88). We proved this relationship for segments in which the equilibrium issuance policy is smooth. For segments over which the issuance policy features jumps, equation (20) shows that for any a, a' in this segment, we have

$$e(a') = \left[e(a) + p - \varphi\right]\frac{a'}{a} - (p - \varphi). \tag{A.94}$$

Taking the first-order derivative with respect to a' and then setting $a' = a$, we obtain equation (A.88).

Now, we establish a contradiction. Suppose first that $a_L = 0$. Thus $p = \varphi$ and $\mathbb{E}[p'(Sa)S] = 0$ for $a \in [a_L, a_U]$. Then, if $p \neq 1$ and thus $\ell(a) = 0$, it is immediate that equations (A.93) and (A.92) are inconsistent. If instead $p = 1$, these two equations imply that

$$(\mu^k - r)\varphi + \ell(a) - \ell'(a)a = 0,$$

but the functional form of $\ell(a)$ cannot be pinned down by these equilibrium conditions because it is a primitive of the problem.

Suppose now that $a_L > 0$. As a first subcase, suppose in addition that $p(a) = p \neq 1$, in which case $\ell(a) = 0$ by definition. Equations (A.93) and (A.92) then imply that

$$\delta'(a) = \frac{r - \mu^k}{ap}\varphi = -\lambda\mathbb{E}[p'(Sa)S]. \tag{A.95}$$

This equation cannot hold, because $r > \mu^k$ while $p' \geq 0$ by Lemma 5. Finally, suppose that $p(a) = 1$. Equations (A.92) and (A.93) imply together that

$$\frac{(\mu^k - r)\varphi + \ell(a) - \ell'(a)a}{a} = \lambda\mathbb{E}[p'(Sa)S]. \tag{A.96}$$

We have

$$\mathbb{E}[p'(Sa)S] = \int_0^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds = \int_{\ln(a/a_L)}^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds = \kappa a^{-(\xi+1)} \tag{A.97}$$

where $\kappa \equiv {a_L}^{\xi+1}\int_0^\infty p'(e^{-s}a_L)\xi e^{-s(\xi+1)}ds$ is a positive constant. To obtain the second line, we use the fact that p is constant over $[a_L, a_U]$. Thus, we must have

$$\ell(a) = \ell'(a)a - (\mu^k - r)\varphi a + \lambda\kappa a^{-\xi} \tag{A.98}$$

for $a \in [a_L, a_U]$. Hence, assuming that the issuance policy is smooth implies that $\ell(a)$ must be the solution of functional equation (A.98). Again, this is a contradiction because $\ell(a)$ is an exogenous function in this problem.

Now, we show that there is only one jump point $a_{jump} \in [a_L, a_U]$ if $e(a)$ is linear over $[a_L, a_U]$. Suppose there are two such jump points (the argument generalizes for more jump points) labeled a^1_{jump} and a^2_{jump}. Then, the single-peak property iii in Assumption 1 ensures that there must be one jump point—say, a^1_{jump}—for which liquidity benefits $A\ell(a)/a$ are larger than at a^2_{jump}. Hence, to maximize its date-0 value, the platform would strictly prefer jumping to a^1_{jump} from any point in $[a_L, a_U]$ rather than to a^2_{jump}.

We are left to show that jumping to a^1_{jump} instead of a^2_{jump} is compatible with the equilibrium issuance policy. By Lemmas 7 and 6, the issuance policy features jumps on $[a_L, a_U]$ only if equity value is linear and price is constant. Hence, from any state a with jump point a^2_{jump}, we have

$$e(a) = \left[e(a^2_{jump}) + p(a^2_{jump}) - \varphi\right]\frac{a}{a^2_{jump}} = \left[e(a^1_{jump}) + p(a^1_{jump}) - \varphi\right]\frac{a}{a^1_{jump}}. \tag{A.99}$$

Hence, jumping to a^1_{jump} is also an optimal equilibrium issuance policy. This equality simply reflects the fact that the platform is indifferent ex post between all points in $[a_L, a_U]$. At date-0, however, the platform would choose jump point a^1_{jump} as the sole jump point. □

Proof of Lemma 7. We first show that if equity value $E(A, C)$ is strictly convex in C over some interval, the issuance policy is smooth in this region. Given any debt level $\hat{C}$, equity holders have the option to adjust the stock of stablecoins to C by issuing $C - \hat{C}$ at the price of $p(A, C)$. Therefore, by optimality of the debt issuance policy, the equity value at $\hat{C}$ must satisfy

$$E(A, \hat{C}) \geq E(A, C) + p(A, C)(C - \hat{C}). \tag{A.100}$$

To show that discrete repurchases are suboptimal, we prove that inequality (A.100) is strict if the equity value is strictly convex with respect to its second argument. Suppose to the contrary that there exists $C' \neq C$ such that $E(A, C') = E(A, C) + p(A, C)(C - C')$. By strict convexity of E, we get that for all $x \in]0, 1[$

$$E(A, xC + (1 - x)C') < xE(A, C) + (1 - x)E(A, C') = E(A, C) + (1 - x)p(A, C)(C - C'). \tag{A.101}$$

Using then condition (A.100) for $\hat{C} = xC + (1 - x)C'$, we obtain

$$E(A, xC + (1 - x)C') \geq E(A, C) + (1 - x)p(A, C)(C - C'), \tag{A.102}$$

which is a contradiction with (A.101). Thus, it must be that

$$E(A, C') > E(A, C) + p(A, C)(C - C'). \tag{A.103}$$

Hence, any discrete issuance with $|C - C'| > 0$ would be suboptimal for shareholders; that is, the debt policy must be smooth if E is strictly convex in C.

Second, we show that there cannot be an equilibrium with positive equity value and a smooth debt policy for all a. For the equilibrium issuance policy to be smooth, it must be that equation (A.88) holds. The platform starts at date 0 if liquidity benefits can be captured in equilibrium. Two cases are possible, given that p is weakly increasing with a. First, there exists an interval $[a_L, a_U]$ over which the price is constant with $p(a) = 1$. Equation (A.88) then implies that e is linear. We can then use Lemma 6 to show that the equilibrium debt policy features jump, a contradiction. The second case is that of a single point $\hat{a}$ for which $p(\hat{a}) = 1$ and such that the platform spends strictly positive time at $\hat{a}$. Such a feature requires that the platform perform a control at $\hat{a}$. The same arguments used in DeMarzo and He (2021), however, show that such a policy cannot be part of an equilibrium in a region in which the equity value is strictly convex.

□

Proof of Lemma 8. From Lemma 5, we know that since the equity value $e(a)$ is weakly convex, there must be a strictly ordered sequence $\{a^{(n)}\}_{n\geq 0}$ such that $a_0 = \underline{a}$ is the liquidation threshold and $\lim_{n\to\infty} a^{(n)} = \infty$ such that on each segment $[a^{(n)}, a^{(n+1)}]$, e is either strictly convex or linear, with different convexity on two consecutive segments.

Our second step is to show that there is at least 1 segment with $e(a)$ strictly convex (possibly empty), and one segment with $e(a)$ linear. We first establish that the equity value cannot be linear on segment $[a^{(0)}, a^{(1)}]$ unless $a^{(0)} = 0$ and $\varphi = 1$. Suppose first that $a^{(0)} > 0$ so that the platform may liquidate itself in equilibrium. If $e(a)$ is linear over $[a^{(0)}, a^{(1)}]$, there is a kink in the equity value at $a^{(0)}$ such that $\lim_{a\downarrow a^{(0)}} e'(a) \neq 0$, which is incompatible with an optimal default decision and the corresponding smooth-pasting condition. Suppose now that $a^{(0)} = 0$ so that the platform never defaults in equilibrium. If $e(a)$ is linear on $[0, a^{(1)}]$, there must be $a^{jump} \in [0, a^{(1)}]$ such that the issuance policy is to jump at a^{jump} from any point in $[0, a^{(1)}]$ by Lemma 6. This implies that for any $a \in [0, a^{(1)}]$

$$e(a) = \left[e(a^{jump}) + p(a^{jump}) - \varphi\right] \frac{a}{a^{jump}} - (p(a^{jump}) - \varphi),$$

with $p(a^{jump})$ constant over $[0, a^{(1)}]$ and $p(a^{jump}) > \varphi$ unless $\varphi = 1$. Hence, when $a \to 0$ limited liability is violated, except in the case $\varphi = 1$. This proves that the equity value is strictly convex over $[0, a^{(1)}]$ unless $\varphi = 1$ and $\underline{a} = 0$. In that case, the equilibrium equity value may be linear for all a.

Second, Lemma 7 implies that there must exist a segment over which $e(a)$ is linear. The last step of the proof is to show that there exists $\bar{a}$ such that the equity value is strictly convex over $[\underline{a}, \bar{a}]$ and linear over $[\bar{a}, \infty)$. Characterization of the equilibrium issuance policy as a targeted Markov policy then follows from Lemmas 5, 6, and 7. Let $\delta(a)$ be a programmable interest rate rule that induces an MPE with strictly positive equity value, and with issuance policy $d\mathcal{G}$ such that there exists a segment $[a^{(2)}, a^{(3)}]$ over which e is strictly convex. We want to show that there exists an alternative rule $\hat{\delta}(a)$ that induces an MPE with issuance policy $d\hat{\mathcal{G}}$ such that $e(a)$ has the desired properties and the date-0 platform value is strictly higher.

We first construct an alternative policy and its induced equilibrium. Let $a^\star$ be the target value in the first linear region $[a^{(1)}, a^{(2)}]$ for equity in the equilibrium induced by the original policy. Construct the alternative policy and the induced equilibrium as follows. Set $\hat{\delta}(a) = \delta(a)$ for all a and $d\hat{\mathcal{G}}(a, C) = d\mathcal{G}(a, C)$ for $a \leq a^\star$ and $d\hat{\mathcal{G}}(a, C) = A/a^\star - C$ for $a \geq a^\star$. Next, set the same liquidation policy $\hat{\underline{a}} = \underline{a}$. Finally, conjecture that in the equilibrium induced by the alternative policy, equity value $\hat{e}(a)$ is linear and price $\hat{p}(a)$ is constant for all $a \in [a^{(1)}, \infty)$.

Next, we argue that the issuance policy $d\hat{\mathcal{G}}(a, C)$ and the liquidation threshold $\hat{\underline{a}}$ are equilibrium policies induced by the alternative rule $\hat{\delta}(a)$. The subspace $[0, a^\star]$ is absorbing for the equilibrium induced by the original policy, because there are only downward jumps to A and the platform jumps to $a^\star$ from any $a \in [a^{(1)}, a^{(2)}]$. Hence, the fact that $d\mathcal{G}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the original interest rate policy implies that $d\hat{\mathcal{G}}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the alternative interest rate policy. The same argument applies to the liquidation threshold $\underline{\hat{a}} = \underline{a}$. This argument also implies that $\hat{e}(a) = e(a)$ and $\hat{p}(a) = p(a)$ for all $a \in [0, a^\star]$. We are thus left to show that $d\hat{\mathcal{G}}(a, C)$ is an equilibrium issuance policy on the rest of the state space, that is, for $a \in [a^{(2)}, \infty)$. This result follows from the observation that $\hat{e}(a)$ is linear over $a \in [a^{(1)}, \infty)$ and $\hat{p}(a)$ is constant. This implies that jumping to any point in $a \in [a^{(1)}, \infty)$, including a^*, can be part of an equilibrium issuance policy, as shown above.

Third, we show that $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$ in the equilibrium induced by the original policy, and thus $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$. Equity value is linear over $[a^{(1)}, a^{(2)}]$ and the equilibrium issuance policy is to jump at $a^\star \in [a^{(1)}, a^{(2)}]$ when $a \in [a^{(1)}, a^{(2)}]$. Hence, the price $p(a) = p$ must be constant over $[a^{(1)}, a^{(2)}]$. Since $[0, a^\star]$ is an absorbing subspace for the equilibrium induced by the original policy, it must be that $p = 1$. If not, investors never enjoy any liquidity benefit for $a \in [0, a^\star]$ and thus $p(a) = e(a) = 0$ for all $a \in [0, a^\star]$, which is a contradiction. To see this, suppose first that $p < 1$. By monotonicity of p, we have $p(a) < 1$ for all $a \in [0, a^{(2)}]$, which implies that investors never enjoy the liquidity benefit. Conversely, if $p > 1$ over $[a^{(1)}, a^{(2)}]$, we have $p(a) = 1$ for a unique $a \in [0, a^{(1)})$ because $p(a)$ is strictly increasing over $[0, a^{(1)})$, since $e(a)$ is strictly convex (see the proof of Lemma 7). With a smooth equilibrium issuance policy on $[0, a^{(1)}]$, this state is not visited with positive probability and thus investors enjoy liquidity benefit with zero probability, which again leads to a contradiction. Hence, $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. This implies $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$ in the equilibrium induced by the alternative policy.

Finally, we can show that the platform value at date 0 is higher under the alternative policy than under the original policy. The platform's value at date 0 is given by equation (10), which we rewrite here for convenience.

$$E_0 = \mathbb{E}\left[\int_0^\tau e^{-rt}\ell(A_t, C_t)C_t\mathbf{1}_{p(A_t,C_t)=1} + (\mu^k - r)\varphi C dt \middle| A_0, C_0 = 0\right]. \tag{A.104}$$

In any equilibrium, liquidity benefits are only enjoyed when $a \in [a^{(1)}, a^{(2)}]$ because $p(a) \neq 1$ for $a \notin [a^{(1)}, a^{(2)}]$. Under the alternative policy, $a^\star \in [a^{(1)}, a^{(2)}]$ is reached immediately at date 0 by design because the equilibrium issuance policy is to jump to $a^\star$ when no stablecoins are outstanding ($a = \infty$). In the equilibrium induced by the original policy, however, the optimal choice at date 0 is some $a^{\star\star} > a^{(2)}$ by design of the original policy. Denote τ_f the first (stochastic) time the platform enters the region $[a^{(1)}, a^{(2)}]$ under the original policy. We have

$$E_0 = \mathbb{E}[E^{-r\tau_f}]\hat{E}_0 + \mathbb{E}\left[\int_0^\infty e^{-rt}(\mu^k - r)\varphi C dt\right] < E_0, \tag{A.105}$$

because no liquidity benefit is enjoyed before the platform reaches $[a^{(1)}, a^{(2)}]$. The inequality follows from the fact that $\mathbb{E}[\tau_f] > 0$ by design of the original policy and $\mu^k < r$.

We have shown that the original policy is strictly dominated. Hence, in an equilibrium induced by an optimal programmable interest rate rule, the issuance policy must belong to the class of TMPs. $\square$

This concludes the proof.

A.8 Proof of Lemma 4

We derive conditions that rule out deviations from a TMP in the target region $[\overline{a}, \infty)$. The conjectured issuance policy features a jump to $a^\star$ from any point in the target region. To verify that this policy to be ex post optimal, we consider "one-shot" deviations, whereby the platform deviates and then follows the equilibrium policy from the value of the demand ratio after the deviation.

Consider state $(A, C_{\text{-}})$ such that $a = A/C_{\text{-}} \geq \overline{a}$. The (conjectured) equilibrium policy for the platform is to jump to $C^\star(A)$, that is,

$$E(A, C_{\text{-}}) = E(A, C^\star(A)) - (1-\varphi)(C_{\text{-}} - C^\star(A)). \tag{A.106}$$

Now consider a jump deviation to some $C \in (0, A/\overline{a})$ after which the platform stays at C for dt and then reverts to the conjectured equilibrium policy. The value from this deviation is

$$\tilde{E}(A, C_{\text{-}}, C) = \hat{E}(A, C) - (\hat{p}(A, C) - \varphi)(C_{\text{-}} - C) \tag{A.107}$$

where $\hat{E}(A, C)$ is the payoff of the platform when it stays at C during dt (instead of jumping to $C^\star(A)$) and $\hat{p}(A, C)$ is the price at (A, C) compatible with this behavior. By definition of the conjectured equilibrium policy, we also have

$$E(A, C_{\text{-}}) = E(A, C) - (1-\varphi)(C_{\text{-}} - C) \tag{A.108}$$

and the deviation payoff is

$$\tilde{E}(A, C_{\text{-}}, C) = \hat{E}(A, C) - (1-\varphi)(C_{\text{-}} - C) + (1 - \hat{p}(A, C))(C_{\text{-}} - C). \tag{A.109}$$

Hence, the deviation is not profitable if, for all $C \leq A/\overline{a}$, we have

$$\hat{E}(A, C) + (1 - \hat{p}(A, C))(C_{\text{-}} - C) \leq E(A, C). \tag{A.110}$$

The value of stay inactive at C during time interval dt before reverting to the equilibrium policy is given by

$$\begin{aligned}\hat{E}(A, C) = \mu^k \varphi C dt - \delta(a)\varphi C dt + (1 - rdt)(1 - \lambda dt)\mathbb{E}\left[E(A + dA, C + \delta(a)Cdt)\right] \\ + (1 - rdt)\lambda dt \mathbb{E}[E(SA, C)].\end{aligned} \tag{A.111}$$

When $a \in [\underline{a}, \infty)$, rewriting (A.106) the equilibrium equity value is given by

$$E(A, C) = \frac{A}{a^\star} e(a^\star) + (p(a^\star) - \varphi)(C^\star(A) - C) = \frac{e(a^\star) + p(a^\star) - \varphi}{a^\star} A - (p(a^\star) - \varphi)C. \tag{A.112}$$

Hence, we get

$$\mathbb{E}\left[E(A+dA, C+\delta(a)Cdt)\right] = E(A,C) + \mu\frac{e(a^\star)+p(a^\star)-\varphi}{a^\star}Adt - (p(a^\star)-\varphi)\delta(a)Cdt. \tag{A.113}$$

Plugging (A.113) into (A.111) and keeping only terms of order at least dt, we obtain

$$\begin{aligned}\hat{E}(A,C) = E(A,C) - (r+\lambda)E(A,C)dt + \mu\big[e(a^\star)+p(a^\star)-\varphi\big]C^\star(A)dt \\ - p(a^\star)\delta(a)Cdt + \mu^k\varphi Cdt + \lambda\mathbb{E}[E(SA,C)]dt.\end{aligned} \tag{A.114}$$

Note that $a = A/C_{\text{-}}$, given that the interest rate policy $\delta(a)$ depends only on state variables $(A, C_{\text{-}})$, and not on (A,C), where C is the jump deviation we consider.

For the price in the deviation, we have

$$\hat{p}(A,C) = \delta(a)dt + (1-rdt)\mathbb{E}[p_{t+dt}] = 1-(r-\delta(a))dt.$$

The second equality obtains because $p_{t+dt} = 1$ since we consider a one-shot deviation. Hence, the deviation considered is not profitable if and only if

$$-(r+\lambda)E(A,C)+\mu\big[e(a^\star)+1-\varphi\big]C^\star(A)-\delta(a)C+\mu^k\varphi C+\lambda\mathbb{E}[E(SA,C)] \le (r-\delta(a))(C-C_{\text{-}}).$$

Note that

$$(r+\lambda-\mu)e(a^\star) = \mu^k\varphi + \mu(1-\varphi) - \delta^\star + \lambda e(Sa^\star)). \tag{A.115}$$

Thus,

$$\begin{aligned}\mu E(A, C^\star(A)) =& (r+\lambda)\left(E(A,C) + (1-\varphi)(C - C^\star(A))\right) \\ & - \mu^k\varphi C^\star(A) - \mu(1-\varphi)C^\star(A) + \delta^\star C^\star(A) - \lambda E(SA, C^\star(A)).\end{aligned} \tag{A.116}$$

Substituting in and rewriting in terms of $a = A/C_-$ and $a' = A/C$, we get

$$\begin{aligned}\Big[(r+\lambda)(1-\varphi)+\mu^k\varphi\Big]\left(1-\frac{a'}{a^\star}\right) \le& \lambda\mathbb{E}[e(Sa^\star)]\frac{a'}{a^\star} - \lambda\mathbb{E}[e(Sa')] \\ & + \delta(a) - \delta^\star\frac{a'}{a^\star} + (r-\delta(a))\left(1-\frac{a'}{a}\right).\end{aligned} \tag{A.117}$$

This concludes the proof.

A.9 Proof of Proposition 5

Substituting $\delta(a)$ by r and $\delta^\star$ by $r - \ell(a^\star(1))$ in equation (A.117), we obtain

$$\left[(r+\lambda)(1-\varphi)+\mu^k\varphi\right]\left(1-\frac{a'}{a^\star}\right) \leq \lambda\mathbb{E}[e(Sa^\star)]\frac{a'}{a^\star} - \lambda\mathbb{E}[e(Sa')] + r - (r-\ell(a^\star(1)))\frac{a'}{a^\star}. \tag{A.118}$$

With $\varphi = 1$, we get $e(a) = e(a^\star)\frac{a}{a^\star}$. Thus,

$$0 \leq (r-\mu^k)C + (\ell(a^\star(1)) - (r-\mu^k))C^\star. \tag{A.119}$$

This condition is satisfied provided that a fully collateralized stablecoin platform is profitable (see Lemma 1). This concludes the proof.

A.10 Proof of Proposition 6

The first part of Proposition 6 follows from the argument in the text. If $p_t > \varphi$, a vault owner can get an infinite profit by issuing stablecoins backed by collateral, which is incompatible with p_t being an equilibrium price.

To obtain the second part of Proposition 6, we derive the dynamic equation for a vault value. From equation (40), the vault value at date t per coin outstanding is $\varphi - p_t$. As mentioned in the main text, the vault value is independent of the amount issued, so we can derive it as if the vault owner issued zero stablecoins. We thus have

$$\varphi - p_t = -\varphi s_t dt + \varphi\mu^k dt + (1-rdt)\mathbb{E}_t[(\varphi - p_{t+dt})]. \tag{A.120}$$

In equation (A.120), the first (second) term corresponds to the collateral cost generated by the platform's fee policy (the return on collateral). Expanding the term of equation (A.120) inside the expectation, we have

$$\varphi - p_t = -\varphi s_t dt + \varphi\mu^k dt + (1-rdt)\mathbb{E}_t\left[\varphi - p_t - dt\frac{dp_t}{dt}\right]. \tag{A.121}$$

Observe that dp_t must be of order dt since there cannot be a jump in the price. Then, keeping only terms of order dt in (A.121) and rearranging, we obtain equation (42). This concludes the proof.

A.11 Proof of Proposition 7

The HJB equation for the price is

$$p_t = \ell(a_t)\mathbb{1}\{p_t = 1\}dt + \mathbb{1}\{p_t = 1\}dt + (1-rdt)\mathbb{E}\left[p_{t+dt}\right]. \tag{A.122}$$

Keeping only terms of order dt, we obtain

$$rp_t = \ell(a_t)\mathbb{1}\{p_t = 1\} + \mathbb{1}\{p_t = 1\} + \mathbb{E}_t\left[\frac{dp_t}{dt}\right]p_t. \tag{A.123}$$

Combining equations (42) for $\varphi = 1$ and (A.123), we get

$$(s_t - \delta_t)p_t = \ell_t\mathbb{1}\{p_t = 1\} + \mu^k - r. \tag{A.124}$$

The maximization problem is then given by

$$E_t = \max_{\tau,s,\delta}\mathbb{E}_t\left[\int_0^\tau e^{-r(s-t)}\Big(\ell_s\mathbb{1}\{p_t = 1\} + \mu^k - r\Big)C_s ds\right] \tag{A.125}$$

subject to $\varphi - p_t \geq 0$. Hence, the platform chooses the policy that maximizes the present discounted value of seigniorage revenues net of collateral costs. This means that the platform seeks to implement the same policy as in the full-commitment outcome of Proposition 1. In particular, it seeks to implement supply rule (44) and interest rate rule $\delta^* = r - \ell(a^\star(1))$.

We are left to shows how the platform can implement the desired policy when it does not directly control issuance. From arbitrage condition (42), their supply function is a step function given by

$$d\mathcal{G}^i = \begin{cases} +\infty & \text{if} \quad s_t < (\mu^k - r)/p_t + r - \mathbb{E}_t\left[\frac{dp_t}{dt}\right], \\ -C^i_{t\text{-}} & \text{if} \quad s_t > (\mu^k - r)/p_t + r - \mathbb{E}_t\left[\frac{dp_t}{dt}\right], \end{cases} \tag{A.126}$$

and it is indeterminate if $s_t = (\mu^k - r)/p_t + r - \mathbb{E}_t\left[dp_t/dt\right]$. To satisfy equation (42) and implement the price peg at the target $a^\star(1)$, we must have $s^\star = \mu^k$. To implement $C^\star(A, 1)$, the platform uses a fee schedule contingent on the amount of stablecoins, whereby vault owners are induced to issue (buy back) stablecoins if $C > C^\star(A, 1)$ ($C < C^\star(A, 1)$). Such a schedule is given by (45). In this case, the only equilibrium supply is $C_t = C^\star(A, 1)$. In particular, we have $s^\star - \delta^\star = \ell(a^\star(1)) + \mu^k - r$. This last equation combined with $s^\star = \mu^k$ implies that $\delta^\star = r - \ell(a^\star(1))$. This concludes the proof.

Online Appendix

A Stablecoins in the Midst of the 2022 Crypto Crash

This appendix provides a short introduction to the variety of stablecoin pegging mechanisms in practice, with an emphasis on their performance during the crypto crunch of May 2022. We review two custodial (USD Coin and Tether), a purely algorithmic (Terra), an overcollateralized (DAI), and a partially collateralized (FRAX) stablecoin platform. At the beginning of May 2022, these five stablecoins accounted for more than 80% of the total stablecoin market.

USD Coin

USD Coin (USDC) is a custodial (fully collateralized) stablecoin managed by the Centre consortium on behalf of the peer-to-peer payment technology Circle headquartered in Boston, MA. USDC effectively acts as a narrow bank by backing its stablecoins exclusively with cash (bank deposits or equivalents) and short-term Treasury securities and providing full redemption. During the May 2022 crypto crash, USDC fared particularly well, as can be seen in Figure 4: It maintained its peg, and the quantity of USDC outstanding increased during that time period. Given its conservative reserves management strategy, USDC presumably benefited from a "flight to safety" because investors were fleeing from fast depreciating cryptocurrencies and other stablecoins.

Tether

Tether (USDT) is another custodial stablecoin that is a native of the Ethereum ledger and issued by Tether Limited company, which is domiciled in Hong Kong under the umbrella of Tether Holdings Limited in the British Virgin Islands. Although Tether claims to be "fully backed by US dollar reserves," its definition of reserves appear to be less restrictive than the one applied by USDC, and also includes privately issued commercial paper and corporate bonds but also volatile cryptocurrencies.[39] Griffin and Shams (2020) report suspicious transaction patterns on the blockchain and suggest that the platform has been using unbacked Tether creation to purchase large quantities of Bitcoin to support its price.

Figure 4 displays the time-series price of Tether and quantities outstanding. We can observe a sharp reduction in supply around the crypto crash of May 2022, along with a temporary depegging. Tether nonetheless re-anchored within a couple of days and proved able to absorb the $5B of redemption it faced.

[39] Since 2021 and a $41MM fine by the Commodity Futures Trading Commission for misleading claims that it was fully backed by the US dollar, Tether Holdings Limited regularly reports a reserves audit from Cayman-based auditing companies.

Terra

Terra (UST) is a prime example of a fully algorithmic (uncollateralized) stablecoin. As described in the main text, algorithmic stablecoins such as Terra are uncollateralized and rely exclusively on quantity adjustments through smart contracts that specify rules for stablecoin issuances and buybacks. In the case of Terra, these are ruled through an external module that allows any investor to exchange 1 unit of stablecoin (Terra) for 1 dollar's worth of governance token (Luna) and vice versa. Between its introduction in early 2020 and the crypto crash of May 2022, Terra was one of the fastest-growing stablecoin platforms. By May 2022, the quantity of stablecoin Terra outstanding was close to $20B, while the governance token Luna had a peak market capitalization of $40B.

As can be seen in Figure 5, the platform completely collapsed between May 7 and May 12, 2022. In the right panel of Figure 5, we see how the platform attempted but failed to defend the peg. On May 12, the platform burnt around 8B of Terra, partly through the issuance of additional Luna at an exponential pace. As can be seen in the left panel, this massive issuance of Luna led to the collapse of its price to zero. Simultaneously, the Terra Foundation liquidated around $3B of Bitcoin it had held in reserves. Given the size of the shock, these adjustments were not sufficient to re-anchor the peg, and the value of Terra eventually also fell very close to zero.

DAI

DAI is a fully decentralized, fully collateralized stablecoin platform. Because of its decentralized nature, DAI is slightly more complex than other stablecoins and requires a longer description. With DAI, every user is able to deposit some Ethereum-based crypto-asset as collateral in a smart contract called a collateralized debt position (CDP). The user can then issue and sell DAI stablecoin tokens against this collateral up to a certain collateralization threshold, while effectively retaining an equity tranche in the CDP. In doing so, CDP users acquire a leveraged position in the collateral asset. Initially, it was only possible to use Ethereum as a collateral asset, but the platform migrated to a multiple collateral system at the end of 2019. Since then, the custodial stablecoin USD Coin (see above) has been used extensively as collateral for DAI. To close the CDP and retrieve the locked collateral, the owner has to repurchase and burn all previously issued DAI from the secondary market.

The platform also issues its own governance token, Maker (MKR). Holding Maker allows the user to vote on key policies of the platform and effectively confers the right to future seigniorage revenues. The platform is able to generate revenues for Maker holders by collecting "stability" fees from CDP owners. These fees accrue to a "buffer" fund up to a certain limit and are then distributed to Maker holders as dividends.

The pegging mechanism in DAI is tied to its collateralization. When the collateral in a CDP falls below the required threshold, the position is automatically liquidated and collateral assets are sold in an auction to burn the corresponding DAI. When auction proceeds are insufficient to repurchase all DAI issued by the CDP, new Makers are automatically issued to cover the shortfall. As shown in Figure 5, we can see that this mechanism was at play during the May 2022 crypto market crash. The platform then liquidated for $3B worth of collateral in CDPs in order to burn more than $2B worth of DAI. This process was nonetheless done in an orderly fashion, and parity was maintained throughout. As can be seen from the right-most panel, no additional Maker was required to be issued.

FRAX

Frax (FRX) is a partially collateralized platform that can be thought of as a hybrid between Terra and DAI. As with Terra, users can exchange the stablecoin FRAX for the platform's governance token Frax Shares (FRS) and the converse. Because the platform is partly collateralized, the swap module requires that users bring both FRS and collateral in a given proportion. For instance, if the collateralization ratio is 90% and Frax is trading for more than 1 USD, users can exchange 90 USD Coins and $10 worth of FRS in exchange for 100 Frax and sell them for a profit. The collateralization ratio in Frax is automatically reduced in expansion and increased in contraction, so that with a large surge in issuance, Frax would converge to a fully algorithmic platform like Terra.

In early May 2022, Frax had a collateralization rate of 86.75%. As can be seen in Figure 5, the platform managed to burn around a $1B without breaking its peg.

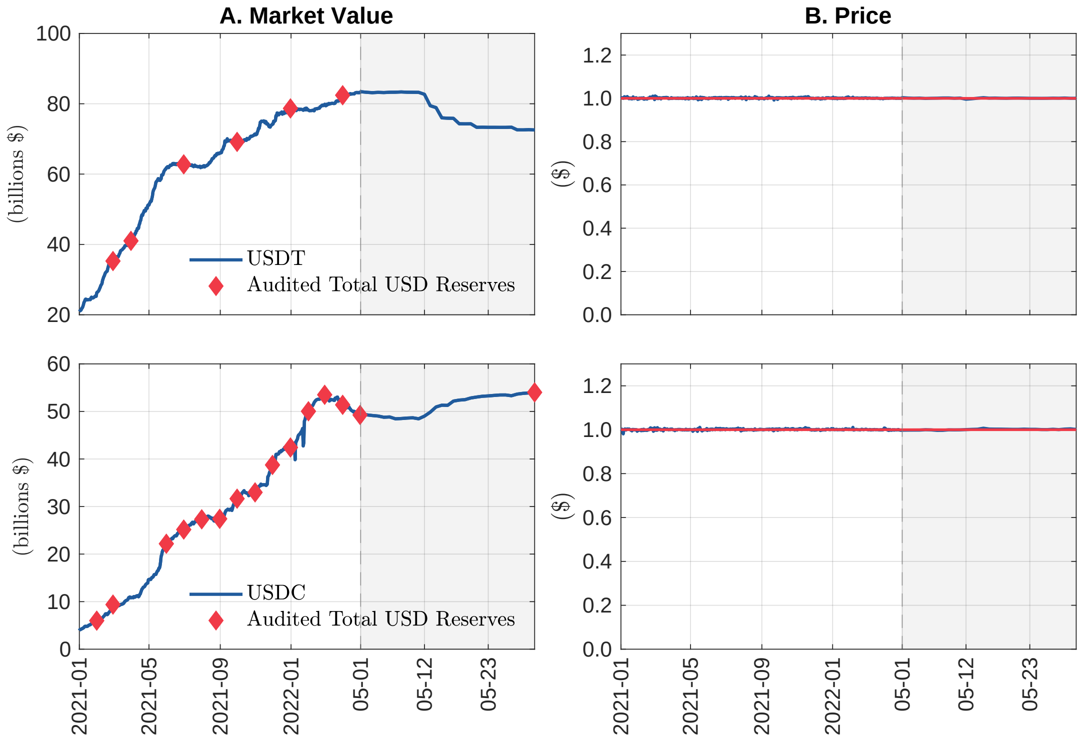

Figure 4: Custodial Stablecoins Time Series. This figure illustrates the daily time series of market capitalization and price for Tether (USDT, first row) and USD Coin (USDC, second row). The first portion of each graph spans the period from January 2021 to April 30, 2022, while the gray shaded area zooms in on May 2022. Pink diamond markers in Panels A illustrate the total USD value of reserves backing the stablecoin, as certified through external audits made available on the platforms' respective web pages. Data sources: Market capitalization and prices are all retrieved through the CoinGecko API.

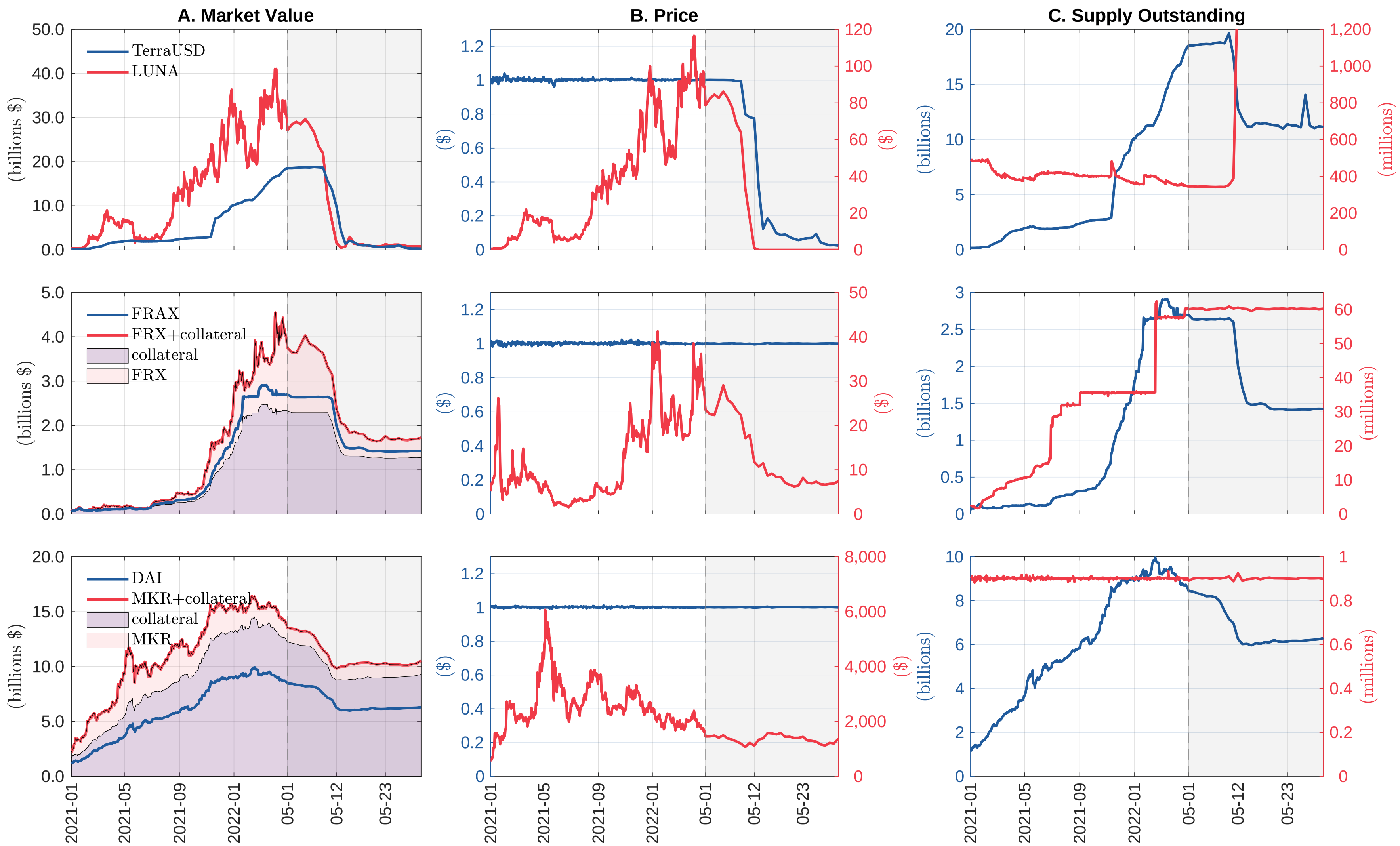

Figure 5: Algorithmic Stablecoins Time Series This figure illustrates the daily time series of market capitalization, price, and circulating supply, as denoted in each column title, for three algorithmic stablecoins. The first portion of each graph spans the period from January 2021 to April 30, 2022, while the gray shaded area zooms in on May 2022. Each row plots the dynamics for a given stablecoin, as labeled in the first column; the blue solid line refers to the stablecoin asset, while the pink solid line (or light pink shaded area in Panels A) refer to the corresponding governance token. Data sources: Market capitalization, prices, and supply outstanding are all retrieved through the CoinGecko API. The total USD value of FRAX collateral, illustrated in light shaded violet in the second row, was manually collected from https://app.frax.finance. The amount of DAI collateral was obtained by aggregating across all collateral assets, using the time-series debt data made available on Dune Analytics by @adcv via https://dune.com/queries/865375; we apply an adjustment factor to account for underestimating measurement error and impute the historical USD value of DAI collateral, illustrated in light violet in the last row, by rescaling the series by the ratio of Total DAI Locked ($) from https://daistats.com#/overview to the aggregated collateral series, both observed as of July 11, 2022, assuming a constant scaling factor.

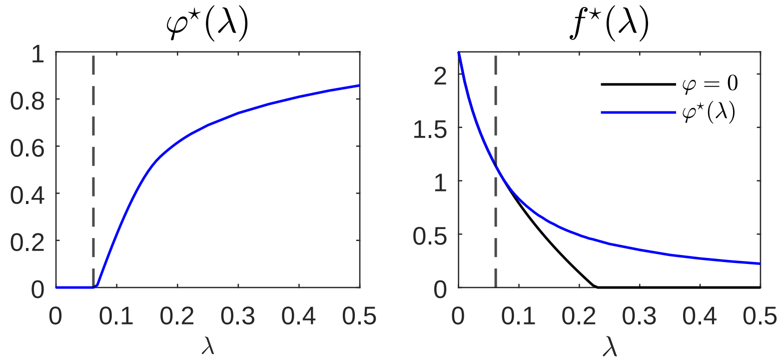

Figure 6: Full-commitment solution with limited liability. The function $\varphi^\star(\lambda)$ represents the optimal collateralization rate $\varphi^\star$ for different levels of large demand shock intensity λ. The function $f^\star(\lambda)$ represents the total platform value $(e(a^\star)+p(a^\star)-\varphi^\star)/a^\star)$ at the optimal target demand ratio $a^\star$ and either optimal collateralization rate $\varphi = \varphi^\star$ (blue) or without collateral $\varphi = 0$ (black) for different levels of large demand shock intensity λ. The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\mu^k = 0.055$, $\sigma = 0.1$, $\ell(A, C) = r\exp(-C/A)$, $\xi = 6$. The numerical solution algorithm is described in the Internet Appendix.

B Partially Collateralized Platforms: Numerical Solution

Partially collateralized platform, $\varphi \in (0, 1)$, do not have analytical solutions. In Figure 6, we solve numerically for the optimal collateralization rule $\varphi^\star(\lambda)$ for different demand shock intensities λ. As λ goes up, the likelihood that limited liability constraint (15) binds increases together with the probability of a large negative shock. Collateral thus becomes more useful because a higher collateralization ratio φ relaxes constraint (15): The platform can finance purchases from collateral holdings to a greater extent when φ is high. In line with Proposition 1, the right panel of Figure 6 shows that collateral is necessary for a stablecoin platform to exist when negative shocks are likely enough (high λ). With full collateralization ($\varphi = 1$), a platform always exists for all values of λ, as shown above. In practice, there exists a large heterogeneity of platform designs, ranging from uncollateralized ones such as Terra-Luna to partially collateralized ones such as FRAX to fully collateralized ones such as DAI. Our model suggests that an optimal collateralization ratio trades off stability with platform profits.

In this appendix, we describe the algorithm to solve the full-commitment problem with collateral. We solve for $f^\star(\lambda, \varphi, a^\star) \equiv (e^\star + p^\star)/a^\star$ for $\{\lambda, \varphi, a^\star\} \in [0, 1] \times [0, 1] \times [1, 4]$ on a $40 \times 20 \times 20$ grid following the pseudo-algorithm below. We note that for the partially collateralized case, $\underline{a} > 0$ and there is a reverting boundary at $\underline{a}$. Indeed, if the platform liquidates, debt holders receive the collateral and thus $p(\underline{a}) = \varphi$. However, if $p(\underline{a}) = \varphi$, then the net cost of repurchasing stablecoins to equity holders is 0 and not doing these repurchases would be a net loss to the date-0 equity value.

Because the ODE is stiff otherwise, we constrain $g(a)$ to be greater or equal to -10. We use the Matlab function ode23. Start with $\underline{a}_0^d = 0$, $\underline{a}_0^u = 1$, $\overline{a}_0 = 1$, $a_0^\star = 1.5$, $\mathbb{E}[p(Sa)]_0 = 1$, $i = 0$, $j = 0$, $k = 0$.

1. Define $\underline{a}_i = (\underline{a}_i^d + \underline{a}_i^u)/2$.

2. Solve for the second order ODE for $p(a)$ on $[\underline{a}_i, \overline{a}_i]$ given in Lemma 2 with $p(\underline{a}_i) = \varphi$ and $p'(\underline{a}_i) = 1e - 6$.

3. If $p(\overline{a}_i) < 1$, set $\underline{a}_{i+1}^u = \underline{a}_i$ and $\underline{a}_{i+1}^d = \underline{a}_i^d$. Otherwise, $\underline{a}_{i+1}^u = \underline{a}_i^u$ and $\underline{a}_{i+1}^d = \underline{a}_i$.

4. If $\underline{a}_{i+1}^u - \underline{a}_{i+1}^d < 1e - 6$, continue to the next step; otherwise, set $i = i + 1$ and go to step 1.

5. Solve for $\mathbb{E}[p(Sa)]_{j+1}$ given the new solution for $p(a)$.

6. If $||\mathbb{E}[p(Sa)]_{j+1} - \mathbb{E}[p(Sa)]_j|| < 1e - 5$, continue to the next step; otherwise, set $j = j + 1$ and go to step 1.

7. Solve for $\overline{a}_{k+1}$ such that $e^\star(\underline{a}_i, \overline{a}_{k+1}, a^\star) = 0$.

8. If $|e^\star(\underline{a}_i, \overline{a}_{k+1}, a^\star) - e^\star(\underline{a}_i, \overline{a}_k, a^\star)| < 1e - 4$, end; otherwise, set $k = k + 1$ and go to step 1.