

Betting on Stocks with Options?*

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Abstract

We examine whether expected stock returns translate into expected option returns as predicted by standard theory. Using machine-learning estimates of expected stock returns, we uncover a pronounced U-shaped relation between expected returns and volatility. Both high and low expected stock returns therefore coincide with elevated volatility, which increases option prices and largely offsets the expected payoff differential. We derive a model-free lower bound on expected option return spreads and show it is strongly violated in the data. As a result, equity options are an inefficient instrument for harvesting stock risk premia. A calibrated Black–Scholes model reproduces these empirical patterns.

Keywords: option returns, stock returns, return prediction, volatility, machine learning

JEL Classification: G10, G12, G13, G14, G17

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1 Introduction

Over the past decades, trading activity in U.S. equity options has grown to the point that notional option trading volume now rivals or even exceeds that of the underlying stocks. Options are highly versatile financial instruments that can be used to implement a wide range of trading strategies. In practice, however, a substantial share of single-stock option trading reflects directional positioning on the underlying rather than sophisticated volatility or hedging motives (e.g., [Lakonishok, Lee, Pearson, and Poteshman, 2007](#)). Consistent with this behavior, a large literature argues that informed investors often prefer to trade in the options market when they possess information about future stock price movements (e.g., [Easley, O’Hara, and Srinivas, 1998](#); [Chakravarty, Gulen, and Mayhew, 2004](#); [Pan and Poteshman, 2006](#); [Ni, Pan, and Poteshman, 2008](#); [Cremers and Weinbaum, 2010](#); [Xing, Zhang, and Zhao, 2010](#); [Ge, Lin, and Pearson, 2016](#)). Yet, from an expected return perspective, the implications of this preference remain surprisingly unclear: If an investor can forecast a stock’s expected return, should that investor trade the stock or its options?

This paper shows that answering this question requires first understanding a strong and previously undocumented joint structure of expected stock returns and volatility: Stocks in both tails of the expected-return distribution are substantially more volatile than stocks with intermediate expected returns, producing a pronounced U-shaped expected return–volatility relation. This relation is essential for understanding how expected stock returns (ESR) map into expected option returns (EOR). In particular, we show that the variation in stock return volatility (VOL) associated with high ESR is strong enough to offset much of the payoff effect of higher ESR. This leads the unconditional ESR–EOR relationship to appear weak—or even absent—despite the amplification mechanism implied by standard option pricing theory.

We begin by constructing ESR for the universe of optionable stocks using the machine-learning framework of [Gu, Kelly, and Xiu \(2020\)](#), which has been documented to deliver strong and stable out-of-sample predictability in the cross section of stock returns. Consistently, we find that a long–short equity strategy based on these ESRs earns a substantial out-of-sample return spread—almost three times the market excess return. We remain agnostic as to whether this predictability reflects compensation for risk or mispricing. Our analysis focuses on predictable variation in expected returns extracted from public information, rather than private or insider signals.

We then document that the ESR–VOL relation is sharply nonlinear: volatility varies systematically across the ESR distribution and is highest in the tails. This *bivariate* relationship is distinct from the well-known finding of [Ang, Hodrick, Xing, and Zhang \(2006\)](#) that *univariate* sorting on VOL predicts subsequent stock returns.

For stock portfolios, this structure is largely irrelevant because, *conditional on expected returns*, the volatility component is predominantly idiosyncratic and diversifies away in long–short portfolios. For options, the same structure is first-order: volatility enters option prices mechanically and is a key determinant of expected option returns.

To understand why this pattern matters, we first study whether ESR-sorted *option* portfolios inherit the large return spreads observed in ESR-sorted *stock* portfolios. We address this question by sorting at-the-money call options each month by the ESR of the underlying stock and forming a long–short portfolio that buys calls on high-ESR stocks and sells calls on low-ESR stocks, holding positions to maturity. [Figure 1](#) presents a key result: While ESRs display substantial and economically meaningful cross-sectional dispersion, the corresponding dispersion in EORs is strikingly weak.

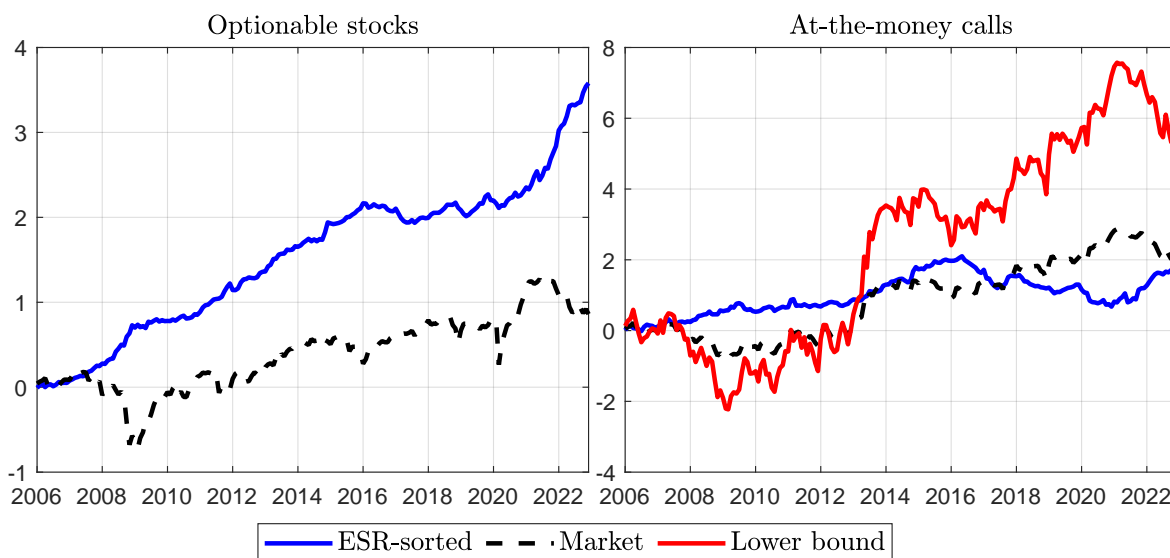
This result is surprising, because standard option pricing theory predicts a strong link between ESRs and EORs. A seminal insight of [Black and Scholes \(1973\)](#) is that the option *price* does not depend on the expected return of the underlying asset. However, because the expected payoff of a call option is strictly increasing in the expected return of the underlying, the expected *return* of the call must also increase. This fundamental property is not specific to the model of [Black and Scholes \(1973\)](#); rather, it holds in virtually all standard option pricing models.

To benchmark how large the spread in ESR-sorted call returns should be, we derive a *model-free lower bound*. Under mild assumptions that hold in virtually all standard option pricing models, the spread in expected option returns between high- and low-expected-return stocks must exceed the spread in expected stock returns, scaled by the ratio of the option market return to the equity premium. The bound does not depend on the level of option returns—only on their relative cross-sectional dispersion. As a result, measurement error in our ESR estimates cannot explain a violation: any attenuation in estimated ESRs affects stock and option spreads symmetrically and cancels out of the bound.

Intuitively, if options provide leveraged exposure to stock returns with no confounding effects, the cross-sectional dispersion in option returns should amplify the dispersion in stock

Figure 1: Cumulative Return of ESR-Sorted Stock and Call Portfolios

This figure shows the cumulative out-of-sample log returns of the expected stock return (ESR) for both stock and call option strategies. The ESR is estimated using the NN3 model of Gu et al. (2020). We sort assets based on the ESR of the (underlying) stock and construct portfolios by going long (short) stocks and at-the-money calls on stocks with high (low) ESR. For the market strategy, we naively buy all options or stocks, respectively, and plot the return in excess of the risk-free rate. For the plot, we adjust the options strategies such that the respective market has the same average volatility as the stock market. All portfolios are equally weighted.



returns. In the data, this bound implies that the return spread for ESR-sorted calls should be at least 2.63 times the call market return. Instead, the observed spread is only about 0.67 times the call market return—a large and statistically significant shortfall. Thus, option returns react far too weakly to predictable differences in expected stock returns—and, as we show, the key to this puzzle lies in a strong and previously undocumented relationship between expected returns and volatility.

Theoretically, two opposing forces shape how ESR translates into EOR. First, holding option prices fixed, higher ESR raises the expected payoff of a call. Second, higher ESR is accompanied by higher VOL. Higher VOL raises call *prices* because it increases the value of the convex call payoff, and, for a given expected payoff, a higher price lowers the expected return; thus this second force pushes expected call returns down (e.g., Galai and Masulis, 1976; Johnson, 2004; Goyal and Saretto, 2009; Cao and Han, 2013; Friewald, Wagner, and Zechner, 2014; Doshi, Jacobs, Kumar, and Rabinovitch, 2019; Hu and Jacobs, 2020; Aretz,

Lin, and Poon, 2022). Thus, even though both tails feature high VOL, the impact of higher VOL on EOR is asymmetric for low- and high-ESR stocks. The key compression mechanism is that, moving from intermediate to high ESR, the payoff effect that should raise EOR is largely offset by the simultaneous rise in VOL. Therefore, EOR increases only weakly—if at all—with ESR, muting the unconditional ESR–EOR relation.

When we explicitly control for VOL using double sorts, two distinct patterns emerge. First, among low-VOL stocks, there is essentially no predictable dispersion in ESR: ESR-sorted stock portfolios within the lowest VOL group exhibit negligible and statistically insignificant return spreads. Consequently, there is little scope for any option-based strategy to exploit ESR in this part of the cross section. Second, predictable differences in ESR are concentrated in medium- and especially high-VOL stocks. In these segments, options written on higher-ESR stocks do tend to earn higher EOR than options written on lower-ESR stocks, consistent with standard option pricing theory. Thus, once VOL is held fixed and high, the theoretical linkage between expected stock and option returns re-emerges. In this sense, the failure of ESR-sorted option strategies is not driven by a breakdown of the basic pricing mechanism, but by the fact that variation in VOL dominates the cross section of option returns.

However, this reconciliation comes at an economic cost. The segments of the cross section in which ESR predictability is strongest are precisely those in which option prices are high and average EOR are low. As a result, even when ESR differences translate into EOR differences after controlling for volatility, the attainable spreads are small in levels and unattractive relative to both stock-based strategies and simpler option strategies that sort directly on VOL. Thus, correcting for volatility restores theoretical consistency but does not restore the economic case for using options to exploit predictable stock returns.

To the best of our knowledge, this particular joint structure of ESR and VOL has not been documented before. It is central for reconciling the weak performance of ESR-based option strategies with the standard theoretical prediction that EOR should co-move with, and strongly amplify, predictable variation in ESR. It also implies that equity options are surprisingly ineffective instruments for an informed investor to harvest the risk premia embedded in predictable differences in ESR.

To discipline this interpretation and to assess its generality beyond portfolio sorts, we predict option returns using machine learning methods based on a comprehensive set of 227

predictors. We find that option returns are indeed predictable, but not by variables that predict stock returns. Even when we include ESR directly, it exhibits very little predictive power. In contrast, when we predict option *prices* instead of returns, the ESR and its components turn out to be very informative. This situation seemingly violates a fundamental property of canonical option pricing models, namely that option prices should be independent of ESR. Yet in the data, option *prices* are explained by ESR while expected option *returns* are not. This again is merely a reflection of the strong dependence between ESR, VOL, and option prices.

In fact, a calibrated version of a simple Black–Scholes model can reproduce these patterns jointly. We discipline the calibration by matching the empirically documented nonlinear joint distribution of ESR and VOL, and then price at-the-money options and compute their implied expected returns. Despite the models canonical pricing mechanism, the calibration generates our documented findings: (i) weak unconditional ESR–EOR relation; (ii) strong dispersion in EOR across VOL-sorted option portfolios; and (iii) substantial explanatory power of ESR for option *prices*. Thus, the full set of empirical findings can arise in a standard option-pricing environment once the nonlinear ESR–VOL structure observed in the data is imposed.

We also study several alternative explanations and find they are unlikely to account for our results. In particular, we find little empirical support for explanations based on joint risk exposures of stocks and options, transaction costs, shorting fees, and biases in option returns. Consistent with the calibration evidence, the results do not rely on market segmentation either, but also arise in a model with perfectly integrated markets. Finally, we show that the documented co-movement between ESR and option prices is not plausibly attributable to option-demand effects.

Related literature. Our paper relates to several strands of the literature.

First, a large body of work (cited above) argues that informed investors prefer to trade in the options market rather than in the underlying stocks. Closely related studies examine the relative trading activity in stock and option markets and provide evidence consistent with this view (e.g., [Roll, Schwartz, and Subrahmanyam, 2010](#); [Johnson and So, 2012](#)). We complement this literature by studying the implications of such behavior from an expected return perspective. In contrast to the widespread notion that options provide an efficient vehicle for exploiting information about stock returns, we find that option-based strategies

sorted on expected stock returns deliver weak performance. We trace this result to a strong comovement between expected stock returns and stock volatility, which offsets the return amplification implied by embedded leverage.

Second, several papers show that option-implied variables predict future stock returns (e.g., [Bali and Hovakimian, 2009](#); [Cremers and Weinbaum, 2010](#); [Xing, Zhang, and Zhao, 2010](#); [An, Ang, Bali, and Cakici, 2014](#); [Ge, Lin, and Pearson, 2016](#)). However, recent work shows that only a few option characteristics have incremental predictive power once a large set of firm characteristics is controlled for ([Neuhierl, Tang, Varneskov, and Zhou, 2025](#)). Our focus is different in that we directly study the link between ESR and EOR.

Third, our paper relates to the literature on expected option returns and their relation to systematic risk. Seminal work by [Coval and Shumway \(2001\)](#) emphasizes that options provide levered exposure to priced risk factors, while subsequent studies highlight the role of variance and higher-moment risk premia in explaining option returns (e.g., [Bakshi, Kapadia, and Madan, 2003](#); [Bakshi and Kapadia, 2003b](#); [Duarte, Jones, and Wang, 2024](#)). While this literature focuses on explaining option returns through compensation for risk, we study whether options amplify predictable cross-sectional differences in expected stock returns. Our model-free bounds allow us to test this implication without specifying a particular factor structure or option pricing model.

Fourth, our work contributes to the growing literature on predicting the cross-section of individual option returns. We build on the comprehensive review in [Bali, Beckmeyer, Moerke, and Weigert \(2023\)](#) and include the full set of commonly used option-based predictors. Unlike most studies, however, we focus on raw, hold-to-maturity option returns, as we are explicitly interested in the link between ESR and EOR. This contrasts sharply with the analysis of delta-hedged option returns, which by construction removes directional exposure to the underlying asset ([Bakshi and Kapadia, 2003a](#)).

Finally, our paper contributes to the literature on machine learning in empirical asset pricing. Following [Freyberger, Neuhierl, and Weber \(2020\)](#) and [Gu et al. \(2020\)](#), a large literature documents substantial predictability in the cross-section of stock returns. We take this predictability as given and ask whether options can be used to exploit it. For option markets, recent studies apply machine learning to delta-hedged single-stock option returns ([Bali et al., 2023](#)) and to index option returns ([Büchner and Kelly, 2022](#); [Fournier, Jacobs, and Orłowski, 2024](#)).

The remainder of the paper is organized as follows. Section 2 develops the theoretical bounds on expected option returns. Section 3 describes the data and the construction of ESR. Section 4 presents the main empirical results and the analysis of VOL as a confounding factor. Section 5 studies robustness and alternative explanations. Section 6 concludes. In the main text, we report and discuss results for call options only. All theoretical and empirical results carry over to put options; treating puts yields no additional insights but complicates the exposition. For brevity, the corresponding put results are deferred to Appendix IA1.

2 Theory: Expected Stock and Option Returns

This section derives theoretical restrictions on expected option returns implied by standard option pricing theory. Section 2.1 shows that, because option *prices* are independent of the expected return of the underlying stock, expected option returns must vary monotonically with expected stock returns. Section 2.2 sharpens this insight by deriving model-free bounds on the magnitude of expected call return spreads.

2.1 Relationship between Expected Stock and Option Return

The object we study in this paper is hold-to-maturity returns of individual stock options in excess of the risk-free rate. The return function can be expressed as

$$R_{i,t+1}^C = \frac{\max(0, S_{i,t+1} - K)}{C_{i,t}} - (1 + R_{f,t}), \quad (1)$$

where $C_{i,t}$ denotes the price of the call on stock i with strike K at time t , $S_{i,t}$ denotes the underlying stock price, and $R_{f,t}$ is the risk-free rate.

A central property of virtually all standard option pricing models is that option prices do not depend on the expected return of the underlying stock S , which we denote by $\mu_i \equiv E_t[R_{i,t+1}^S]$. Formally,

$$\frac{\partial C_{i,t}}{\partial \mu_i} = 0. \quad (2)$$

This property holds in a wide class of models, including Black–Scholes and jump–diffusion models such as Black and Scholes (1973), Merton (1976), Hull and White (1987), Heston (1993), Bates (1996), Bates (2000), and Eraker (2004). A simple way of verifying Equation

(2) is that, in all these models, option prices do not depend on the parameter(s) governing μ_i .¹

In contrast, expected option payoffs are monotonic in μ_i . The expected payoff of a call option is strictly increasing in μ_i :

$$\frac{\partial \mathbb{E}_t [\max(0, S_{i,t+1} - K)]}{\partial \mu_i} > 0. \quad (3)$$

Since option prices do not depend on μ , this immediately implies that expected option returns inherit this monotonicity:

$$\frac{\partial \mathbb{E}_t [R_{i,t+1}^C]}{\partial \mu_i} > 0. \quad (4)$$

More broadly, stocks and options are claims on the same underlying cash flows and should therefore be exposed to the same fundamental sources of variation in expected returns. As a result, variables that predict equity returns should also contain information about expected option returns (see, e.g., [Bali et al., 2023](#); [Chen, Roussanov, Wang, and Zou, 2024](#)).²

2.2 Bounds on Expected Option Returns

The previous subsection establishes a qualitative implication of standard option pricing theory: expected option returns must be monotonic functions of the expected return of the underlying stock. We now sharpen this implication quantitatively.

Specifically, we derive model-free bounds on the magnitude of expected option return spreads that must arise when options are written on stocks with sufficiently different expected returns. These bounds allow us to assess whether option returns react “too weakly” to variation in expected stock returns, without committing to a particular option pricing model.

We consider the following thought experiment. Take two stocks whose expected excess returns differ substantially, and compare the expected returns of otherwise comparable options written on these stocks. If option prices do not depend on expected stock returns—as

¹Discrete-time GARCH option pricing models allow μ_i to affect option prices through risk-neutral dynamics (e.g., [Heston and Nandi, 2000](#)). However, this effect is quantitatively negligible.

²Note that our arguments do not extend to any unpredictable (idiosyncratic) part of the option return, and similarly not to trading on private (insider) information.

implied by virtually all canonical models—then differences in expected option returns must arise *mechanically* from differences in expected option payoffs. The question is not whether such differences exist, but how large they must be.

In the following, let M denote the aggregate stock market.

Proposition 1 (Lower Bound on Expected Call Returns) *Assume that:*

1. *Call prices do not depend on the expected stock return μ of the underlying stock;*
2. *There exist four stocks with $\mu_H > \mu_M > R_f \geq \mu_L$ such that $\mu_H + \mu_L \geq \mu_M + R_f$;*
3. *There exist call options with identical relative strike (K/S) and maturity for all stocks;*
4. *Stock return distributions are identical up to differences in μ .*

Then, the expected return spread between calls written on high- and low- μ stocks satisfies:

$$\mathbb{E} [R_H^C - R_L^C] > \frac{\mu_H - \mu_L}{\mu_M - R_f} \mathbb{E} [R_M^C - R_f^C], \quad (5)$$

where $R_H^C, R_M^C, R_L^C, R_f^C$, are the returns of call options on stocks with expected returns μ_H, μ_M, μ_L , and R_f , respectively.

Proof: See Appendix A.1. ■

Proposition 1 delivers a simple but powerful implication. The expected return spread between calls written on high- and low- μ stocks must be at least as large as the product of two terms. The first term, $\frac{\mu_H - \mu_L}{\mu_M - R_f}$, measures how large the cross-sectional spread in expected stock returns is relative to the equity market premium. The second term, $\mathbb{E}[R_M^C - R_f^C]$, is the average excess return on a representative call option written on a stock with expected return equal to the market return. We refer to this quantity as the *call market return*. Intuitively, if expected stock returns vary strongly in the cross section, and if call prices do not absorb this variation, then expected call returns must amplify these differences relative to the benchmark provided by the call market return.

This restriction does not depend on the level of expected option returns. While specific option pricing models make quantitative predictions about the *level* of expected returns, these level effects cancel out in the bound. What remains is a restriction on *relative* expected returns across options written on stocks with different expected returns: even though the absolute sensitivity of expected call returns to μ may be model dependent, their response

relative to the call market return cannot be arbitrarily weak when dispersion in expected stock returns is large.

If the bound in Proposition 1 is violated in the data, at least one of the underlying assumptions must fail. Assumption 1 holds in essentially all standard option pricing models and therefore provides a natural theoretical benchmark. Assumption 2 requires that the cross-sectional dispersion in expected stock returns is sufficiently large relative to the equity market premium, a condition we verify empirically. Assumption 3 concerns option contract comparability and is addressed by focusing on options with similar maturity and relative moneyness.

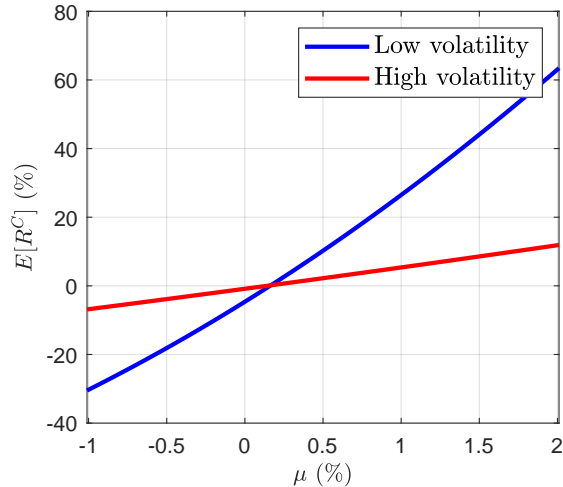
Assumption 4 is the most restrictive. It rules out systematic differences in higher moments of stock return distributions—most importantly volatility—across stocks with different expected returns. This assumption is likely violated—for example, any standard factor model would imply that ESR and volatility are related. Nevertheless, Proposition 1 provides a useful benchmark for what ESR amplification we should expect in options absent confounding channels.

To build intuition and clarify the role of the assumptions, Figure 2 illustrates the theoretical results using the Black–Scholes model. Holding volatility σ fixed, expected call returns are increasing in μ , reflecting the fact that option prices are independent of μ whereas expected payoffs are not. Moreover, expected call returns are convex functions of μ , a property that underlies the lower bound in Proposition 1. A graphical illustration of the convexity argument underlying Proposition 1 is provided in Figure A.1 in the Appendix.

The figure also highlights that volatility is a first-order determinant of expected option returns even when μ is held constant, a well-known result in the literature (see, e.g., Galai and Masulis, 1976; Johnson, 2004; Cao and Han, 2013; Friewald et al., 2014; Doshi et al., 2019; Hu and Jacobs, 2020; Aretz et al., 2022). Assumption 4 of Proposition 1 abstracts from such differences by requiring identical return distributions up to the mean. A systematic relationship between μ and σ therefore provides a natural channel through which the bound may fail, a mechanism we investigate in detail below.

Figure 2: **Expected Call Returns as a Function of μ and σ**

This figure displays the expected call return in the Black-Scholes model as a function of monthly expected stock return μ , for low volatility ($\sigma = 15\%$ p.a.) and high volatility ($\sigma = 80\%$ p.a.) stocks. Time-to maturity is fixed at one month, the risk-free rate at 2% p.a., and $K/S = 1$.



3 Data and Variable Definitions

We outline our data sources used in the empirical analysis and provide summary statistics for both the sample of options and the sample of underlying optionable stocks.

3.1 Stock and Firm Data

We obtain historical price and trading volume data for the underlying stocks from CRSP and accounting data from Compustat. As is common in the literature, we restrict our analysis to common stocks (share codes 10 or 11) of firms listed on the NYSE, Amex, or NASDAQ (exchange codes 1, 2, 3, 31, 32, and 33). Our stock-level variables span the period from January 1996 to December 2022, featuring data from 22,123 stocks, with an average of 6,829 stocks per month. We supplement this data with the risk-free rate provided by OptionMetrics.

3.2 Options Data

Our option sample consists of all available individual stock options from OptionMetrics IvyDB US for the period from January 1996 until December 2022. The data contain information on the entire exchange-traded U.S. individual stock options market and include the daily closing bid and ask quote as well as implied volatility, option Greeks, trading volume, and open interest. We also use the interpolated implied volatility surface data from OptionMetrics, but only for constructing option-based characteristics. We then apply a series of standard options data filters established in the literature.³ Moreover, since individual stock options are of the American type, we exclude options on stocks that pay a dividend during the holding period to eliminate effects from a potential early exercise. Finally, we use the risk-free rate provided by OptionMetrics for option returns and characteristics, as [Van Binsbergen, Diamond, and Grotteria \(2022\)](#) show this rate is both convenience-yield-free and effectively credit-risk-free.

3.3 Option Returns

Our main variable of interest are monthly, hold-to-maturity returns of individual stock call options in excess of the risk-free rate, as defined in Equation (1). While many papers study so-called delta-hedged option returns, we deliberately deviate. Delta-hedged returns are designed to *eliminate* the effect of the underlying stock return on the option return ([Bakshi and Kapadia, 2003a](#)) and isolate the effects of higher moments on option risk premia. We, instead, are explicitly interested in studying the relationship between ESR and EOR.

We construct a panel of monthly, non-overlapping option returns from the first trading day after the third Friday in a given month until the third Friday in the next month (which is the expiration date following the standard monthly expiration cycle). We focus on these options, since they usually represent the most liquid contracts. The options are bought on the first trading day after each third Friday (typically a Monday) to avoid expiration

³Specifically, we exclude all option prices that violate standard no arbitrage bounds (which corresponds to the options where OptionMetrics does not provide an implied volatility or Greeks, see [Duarte et al., 2024](#)). Moreover, we follow [Goyal and Saretto \(2009\)](#), [Cao and Wei \(2010\)](#), [Muravyev \(2016\)](#), [Christoffersen, Goyenko, Jacobs, and Karoui \(2018\)](#), and [Duarte et al. \(2024\)](#), and require options to have a strictly positive best bid, a mid-price of at least \$0.10, a relative bid-ask spread not exceeding 50% of the midpoint, and we exclude options that have zero open interest.

day effects. Note that our return calculation only requires option prices at the time of the portfolio formation and none thereafter, which alleviates concerns regarding a look-ahead bias as discussed in Duarte, Jones, Khorram, Mo, and Wang (2025). We consider call options with moneyness in the range of $K/S \in [0.9, 1.2]$. For each stock and month, we maintain the contract that has moneyness closest to 1 (at-the-money; ATM) and 1.1 (out-of-the-money; OTM), respectively.

3.4 Summary Statistics on Option and Stock Returns

Table 1 provides summary statistics for the call options in our final sample from January 1996 to December 2022. Our sample comprises 7,861 unique optionable stocks, with an average of approximately 1,747 stocks per month, totaling around 564,000 firm-month observations. On average, we have 1,460 ATM calls per month in our sample, and 1,198 ATM puts. The ratio of ATM calls to puts is roughly 55% to 45%, and for OTM it is 54% to 46%, which reflects the well-known fact that for individual stock options, calls are more actively traded than puts. In the main analysis we focus on the results for calls, while the analogous results for puts are presented in Appendix IA1.

Panel A of Table 1 shows that the average excess return for ATM call options is large at 6.75% per month, whereas the median monthly excess return is essentially a total loss of -98.04% . The average moneyness is 1.01, the average delta is 0.51, and the average implied volatility is 45.96%. For out-of-the-money call options, the average return is 0.64% and the median return is -100.04% . The average moneyness is 1.12, and the average implied volatility is higher compared to ATM options, at 51.52%. Finally, Panel C shows that stocks which have an option written on them in our sample have an average monthly excess return of 0.58%. In comparison, ATM option returns are more than an order of magnitude larger than stock returns but also an order of magnitude more volatile. ATM Calls have higher skewness and kurtosis than stocks. For OTM options, both skewness and kurtosis are substantially higher, driven by the fact that most options expire worthless while a few yield very large returns. The typical time-to-maturity is either 25 or 32 days, with a few exceptions around holidays.

Figure 3 illustrates the evolution of the sample over time. Subfigure A shows that the number of underlying stocks has increased considerably over time. Similarly, the market

Table 1: **Summary Statistics for Option and Stock Returns**

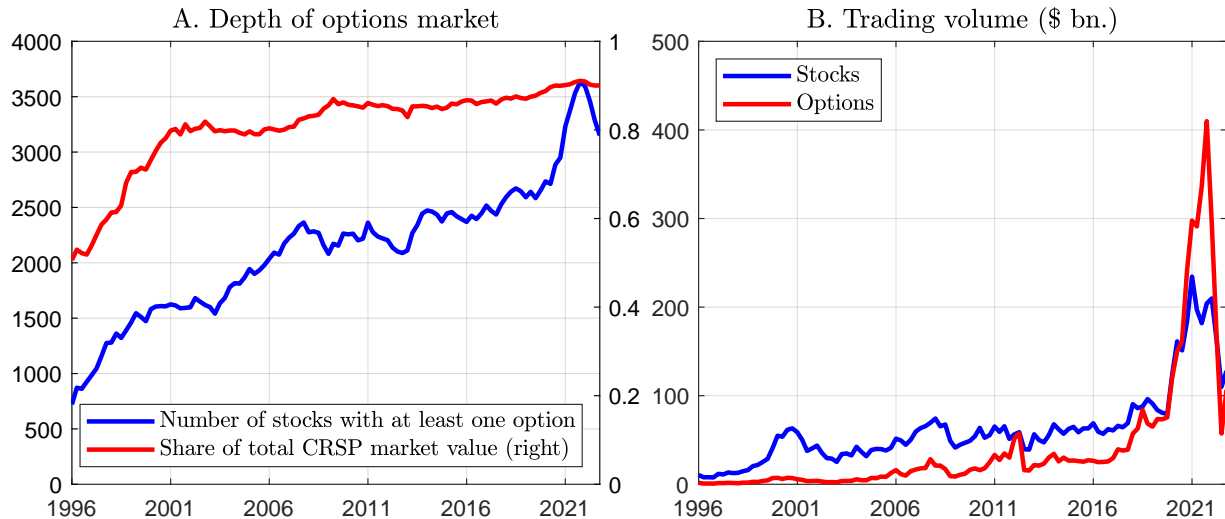
This table reports descriptive statistics for monthly option and stock returns (in %, per month) for the period from 1996 to 2022. All returns are in excess of the risk-free rate. Hold-to-maturity option returns are measured from the first trading day after a third Friday until the next third Friday. Panel A reports returns and option characteristics for at-the-money call options ($N \times T = 471,565$), Panel B reports returns for out-of-the-money call options ($N \times T = 313,115$). We consider call options with moneyness in the range of $K/S \in [0.9, 1.2]$. For each stock and month, we maintain the contract that has moneyness closest to 1 (at-the-money) and 1.1 (out-of-the-money), respectively. Option implied volatility (expressed in %, p.a.) and deltas are provided by OptionMetrics. Lastly, Panel C presents descriptive statistics for stock returns of optionable stocks ($N \times T = 564,244$) in our sample.

	Mean	SD	10%	25%	50%	75%	90%	Skew.	Kurt.
<i>A. At-the-money calls</i>									
Option Return	6.75	180.36	-100.35	-100.10	-98.04	63.87	222.02	4.01	44.22
Time-to-Maturity	27.97	3.39	25.00	25.00	26.00	32.00	33.00	0.59	-1.52
Moneyness	1.01	0.04	0.96	0.99	1.00	1.03	1.06	-0.01	0.17
Implied Volatility	45.96	25.34	21.09	28.38	39.96	56.78	78.26	1.85	6.19
Delta	0.51	0.13	0.34	0.43	0.51	0.58	0.66	-0.06	0.41
<i>B. Out-of-the-money calls</i>									
Option Return	0.64	405.53	-100.45	-100.20	-100.04	-100.01	196.47	17.52	898.38
Time-to-Maturity	28.01	3.41	25.00	25.00	26.00	32.00	33.00	0.54	-1.58
Moneyness	1.12	0.04	1.06	1.08	1.11	1.15	1.18	0.25	-1.14
Implied Volatility	51.52	26.43	25.25	33.04	45.33	63.25	85.43	1.78	5.74
Delta	0.22	0.11	0.08	0.13	0.20	0.29	0.37	0.42	-0.58
<i>C. Optionable stocks</i>									
Stock Return	0.58	15.49	-15.74	-6.71	0.53	7.38	15.99	1.49	23.42

share of the underlying stocks in our final sample relative to total CRSP market value has increased, and is well above 80% for most of the sample. Moreover, Graph B of Figure 3 shows that trading volume in the options market has increased tremendously over time, and even more so than stock market trading. Strikingly, since 2017, the two values are roughly at par, with options market activity exceeding stock market activity in several quarters. Table IA.10 presents the summary statistics separately for the initial training and validation sample (1996–2005) and the out-of-sample testing period (2006–2022).

Figure 3: **Evolution of Option and Stock Market Size**

This figure shows the evolution of the options market relative to the stock market over time at quarterly frequency. Panel A shows the number of unique underlying stocks in our sample over time (before removing dividend paying stocks), together with the share of total CRSP market value. Panel B shows the notional trading volume across all US listed equity options markets in our sample (sourced from OptionMetrics), together with the corresponding trading volume of the underlying stocks from CRSP.



3.5 Option and Stock Characteristics

We follow the growing literature on the application of machine learning algorithms for predicting asset returns and construct a comprehensive set of return predictors. Specifically, we construct the 80 option characteristics used by [Bali et al. \(2023\)](#), which are taken from the literature on the cross section of option returns. [Table IA.9](#) provides an overview of the option-based characteristics.

Moreover, we include the 94 stock characteristics proposed by [Green, Hand, and Zhang \(2017\)](#), which are taken from the literature on the cross section of stock returns. We expand this set of variables by adding 48 industry dummies, based on the definition of [Fama and French \(1997\)](#), and a few more variables that are known to relate to option returns, such as skewness and kurtosis of the stock returns ([Conrad, Dittmar, and Ghysels, 2013](#)), price delay ([Hou and Moskowitz, 2005](#)) and CAPM-beta calculated using one year of daily returns (in addition to the already included CAPM-beta from three years of weekly returns). [Table IA.8](#) provides an overview of the stock-based characteristics. All together, we have 147 stock characteristics, 80 option characteristics, and 227 variables in total.

3.6 Estimating Expected Stock Returns

A “stock characteristic” central to our analysis is the ESR. In our benchmark analysis, we estimate ESRs following Gu et al. (2020), using a neural network with three layers, which is found to perform best across various metrics. The model is trained to maximize the out-of-sample (OOS) R^2 in the test sample, as in Gu et al. (2020). We train the model on the sample of optionable stocks using the initial training and validation sample (1996–2005) and subsequently expand the window each calendar year. For the monthly stock returns, we use the same timing as for the option returns. For predicting stock returns we rely only on the 147 stock characteristics, and not on option characteristics.

3.7 Estimating Expected Option Returns

We estimate EORs using a neural network and the full set of 227 stock and option characteristics, following the same expanding-window procedure as for ESRs. These forecasts serve as a flexible empirical benchmark for the maximal dispersion in EORs, against which we compare the implications of ESRs.

4 Empirical Results

In this section, we show that ESR has little unconditional power for EORs, and that the apparent disconnect is resolved once we control for VOL. Because high-ESR stocks are predominantly high-VOL, options are most expensive precisely where ESRs are highest, leaving little scope to harvest the equity premia through option positions. Finally, we show that a simple Black–Scholes model can quantitatively reproduce these patterns.

4.1 ESR-Sorted Stock and Option Portfolios

To first evaluate the out-of-sample accuracy of our ESRs, we form monthly, equally-weighted decile portfolios of optionable stocks sorted on ESRs. We also form a high-minus-low (H–L) portfolio that buys decile 10 and sells decile 1 (the ESR-stock strategy). Table 2 shows that realized stock returns are largely monotonic in the ESR rank. The H–L portfolio earns 184 bps per month with an annualized Sharpe ratio of 1.64, implying that our ESRs identify

Table 2: **ESR-Sorted Stock Returns**

This table reports average stock portfolio returns of optionable stocks, sorted by ESR based on the neural network model described in Section 3.6 for the period 2006 to 2022. All returns are in excess of the risk-free rate. Sharpe ratios are denoted in annual terms. Absolute t -statistics are reported in parentheses. All portfolios are equally-weighted. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
Mean	-0.59 (0.77)	0.34 (0.58)	0.63 (1.20)	0.73 (1.51)	0.89** (2.02)	0.91** (2.19)	0.92** (2.11)	0.86* (1.97)	1.07** (2.18)	1.25** (2.16)	1.84*** (4.25)	0.70 (1.41)
SD	9.63	7.83	6.97	6.51	6.49	6.50	6.67	6.88	7.64	8.54	3.90	7.19
SR	-0.21	0.15	0.31	0.39	0.47	0.48	0.48	0.43	0.49	0.51	1.64	0.34

economically meaningful dispersion in ESRs out of sample.⁴ Compared to the equal-weighted market excess return over the same period of 70 bps per month, the ESR-strategy returns about $184/70 \approx 2.63$ times the equity market premium.

We next ask whether the spread in ESRs translates into spreads in EORs. Each month, we sort calls into deciles based on the ESR of the underlying stock and form the corresponding H-L portfolio (the ESR-option strategy). If option prices are independent of ESRs, differences in EORs should arise mechanically from differences in expected option payoffs.

Table 3 shows the opposite. For calls, the H-L spread is small relative to the average call return (which we refer to as the call market return) despite the large spread in the underlying stocks. Thus, option returns react far more weakly to predictable variation in ESRs than the behavior of the underlying stocks would suggest.

4.2 Test of Option Return Bounds

We next assess whether the weak ESR-option spreads are merely small in economic magnitude or whether they violate the quantitative restrictions implied by standard option pricing

⁴In comparison with Gu et al. (2020), the performance is somewhat worse than their equal-weighted NN3 strategy, which returns an average H-L spread of 258 bps per month, with a standard deviation of 3.27% and a Sharpe ratio of 2.36 (see their Table A.9). This lower performance is driven by our restriction to the sample of optionable stocks, as small and illiquid stocks—which are typically not optionable—tend to have larger predictive spreads (e.g., Avramov, Cheng, and Metzker, 2023). Moreover, Gu et al. (2020) generally find better performance in the sample pre-2006 (see their Figure 9). Interestingly though, Figure 1 shows that the performance of the H-L portfolio increases post publication of Gu et al. (2020), at least relative to the rest of our sample.

Table 3: **ESR-Sorted Call Option Returns**

This table reports average call option portfolio returns, sorted by expected stock return based on the neural network model described in Section 3.6 for the period from January 2006 to December 2022. Sharpe ratios are denoted in annual terms. Absolute t -statistics are reported in parentheses. All portfolios are equally-weighted. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
<i>A. At-the-money calls</i>												
Mean	0.41 (0.08)	7.63 (1.48)	8.51 (1.56)	12.64** (2.01)	15.64*** (2.53)	14.75*** (2.52)	9.57* (1.78)	10.01* (1.87)	11.69** (2.31)	6.62 (1.67)	6.21* (1.75)	9.75* (1.94)
SD	65.91	71.34	72.86	80.73	85.64	85.21	79.72	81.16	80.73	68.80	40.87	72.35
SR	0.02	0.37	0.41	0.54	0.63	0.60	0.42	0.43	0.50	0.33	0.53	0.47
<i>B. Out-of-the-money calls</i>												
Mean	-5.61 (0.99)	3.64 (0.67)	-0.22 (0.04)	4.37 (0.58)	7.68 (0.94)	5.24 (0.82)	1.00 (0.18)	11.02 (1.70)	7.62* (1.31)	5.65 (1.30)	11.26*** (2.70)	4.03 (0.76)
SD	76.98	92.94	85.72	90.42	106.72	100.94	90.24	100.63	96.92	84.48	67.60	81.17
SR	-0.25	0.14	-0.01	0.17	0.25	0.18	0.04	0.38	0.27	0.23	0.58	0.17

theory. To this end, we test the model-free bounds derived in Section 2.2. Operationally, we map the theoretical objects to portfolio returns: R_H^S and R_L^S are the returns on the ESR stock deciles 10 and 1, and R_H^C and R_L^C are the returns on options written on those stocks.

While proxying for R_M^S is straightforward, our theory requires the option counterpart to be written on a stock with $\mu = R_M^S$. We proxy for this using the simple equal-weighted average of all ATM call returns, as this aligns with our usual intuition of a naïve benchmark. This is a conservative choice: natural alternatives would be options on stocks from ESR portfolios 5 and 6, which ex ante have expected returns closest to the market, or portfolio 4, whose realized return is closest to the market return (Table 2). All of these have even higher average call returns than our chosen benchmark (Table 3), making the bound harder to satisfy.

A further complication is that the expected option return on a stock with $\mu = R_f$ generally differs from R_f in the presence of non-equity risk premia. We therefore proxy for this object using options written on low-ESR stocks. In our sample, the average monthly risk-free rate lies between the realized returns of ESR portfolios 1 and 2, and we use portfolio 1 as our benchmark for simplicity. Using the average of portfolios 1 and 2 instead yields virtually identical results. It is easy to verify that under these specifications, the realized

Table 4: **Test of Option Return Bounds**

This table reports the results for tests of the lower bound for call returns in Equation (5). R_M^C , R_f^C , R_H^C , and R_L^C are the average (“market”) call return, and the average call returns on stocks with an ESR equal to R_f , high, and low ESR, respectively. The null hypothesis H_0 is that the lower bound holds, and the corresponding p -value (in %) is for a one-sided test. All values are in percent.

	ATM	OTM
$R_M^C - R_f^C$	9.45	9.76
$R_H^C - R_L^C$	6.21	11.26
Lower bound	24.92	25.72
$(R_H^C - R_L^C)$ –Lower bound	–18.71	–14.46
t -statistic	–3.72	–2.43
p -value (in %)	0.01	0.80

stock returns satisfy Assumption 2. Moreover, Table IA.11 shows that moneyness varies little across ESR deciles, confirming that Assumption 3 is closely satisfied.

The bound links four observable quantities: the H–L spread in ESR-sorted option returns, the average option market return, the H–L spread in ESR-sorted stock returns, and the equity market premium. The bound requires that the option H–L spread must be at least as large as the stock H–L spread scaled by the ratio of the option market return to the stock market return. This formulation makes clear that the test is inherently relative: regardless of the overall levels of option returns, the cross-sectional option spread must scale with the dispersion in expected stock returns.

Table 4 reports the results. The observed call H–L spread falls far short of the theoretical lower bound for both ATM and OTM calls, and the violations are highly statistically significant. If stock return distributions differ only in their mean—as assumed by our bound—the return spread for ESR-sorted calls should be at least 2.63 times the call market return. The observed spread is only 0.67 times the call market return, which constitutes a significant violation, indicating that option returns react far too weakly to predictable differences in expected stock returns relative to what theory requires.⁵

One concern is that measurement error in estimated ESRs may attenuate option return

⁵We assess statistical significance using Newey–West adjusted t -statistics with four lags and conduct one-sided tests, since the theoretical restriction is an inequality. To address concerns about the extreme distributional nature of option returns, we have verified that pairwise block bootstrapped p -values are very similar to those based on the t -statistics.

spreads. However, such errors affect stock and option strategies symmetrically: a more accurate ESR forecast would mechanically increase both the ESR-sorted stock spread and, under the null, the corresponding ESR-sorted option spread. Since our bounds scale option return spreads by the observed stock return spread, attenuation from ESR noise cannot explain violations of the bound.

Moreover, stock and option returns are strongly correlated. The average correlation between individual stock returns and ATM call returns is 69%, and the correlation between ESR-sorted stock and option portfolio returns is 73%. Thus, lack of comovement between stock and option returns cannot account for the weak response of option return spreads to variation in expected stock returns.

Rather, the weak response likely reflects that high-ESR stocks are systematically associated with characteristics that raise option prices—offsetting the higher expected payoffs—which we investigate next.

4.3 The Confounding Role of Stock Volatility

The violations of the call bound in the previous section imply that at least one of the assumptions underlying Propositions 1 fails in the data. Given that Assumptions 2 and 3 are closely satisfied empirically, two channels remain. First, option prices may comove with expected stock returns through correlated state variables, violating Assumption 1. Second, stocks with different ESRs may differ systematically in higher moments—most importantly VOL—violating Assumption 4. We start with the second channel because VOL is a first-order determinant of option prices and EORs.

Table 5 documents a pronounced U-shaped relationship between ESR and VOL.⁶ Stocks in both the lowest and highest ESR deciles exhibit substantially higher VOL than stocks in the middle of the ESR distribution, with VOL at the extremes being nearly twice as high as in the center portfolios.⁷ Thus, high-ESR stocks tend to be volatile, but so do low-

⁶We also studied differences in (both option-implied and past realized) skewness and kurtosis, and, after controlling for volatility, did not find any systematic differences that can explain our results.

⁷Our findings are related to evidence that stock return predictability is strongest among small and illiquid firms (e.g., Hou, Xue, and Zhang, 2020; Avramov et al., 2023). The mechanisms are, however, distinct. First, while stock size and liquidity are correlated with stock volatility, those studies do not imply the pronounced U-shaped relation between expected returns and volatility that we document. Second, they emphasize microcap stocks, which are largely absent from our sample of optionable firms, as exchange listing rules

Table 5: VOL across ESR-Sorted Portfolios

This table reports average values for several measures of VOL across the 10 ESR-sorted stock portfolios for the period from 2006-2022. VOL denotes the stock volatility measured using the past 21 day returns, IVOL is the idiosyncratic volatility measured as in [Ang et al. \(2006\)](#), and IV is the implied volatility from the option price. For each measure, we first compute the average within a portfolio each month, and then the average across months. All volatility values are annualized, in percent.

	Low	2	3	4	5	6	7	8	9	High	Mean
VOL	62.8	45.8	38.1	34.6	34.0	34.8	36.0	38.4	41.9	50.3	41.7
IVOL	55.3	41.4	34.1	30.9	30.1	30.6	32.0	34.0	37.5	47.6	37.3
IV	61.8	48.0	40.7	37.4	36.6	37.4	38.8	40.6	44.2	53.9	43.9

ESR stocks, while medium-ESR stocks are comparatively low VOL. This volatility pattern has little consequence for ESR-sorted stock portfolios, because most stock-level volatility differences diversify away in large stock portfolios.

However, the same is not true for the corresponding option portfolios. VOL is a first-order determinant of option prices and of EORs. Options written on both high- and low-ESR stocks are priced at substantially higher implied volatilities than options written on medium-ESR stocks, potentially affecting EORs at both ends of the ESR distribution. In fact, [Hu and Jacobs \(2020\)](#) show both theoretically and empirically that EORs of calls are decreasing in the underlying’s volatility (while the reverse is true for puts).⁸ This difference in the impact of VOL on stock and options is crucial for interpreting our results.

To illustrate, we replicate the result of [Hu and Jacobs \(2020\)](#) and [Aretz et al. \(2022\)](#) in Table 6. The results show that VOL is one of the strongest predictor variables for ATM call returns: we find a H–L return spread of 23.4% per month for calls. These magnitudes dwarf the ESR-sorted option spreads in Table 3, indicating that even moderate dependence between ESR and VOL can dominate ESR-sorted option portfolio returns. To put these numbers into perspective: when we use a NN3 and the full set of 227 predictor variables to predict ATM call option returns in Section 4.7, we obtain a H–L return spread of 32.2% per month (see Table 10). Hence, VOL alone delivers more than 72% of the predictable spread

impose minimum requirements on stock price, shares outstanding, and trading activity for option eligibility.

⁸Theoretically, what matters for the effect we describe is idiosyncratic volatility, i.e., the part of the return volatility that is not related to ESRs (see, e.g., [Hu and Jacobs, 2020](#); [Aretz et al., 2022](#)). In practice, however, idiosyncratic volatility measures such as the one from [Ang et al. \(2006\)](#) are almost perfectly correlated with total volatility. We hence use total volatility for simplicity. Appendix IA3.3 shows that all our results also hold when using the idiosyncratic volatility of [Ang et al. \(2006\)](#) or implied volatility from options.

Table 6: Call Option Returns Sorted on VOL

This table reports average ATM call option returns (per month, in %) for stock volatility (VOL; historical volatility over the past 21 trading days) for the period from 2006-2022. t -statistics (in parentheses) are adjusted for heteroscedasticity and autocorrelation (Newey and West, 1987) with four lags. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively. Sharpe ratios (SR) are annualized.

Low	2	3	4	5	6	7	8	9	High	H-L	SR
20.0***	17.0***	12.4**	13.5**	12.0**	11.7**	5.2	5.4	3.4	-3.4	23.4***	1.05
(3.21)	(2.89)	(2.06)	(2.27)	(2.19)	(2.09)	(1.04)	(1.14)	(0.81)	(-0.88)	(4.08)	

that state-of-the-art methods can detect for calls.

These two effects together—the ESR-VOL relation and the effect of VOL on EORs—have strong implications for the naïve ESR-sorted option returns. Because high VOL mechanically lowers expected call returns, the U-shaped relationship between ESR and VOL induces a hump-shaped pattern for average call returns across ESR deciles observed in Table 3.

4.3.1 Disentangling ESR and VOL

To further dissect the strong ESR-VOL dependence, we perform an independent double sort. Figure 4 shows that the joint distribution of ESR and VOL is highly non-uniform: low-VOL stocks are concentrated in the middle ESR deciles, whereas the extreme ESR deciles are dominated by high-VOL stocks. A concentration on the main diagonal of Figure 4 would be consistent with a positive risk-return trade-off, and the observed concentration slightly below the actual main diagonal is roughly consistent with that. However, the puzzling “outlier” is the top right corner, i.e., the stocks with both high VOL and low ESR. This subset of stocks also gives rise to the finding of Ang et al. (2006) of a hump-shaped return pattern when sorting stocks on VOL, with a negative H-L spread—which is equally present in our sample.⁹

From an expected option return perspective, the economically attractive combination would be high ESR and low VOL, which is extremely rare, making *independent* double sorts not a feasible empirical strategy. We therefore use *dependent* double sorts, sorting first on VOL and then on ESR within VOL buckets.

⁹Figure IA.2 shows that the U-shaped relation is equally present in the full sample of CRSP stocks, and Figure IA.3 shows that the results are robust to using either IVOL or IV as alternative measures of VOL.

Figure 4: **Dependence between ESR and VOL**

This figure shows the average number of stocks per month in each portfolio for a 10x10 independent double sort on ESR and stock volatility (VOL; historical volatility over the past 21 trading days) for the period from 2006-2022. The average number of total stocks/month is 1,747.

Average number of stocks per portfolio

L	5.9	6	6.3	7.2	8.4	11	16	24	36	58
2	17	13	12	13	14	16	19	22	24	24
3	28	21	18	16	16	15	15	16	15	13
4	34	26	21	18	15	14	14	12	11	8.8
5	31	27	23	20	17	15	12	11	10	7.7
6	24	26	25	21	18	16	14	11	10	7.8
7	17	23	24	23	21	19	16	13	10	7.9
8	12	18	21	23	23	21	19	16	13	9.3
9	6.2	11	17	21	23	24	23	20	17	12
H	2.5	4.5	8	13	18	23	26	28	27	25
	L	2	3	4	5	6	7	8	9	H
	VOL portfolio									

The results for the dependent double sorts of ATM calls are in Table 7. Panel A reports average VOL within each dependent double-sort cell. By construction, VOL varies strongly across the VOL portfolios, but, within each VOL bucket, variation across ESR quintiles is substantial only in the highest-VOL group. This confirms that the dependent sort largely purges the mechanical U-shaped relation between ESR and VOL, except among the most volatile stocks, where low-ESR stocks remain systematically more volatile.

Panel B reports the difference between VOL and option-implied volatility (IV), which can be interpreted as a measure of option expensiveness relative to fundamentals (Goyal and Saretto, 2009). Two patterns stand out. First, options written on low-VOL stocks tend to be priced at IVs exceeding VOL, consistent with a positive variance risk premium for single stocks (Duarte et al., 2024). Second, within most VOL buckets, VOL-IV is weakly lower for high-ESR stocks, implying that options on high-ESR stocks are systematically more expensive relative to their risk than options on low-ESR stocks. This pattern might further depress EORs exactly where ESRs are highest, reinforcing the attenuation of ESR-based

Table 7: **Dependent Double Sorts on ESR and VOL**

This table reports results for dependent double sorts on ESR and volatility (historical stock volatility from daily returns of the past 21 trading days) for the period from 2006 to 2022. t -statistics (in parentheses) are adjusted for heteroscedasticity and autocorrelation (Newey and West, 1987) with four lags. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively. Returns are in %, per month, volatilities are in %, p.a., and Sharpe ratios (SR) are annualized.

	Low	2	3	4	High	Low	2	3	4	High
<i>A. VOL</i>						<i>B. VOL – IV</i>				
Low-ESR	19.21***	28.19***	36.28***	47.53***	85.53***	-6.02***	-6.44***	-6.95***	-6.67***	9.65***
2	19.18***	27.99***	35.91***	46.78***	78.17***	-5.42***	-4.39***	-3.88***	-3.29***	9.19***
3	19.45***	28.01***	35.87***	46.50***	75.30***	-5.75***	-4.73***	-3.58***	-1.90**	10.22***
4	19.93***	28.24***	35.97***	46.47***	73.88***	-6.29***	-5.37***	-4.02***	-1.30*	11.18***
High-ESR	20.77***	28.55***	36.28***	46.88***	75.01***	-8.04***	-8.02***	-7.41***	-5.39***	8.04***
H-L	1.57*** (10.68)	0.36*** (8.76)	-0.00 (-0.05)	-0.64*** (-6.47)	-10.51*** (-9.47)	-2.02*** (-6.14)	-1.58*** (-4.44)	-0.47 (-1.06)	1.28** (2.12)	-1.62** (-1.99)
<i>C. Stock returns</i>						<i>D. Call option returns</i>				
Low-ESR	0.76	0.69	0.40	0.17	-0.80	13.84***	10.82**	7.29*	8.06*	-5.73
2	0.73	0.84	0.85	0.49	-0.02	18.04***	15.23***	12.72***	4.16	-2.07
3	0.82	0.85	0.99	0.74	0.76	21.30***	14.80***	12.29***	3.57	3.14
4	0.75	0.88	0.94	0.69	0.72	20.32***	11.56***	13.45***	4.30	0.03
High-ESR	0.77	0.97	1.17*	1.08	1.21*	19.14***	12.52***	13.44***	6.46	4.66
H-L	0.01 (0.03)	0.28 (1.48)	0.77*** (3.08)	0.91*** (2.72)	2.01*** (5.04)	5.30 (1.05)	1.71 (0.40)	6.14 (1.63)	-1.60 (-0.47)	10.39*** (3.25)
SR	0.01	0.41	0.90	0.85	1.52	0.26	0.11	0.43	-0.12	0.85

option strategies.

Panel C shows the H-L spread in stock returns within each VOL quintile. Among the two lowest volatility groups, stock return spreads are close to zero and statistically insignificant, indicating little predictable variation in ESRs. In contrast, ESR spreads are economically and statistically significant in VOL quintiles 3 and 4 and largest in quintile 5. Thus, predictable differences in ESRs are concentrated in medium- and especially high-VOL stocks.

Panel D reports the corresponding option return spreads. Despite sizable stock-level predictability in VOL quintiles 3 and 4, ESR-sorted option return spreads remain small and statistically insignificant. Only in the highest VOL quintile does the ESR-option strategy deliver a significant spread.

Nevertheless, this is a pyrrhic victory for ESR-sorted option strategies. The only segment of the cross section in which ESR predicts option returns is the highest volatility quintile.

However, here average option returns are low in levels because high VOL substantially increases option prices and reduces the embedded leverage of option positions. As a result, even when the ESR-sorted option spread is statistically significant, it translates into only modest absolute returns and unattractive risk-adjusted performance.

Quantitatively, in the high-VOL subsample, the ESR-sorted stock strategy earns an average return spread of 201 basis points per month. The corresponding option strategy delivers a spread that is only about five times as large, even though unconditional average ATM call returns are more than ten times larger than stock returns. At the same time, call returns are roughly ten times more volatile. As a result, the Sharpe ratio of the option strategy is only about half that of the corresponding stock strategy (0.85 vs. 1.52). Moreover, an even simpler strategy that sorts options directly on volatility (Table 6) produces a return spread more than twice as large and a clearly higher Sharpe ratio.

Thus, even in the parts of the cross section where leverage should, in theory, generate the strongest amplification, calls do not provide an economically efficient way to exploit predictable variation in ESRs. The ability of ESR to forecast stock returns therefore appears largely disconnected from the empirical drivers of option risk premia, and once volatility is controlled for, little additional performance remains to be extracted via options.

4.4 Option Return Bound Conditional on Volatility

We next repeat the test of the option return bounds while controlling for VOL using dependent double sorts. For volatility portfolios 1–4 the mean-preserving spread condition in Assumption 2 is violated in the data, based on realized returns in Tables 7 and IA.5. We therefore restrict attention to the highest-VOL quintile, where both the theoretical assumptions are more plausible and ESR-sorted stock return spreads are economically large.

The results in Table 8 show that the lower bound is no longer rejected in the high-VOL subsample. The economic reason is straightforward: average call returns in this subsample are low, reflecting the high option prices associated with volatile underlying stocks. Consequently, even a modest ESR-sorted option return spread can satisfy the bound, which scales the option spread by the ratio of the option market return to the stock market return. Thus, relative to the low market return on calls, the observed ESR-option spread is sufficiently large to satisfy the theoretical restriction.

Table 8: **Test of Call Option Return Bounds – Controlling for VOL**

This table is analogous to Table 4, but uses only the subset of high volatility stocks (top 20%). Within this subset, we perform a sort on ESR and test the lower bound for call returns in (5). The null hypothesis H_0 is that the lower bound holds, and the corresponding p -value (in %) is for a one-sided test. All values are in percent.

	ATM	OTM
$R_M^S - R_f$	0.37	0.44
$R_H^S - R_L^S$	2.01	2.02
$R_M^C - R_f^C$	-0.18	0.93
$R_H^C - R_L^C$	10.39	12.92
Lower bound (LB)	-0.95	4.24
$(R_H^C - R_L^C) - LB$	11.34	8.68
t -statistic	4.17	1.99
p -value (in %)	100.00	97.63

Thus, conditioning on volatility restores consistency with the model-free restriction, but only because the benchmark option return against which the spread is evaluated is itself small. In other words, the bound holds in relative terms, yet this does not imply that option returns strongly reflect predictable variation in expected stock returns in economically meaningful magnitudes: the theoretical amplification through option leverage holds only where there is little economic value to be harvested. This reinforces the interpretation that volatility, rather than expected stock returns, is the dominant driver of cross-sectional variation in option risk premia.

4.5 Relation to Firm Leverage

A natural question is what drives the complex ESR-VOL relationship documented above. A large literature emphasizes the role of firm leverage in shaping equity volatility: a decline in equity value raises the debt-to-equity ratio, mechanically increasing equity risk (Black, 1976; Christie, 1982), a channel that Choi and Richardson (2016) confirm empirically using market values of corporate debt and Doshi et al. (2019) show matters for the cross section of expected equity returns. In our setting, leverage is a natural candidate because, holding all else constant, higher leverage implies both higher equity volatility and higher expected equity returns. To investigate, Table 9 reports average leverage, defined as the face value of

Table 9: **Firm Leverage – Dependent Double Sorts on ESR and VOL**

This table reports the average leverage per firm for dependent double sorts on ESR and volatility (historical stock volatility from daily returns of the past 21 trading days) for the period from 2006 to 2022. Leverage is defined as face value of debt over market value of equity.

	Low-VOL	2	3	4	High-VOL
Low-ESR	0.38	0.43	0.48	0.59	1.23
2	0.35	0.40	0.45	0.54	0.92
3	0.37	0.41	0.46	0.56	0.86
4	0.39	0.45	0.50	0.60	0.82
High-ESR	0.44	0.51	0.58	0.67	1.03

debt divided by the market value of equity, for the dependent ESR-VOL double sort. The leverage values strongly comove with VOL observed in Panel A of Table 7, suggesting that firm leverage at least partially explains the observed pattern.

To assess this channel more directly, we construct de-levered *asset* returns. Specifically, we apply Merton (1974)’s model to unlever stock returns and compute asset volatility, then use a NN3 network and the procedure from Section 3.6 to compute expected *asset* returns. Due to data availability, the sample shrinks from an average of 1,747 firms per month to 1,550. Further details are provided in Appendix IA4.

Figure 5 shows the number of firms per portfolio for a 10×10 independent double sort on expected asset return and asset volatility. Compared with Figure 4, the correlation across bins is visibly reduced, firms are distributed more uniformly across portfolios, and the puzzling concentration in the top-right corner of the grid shrinks considerably. The main pattern nevertheless persists, indicating that leverage only partially accounts for the ESR-VOL relationship.

4.6 Our Result in a Black–Scholes Model

The Black–Scholes model, despite its well-known limitations for option pricing, provides a transparent laboratory for isolating the forces behind our empirical findings.

We select ESR values to match the average portfolio returns in Table 2, with one representative stock per portfolio, and calibrate volatility levels so that model-implied option returns match the data. We also consider a second calibration in which stock volatility is

Figure 5: **Dependence Between Expected Asset Return and Asset Volatility**

This figure shows the average number of firms per month in each portfolio for an 10x10 independent double sort on expected asset return and asset volatility for the period from 2006-2022. The average number of firms/month is 1,550.

Average number of firms per portfolio

Expected asset return portfolio	L	9	8.8	8.6	9.4	11	13	16	21	27	32
	2	15	12	12	12	12	13	14	14	15	13
	3	21	17	13	12	11	11	11	12	11	10
	4	22	20	17	14	13	12	11	10	10	10
	5	24	22	19	17	15	13	11	10	8.5	7.6
	6	21	23	22	20	17	15	12	10	8.3	6.3
	7	18	21	22	21	19	17	14	12	10	8.1
	8	13	16	19	21	21	20	18	16	13	11
	9	8.5	11	15	18	21	22	22	22	19	17
	H	3.6	4.9	7.4	11	15	19	23	28	33	41
		L	2	3	4	5	6	7	8	9	H
		Asset volatility portfolio									

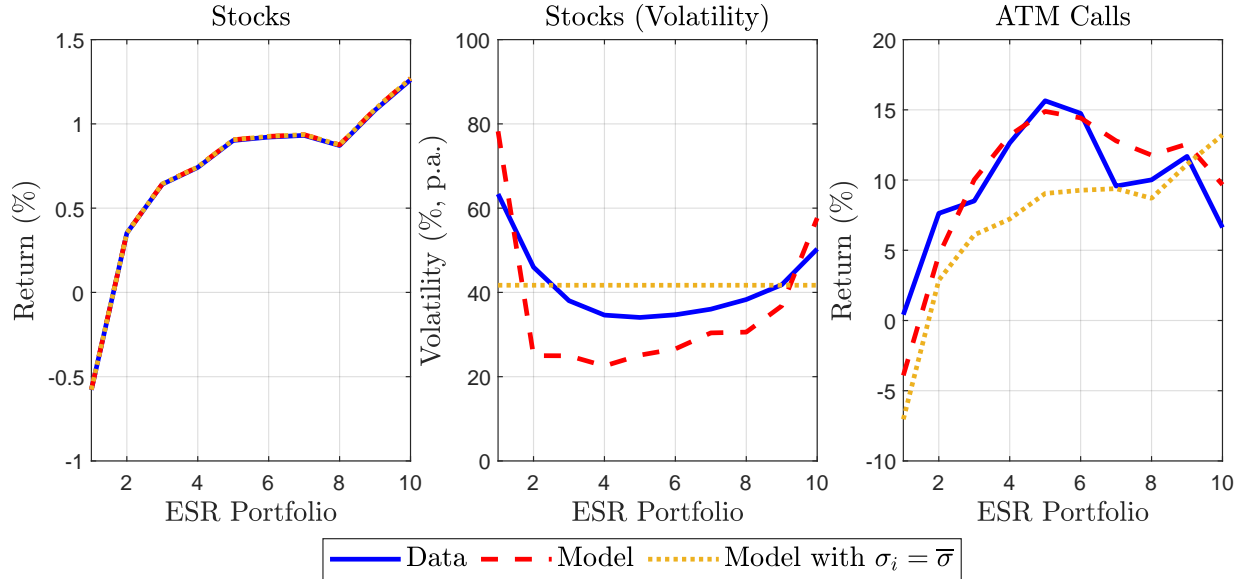
held constant at the average empirical level of 41.7% p.a., so that all assumptions of Proposition 1 are satisfied. In both cases, we set $R_f = 0.11\%$ per month, maturity to one month, and moneyness $K/S = 1$.

Figure 6 presents the results. The first panel confirms that expected stock returns line up with the data by construction, while the calibrated volatility levels are qualitatively consistent with their empirical counterparts. A richer model incorporating, for example, a variance risk premium would likely improve the fit further.

The middle and right panels show that the calibrated model can reproduce the data well, both qualitatively and quantitatively. Expected call returns are hump-shaped across ESR portfolios. Under constant volatility, by contrast, expected call returns increase monotonically in ESR, as implied by Proposition 1. Comparing the two calibrations isolates the key mechanism: the U-shaped volatility pattern across ESR deciles generates the non-monotonicity in option returns. At low ESRs, high volatility compresses option returns toward zero. At intermediate ESRs, low volatility amplifies returns relative to the constant-

Figure 6: **Calibrated Black–Scholes Model**

This figure shows expected stock returns, stock volatilities, and expected call returns in a calibrated Black–Scholes model with one representative stock per portfolio. ESR values are set to match the average portfolio returns in Table 2, volatility levels are chosen such that the model-implied option returns match the data. The second model calibration (dotted yellow line) sets the stock volatility constant at the average empirical level of $\bar{\sigma} = 41.7\%$ p.a. We set $R_f = 0.11\%$ per month, maturity to one month, and moneyness $K/S = 1$.



volatility benchmark. At high ESRs, high volatility again compresses returns, offsetting the large ESRs and producing the non-monotonic patterns observed in the data.

4.7 Corroborating Evidence from Machine-learned Option Returns

4.7.1 Feature Importance

We next examine which characteristics matter most for predicting option returns within our machine-learning framework. Following Gu et al. (2020), we measure variable importance as the reduction in out-of-sample R^2 when a given predictor is set to its median value of zero while all other predictors are held fixed.

Figure 7 reveals a striking dichotomy. Option return predictability is dominated by option-specific and volatility-related characteristics: moneyness, trading volume, implied

and realized variance, and liquidity measures account for most of the explanatory power, whereas stock characteristics known to predict equity returns play a negligible role. Panel A confirms that when predicting stock returns, the top predictors closely match those in Gu et al. (2020), yet these same variables are almost entirely absent among the leading predictors of option returns. Of the top 30 stock return predictors, only a handful (in bold in Panel B) also rank among the top option return predictors.

One concern is that variables related to ESRs may affect option returns only through nonlinear transformations that our baseline specification fails to capture. To address this, we explicitly include ESR as an additional predictor. Even then, ESR ranks low in predictive importance for option returns—106th out of 227 in Panel B—contributing little to out-of-sample fit despite being a strong cross-sectional predictor of stock returns (Table 2). The information driving ESRs is thus largely orthogonal to the information that prices option risk. These conclusions do not change if the information set used to predict option returns either excludes ESR or stock features (See Figure IA.4 for details).

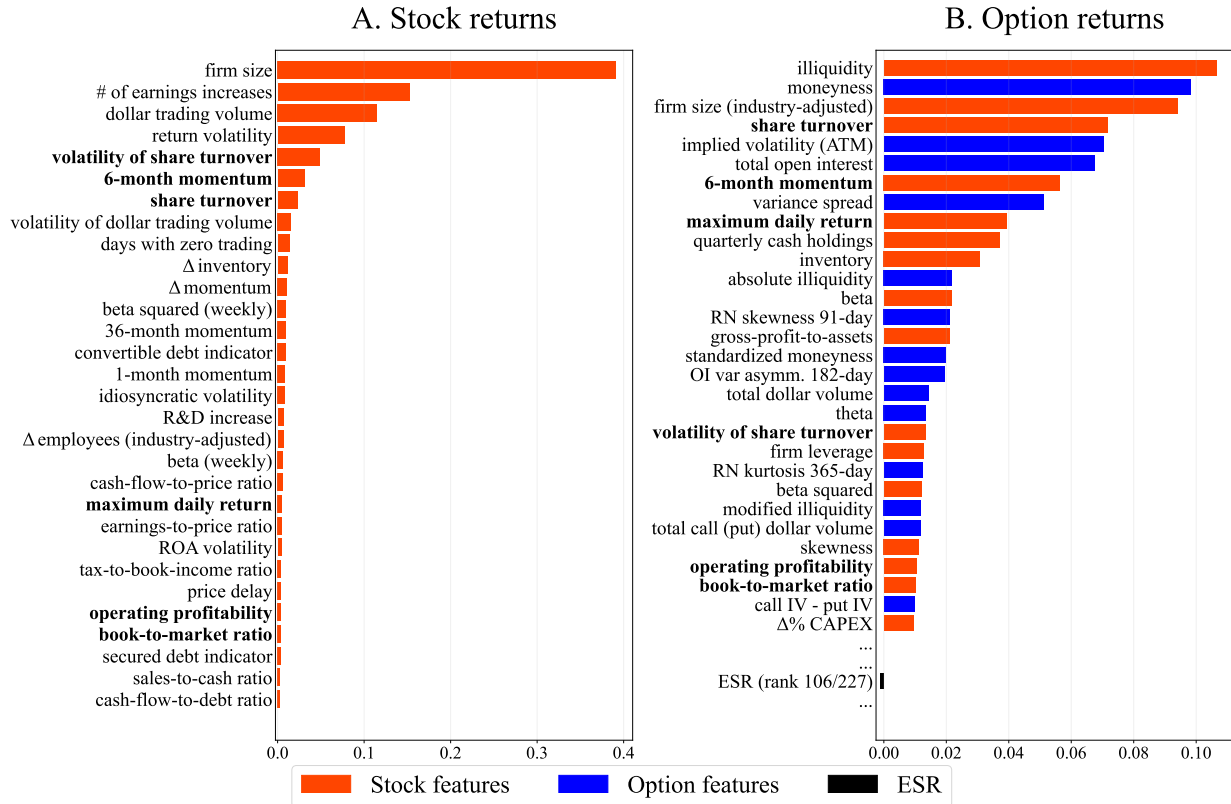
These results reinforce the portfolio-based findings: predictable variation in option returns is driven primarily by volatility and liquidity, not by ESRs. This is consistent with the mechanism documented in Section 4.3, where the dependence between ESR and VOL mechanically offsets the effect of higher expected payoffs on EORs.

4.7.2 EOR sorted Option Portfolios

Table 10 reports average out-of-sample returns for call portfolios sorted by EOR obtained from the trained NN3 model. The three panels vary the information set used to predict option returns: the full set of 227 stock and option features (Panel A), only option features (Panel B), and option features augmented with ESR (Panel C). The results in Panel A show that option returns are well predicted by the NN3 model, generating a substantial H–L spread of 32.2% per month with a Sharpe ratio of 3.03 p.a. Restricting the information set to option features alone in Panel B reduces the spread modestly to 25.7%. Adding ESR as an additional feature in Panel C has essentially no effect on the H–L spread. Both findings are consistent with the feature importance results in Figure 7: stock characteristics and ESR are largely irrelevant for predicting option returns. Option returns are thus well predictable, but not by ESR or its components.

Figure 7: Feature Importance for Option and Stock Return Prediction

This figure shows the top 30 features and ESR ranked by feature importance for the out-of-sample period from 2006 to 2022, measured as the average decrease in out-of-sample R-squared. Following Gu et al. (2020), we determine the influence of feature j based on the reduction in out-of-sample R-Squared from setting all values of predictor j to their median, while holding the remaining model estimates fixed. Panel A displays the feature importance when predicting stock returns using stock features. Panel B displays the feature importance when predicting call option returns using both option and stock features and ESR measure. Variable importance within each model is normalized to sum to 1.



4.7.3 Predicting Option Prices

As a final exercise, we modify the neural network to predict option *prices*—transformed into implied volatility levels—rather than option *returns*. From the full variable set, we retain only stock-level variables and moneyiness, since all other option-based variables are direct or indirect functions of option prices. Figure 8 presents the top 30 most important features for ATM call options. In sharp contrast to the option return results, many features that predict stock returns also predict option prices, and ESR itself ranks among the top variables. This

Table 10: **EOR Sorted Call Returns**

This table reports average option portfolio returns, sorted by expected option return (EOR) based on the neural network model described in Section 3.7 for the period from 2006 to 2022. Sharpe ratios are denoted in annual terms. Absolute t -statistics are reported in parentheses. All portfolios are equally-weighted. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
<i>A. Option and stock features</i>												
Mean	-9.92** (-2.26)	0.57 (0.13)	3.53 (0.85)	5.91 (1.36)	6.65 (1.55)	7.98* (1.95)	8.44** (2.09)	10.48** (2.43)	11.11** (2.54)	22.29*** (4.13)	32.22*** (12.90)	9.75* (1.94)
SD	71.20	65.71	63.52	63.24	61.51	61.55	62.37	64.59	66.77	77.91	36.97	72.35
SR	-0.48	0.03	0.19	0.32	0.38	0.45	0.47	0.56	0.58	0.99	3.03	0.47
<i>B. Option features</i>												
Mean	-6.48 (-1.42)	2.05 (0.51)	4.50 (0.88)	6.55 (1.31)	7.20 (1.64)	7.85* (1.95)	8.37** (2.07)	9.62** (2.39)	9.88** (2.43)	19.85*** (3.88)	25.70*** (11.10)	9.75* (1.94)
SD	71.92	62.05	63.44	63.12	61.45	61.63	62.17	63.78	65.23	77.58	36.92	72.35
SR	-0.31	0.11	0.25	0.36	0.41	0.44	0.47	0.52	0.52	0.89	2.41	0.47
<i>C. Option features and ESR</i>												
Mean	-6.12 (-1.46)	2.18 (0.53)	4.63 (0.90)	6.72 (1.32)	7.33* (1.68)	8.06** (1.97)	8.52** (2.09)	9.78** (2.43)	10.08** (2.47)	20.18*** (3.95)	26.30*** (11.14)	9.75* (1.94)
SD	71.38	61.73	63.21	62.89	60.97	61.14	61.52	63.08	64.82	77.36	36.45	72.35
SR	-0.30	0.12	0.25	0.37	0.42	0.46	0.48	0.54	0.54	0.90	2.50	0.47

stands in stark opposition to Figure 7, where there is virtually no overlap between stock return and option return predictors, and where ESR ranks low.

At face value, this would suggest that option prices increase in ESR, offsetting the higher expected payoffs. However, as we discuss below in Section 5.2, this is not the correct interpretation. Instead, the result again reflects the strong dependence between volatility and ESR: this dependence is sufficiently pronounced that the neural network sometimes selects ESR as a more informative proxy for stock volatility than realized volatility itself.

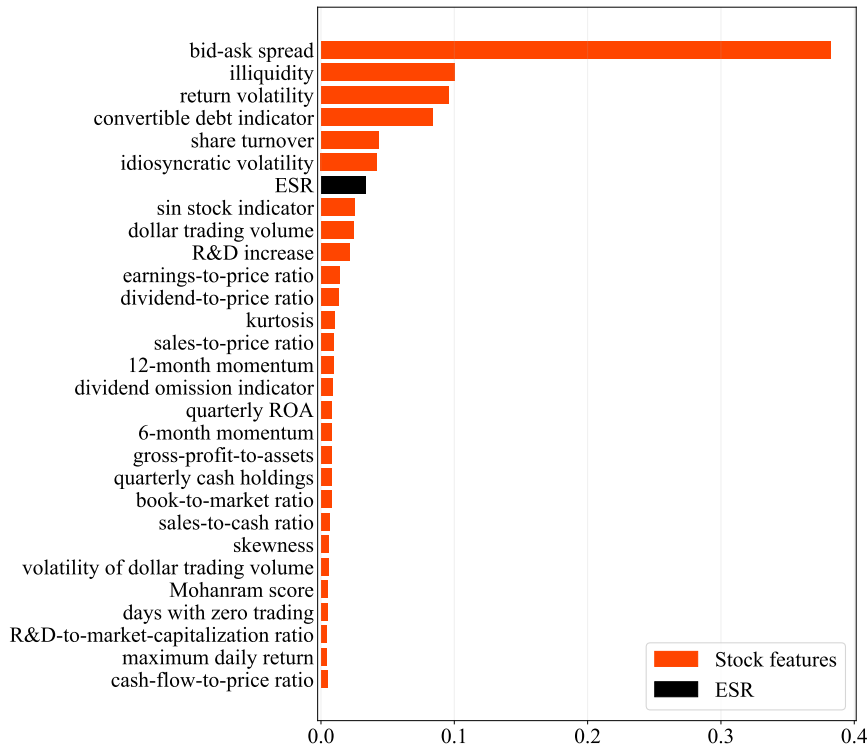
5 Other Potential Explanations and Robustness

5.1 Joint Risk Factors of Stocks and Options

It is conceivable that risk factors that drive option prices are also present in stock returns. For example, it is well-known that options contain a large variance risk-premium, but also

Figure 8: **Feature Importance for Predicting Implied Volatilities**

This figure shows the feature importance when predicting the level of implied volatilities of ATM call options via NN3. We plot the top 30 features and ESR ranked by feature importance for the out-of-sample period from 2006 to 2022, measured as the average decrease in out-of-sample R-squared. Following Gu et al. (2020), we determine the influence of feature j based on the reduction in out-of-sample R-Squared from setting all values of predictor j to their median value of zero, while holding the remaining model estimates fixed. We use stock features and ESR as predictor variables. Variable importance within each model is normalized to sum to 1.



stocks with higher exposure to variance risk have higher expected returns (Ang et al., 2006). In the case of calls, this could lead to a positive co-movement of call prices and ESR, that could generate our result at least qualitatively (and would violate Assumption 1). For puts, however, we would expect the opposite: puts on stocks with high ESR (high variance risk exposure) should also be more expensive, which should further increase the ESR-sorted put return spread, instead of decrease it. More generally, the same line of reasoning would apply to any other risk factor we can think of, say jumps, liquidity, etc. Hence, the fact that we find the ESR-sorted option return spread being too low for *both* calls and puts (see Appendix IA1) is difficult to reconcile with joint risk-factor explanations.

5.2 Direct Dependence of Option Price on ESR

We document that stock volatility co-moves with ESR, which in turn induces a co-movement between option prices and ESR. In principle, however, this relation need not be driven exclusively by the volatility channel. Option prices could also co-move with ESR for non-fundamental reasons, such as demand pressures (Bollen and Whaley, 2004; Garleanu, Pedersen, and Poteshman, 2008). Such pressures could contaminate the empirical mapping from expected stock returns into expected option returns (and would violate Assumption 1), even if they are not the primary force behind the patterns we document.

A direct demand-based explanation, however, faces a sharp no-arbitrage restriction. For a given stock, strike, and maturity, call and put prices cannot move independently, since put-call parity implies

$$C_{i,t} - P_{i,t} = S_{i,t} - PV(D) - K/(1 + R_{f,t}). \quad (6)$$

Accordingly, any ESR-related demand distortion that affects calls and puts asymmetrically (to match the low ESR-sorted option spread for both, see above) must show up as a corresponding deviation from (6). For example, if calls on high-ESR stocks were relatively expensive and the corresponding puts relatively cheap, then $C_t - P_t$ would be too high relative to the spot-forward benchmark on the right-hand side of (6). Hence deviations from put-call would need to be highly related to ESR¹⁰, and they should also predict future stock returns. However, once non-synchronous data issues are properly controlled for, there is little such evidence in the data.¹¹

¹⁰An earlier version of this paper documented a positive relationship between ESR and put-call parity deviations. This relationship becomes insignificant, however, once we jointly control for the U-shaped ESR-VOL structure and shorting fees (Muravyev, Pearson, and Pollet, 2022), suggesting it was a spurious reflection of the volatility channel rather than evidence of a genuine demand distortion.

¹¹Ofek, Richardson, and Whitelaw (2004) document violations of put-call parity, and Cremers and Weinbaum (2010) find that such deviations predict future stock returns. Both results have been substantially qualified: Battalio and Schultz (2006) show that most violations in the first study reflect non-synchronous prices in OptionMetrics rather than genuine arbitrage opportunities, while Honarvar and Howard (2024) show that the predictability in the second study is confined to the period before March 2008, when OptionMetrics began recording synchronized option and stock prices. All our results hold in the post-March 2008 sample. Bogousslavsky and Muravyev (2023) further document that closing auction stock price deviations account for a substantial share of apparent parity violations and that correcting for this measurement error largely eliminates their predictive power for future stock returns.

5.3 Segmented Markets

A natural concern is that our results are driven by segmentation between stock and option markets. In our framework, however, Proposition 1 implies that higher ESR mechanically raises EOR unless option prices adjust so as to offset the higher expected payoff. A segmentation story can therefore account for our findings only if the resulting pricing wedge is itself systematically related to ESR, beyond the volatility channel. Random deviations of option prices from their benchmark values would not deliver this effect; they would simply wash out in the cross section. In that sense, the required mechanism is not mere segmentation, but a structured option-pricing distortion aligned with stock-market risk premia. In any case, Section 4.6 shows that the main qualitative patterns already arise in a calibrated Black–Scholes model with fully integrated markets.

5.4 Shorting Fees

Muravyev, Pearson, and Pollet (2025) argue that shorting fees are related to ESR: stocks with low ESRs tend to be expensive to short, whereas stocks with high ESR tend to have lower shorting fees. To the extent that these fees reduce the profitability of short positions in low-ESR stocks, they may mechanically compress the return spread of ESR-sorted stock portfolios. By contrast, option prices—and therefore option returns—already incorporate shorting fees.

To evaluate this possibility, we construct the shorting-fee measure of Muravyev et al. (2022). Over our 2006–2022 sample, the average monthly cross-sectional correlation between shorting fees and ESR is only -0.07. Moreover, the average monthly difference in shorting fees between the long and short legs of the ESR-sorted stock portfolio is less than 10 basis points, which is small relative to the corresponding stock return spread of 184 basis points reported in Table 2. Consistent with this, shorting fees (“shrtfee”) also receive little importance in the machine-learning analysis shown in Figure 7.

Taken together, these results suggest that shorting fees move in the right direction to attenuate differences in expected stock returns, but only to a limited extent. They may provide a partial offset at the margin, but they are far too small to account for the magnitude of the patterns we document.

5.5 Transaction Costs

Trading options instead of stocks could be attractive if the former entailed lower trading costs. To assess this possibility, we recompute both stock and option returns net of transaction costs.

5.5.1 Transaction Costs of Stock Portfolios

For stocks, we measure transaction costs using quoted bid-ask spreads. Let $\kappa_{i,t}^S$ denote the quoted bid-ask spread of stock i at time t . Following the standard approach in the literature, we assume that the effective one-way trading cost equals one half of the quoted spread (e.g. [Heston, Jones, Khorram, Li, and Mo, 2023](#)). Thus, the transaction cost of trading stock i is

$$\tau_{i,t}^S = 0.5 \times \kappa_{i,t}^S. \quad (7)$$

We then adjust stock portfolio returns using the usual linear approximation, deducting transaction costs from raw returns.

5.5.2 Transaction Costs of Option Portfolios

For options, we follow [Heston et al. \(2023\)](#) and assume that the effective spread is 20.3% of the quoted option spread. Let $\kappa_{i,j,t}^O$ denote the quoted bid-ask spread of option j written on stock i at time t . We assume that traders pay one half of the effective spread when entering or exiting a position, so that the transaction cost of trading the option is

$$\tau_{i,j,t}^O = 0.5 \times 0.203 \times \kappa_{i,j,t}^O. \quad (8)$$

Because we study hold-to-maturity option returns, we compute net long and short returns directly rather than relying on the linear approximation typically used for stocks. In particular, we allow transaction costs to affect long and short positions asymmetrically:

$$R^{C,Long} = \frac{(S_T - K)^+(1 - \tau^S)}{C_0(1 + \tau^O)} - (1 + R_f), \quad R^{C,Short} = \frac{(S_T - K)^+(1 + \tau^S)}{C_0(1 - \tau^O)} - (1 + R_f), \quad (9)$$

$$R^{P,Long} = \frac{(K - S_T)^+(1 - \tau^S)}{P_0(1 + \tau^O)} - (1 + R_f), \quad R^{P,Short} = \frac{(K - S_T)^+(1 + \tau^S)}{P_0(1 - \tau^O)} - (1 + R_f). \quad (10)$$

Table 11: **ESR-Sorted Optionable Stock Returns – Adjusted for Transaction Costs**

This table reports average stock and option portfolio returns after adjustment for transaction costs based on the procedure outlined in Sections 5.5.1 and 5.5.2 for the period from 2006 to 2022. ESRs are built using the neural network model described in Section 3.6. Returns are denoted in % per month. Returns of a short position are calculated such that a negative value indicates a loss, and a positive value a gain. Values that enter the respective H-L column are highlighted in bold.

ESR Decile:		Mean Return										H-L	t-stat
		Low	2	3	4	5	6	7	8	9	High		
Stocks	Long	-1.03	-0.25	0.07	0.20	0.36	0.38	0.37	0.31	0.51	0.79	0.94**	(2.20)
	Short	-0.16	0.93	1.18	1.26	1.41	1.44	1.46	1.42	1.63	1.71		
ATM Calls	Long	-5.34	1.89	3.38	7.29	10.46	9.35	4.43	4.72	5.92	0.62	-5.27	(1.36)
	Short	5.89	13.20	14.26	18.41	21.82	20.91	15.41	15.83	17.62	12.75		

Here, τ^S denotes the transaction cost associated with trading one unit of the underlying stock, computed as above, and τ^O denotes the transaction cost of trading one unit of the option, and we have omitted all subscripts for simplicity.

5.5.3 ESR-Sorted Portfolio Returns with Transaction Costs

Table 11 reports the returns on ESR-sorted portfolios of stocks, calls, and puts after accounting for transaction costs. For the H-L portfolios, we take the long side to be the high-ESR decile and the short side to be the low-ESR decile for stocks and calls, and the reverse for puts, as indicated by the bold entries in the table.

Transaction costs strengthen rather than overturn our main result. The ESR-sorted ATM call strategy is already weak in gross returns: although its average H-L spread is positive, it underperforms the naïve option market strategy and falls well short of the stock-side evidence. Once transaction costs are taken into account, this weak gross performance turns into a negative net return, while the corresponding ESR-sorted stock strategy remains strongly profitable. On average, transaction costs reduce monthly returns by about 0.5% for stock portfolios and by about 5.7% for option portfolios, with these effects being broadly similar across deciles. In proportional terms, these magnitudes are not far apart, since option returns are themselves roughly an order of magnitude larger than stock returns. The main reason option strategies perform worse net of costs is not disproportionately high per-trade costs, but the fact that option positions must be re-established every month, whereas average

turnover in each leg of the stock strategy is only about 15%.

5.6 Bias-Adjusted Option Returns

Duarte et al. (2024) point out that measurement error in option prices can lead to biases in returns and this in turn might lead to incorrect conclusions. To check the relevance for our tests, we hence implement the bias corrections recommended by Duarte et al. (2024). First, all option data filters are applied to the day prior to portfolio formation to mitigate sample selection bias. Second, when computing average returns, we weight option returns by their lagged gross return from the day before portfolio formation. This addresses the mean return (MR) bias and the regression coefficient bias, respectively. For consistency and comparability of the two strategies, we also apply the both the sample selection and the weighting scheme to the matching corresponding stocks.

Table 12 reports the results for ATM calls and the corresponding stock returns. In comparison with the respective unadjusted returns in Table 2 and Table 3 we find only small differences in return levels. The ESR-sorted call spread increases slightly from 6.21% to 7.51%, while the “market” (weighted average) drops slightly from 9.75% to 9.04%. Nevertheless, the main result that the ESR-sorted option returns underperforms the naïve options “market” remains unchanged. Moreover, the corresponding test of the bound results in a t -statistic of 3.86, which is even higher than the analogous value in Table 4 (due to lower volatility). Lastly, Table IA.12 in the appendix shows that the bias-adjustment has little effect on the option returns from the double-sort on ESR and volatility.

6 Conclusion

This paper asks whether options are an effective vehicle for exploiting predictable cross-sectional differences in expected stock returns. They are not. Although high-ESR stocks earn substantially higher returns than low-ESR stocks, the corresponding option strategies generate much weaker spreads and fail to outperform a naïve strategy that simply buys the broad option market.

This finding is difficult to reconcile with standard intuition. We derive a model-free lower bound on the expected option return spread implied by the stock return spread and

Table 12: **Bias-Adjusted ESR-Sorted Call and Stock Returns**

This table reports average bias-adjusted call option and stock portfolio returns, sorted by expected stock return based on the neural network model described in Section 3.6 for the period from 2006 to 2022. The last column presents average weighted returns. Absolute t -statistics are reported in parentheses. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
<i>A. At-the-money calls</i>												
Mean	-0.06	6.75	7.90	12.44	13.98**	13.00**	9.62*	9.08*	10.55**	7.46*	7.51**	9.04*
	(0.01)	(1.30)	(1.45)	(1.93)	(2.43)	(2.22)	(1.82)	(1.75)	(2.09)	(1.86)	(2.20)	(1.83)
<i>B. Matching stocks</i>												
Mean	-0.47	0.28	0.57	0.74	0.90**	0.90**	0.85*	0.79*	0.96**	1.24**	1.71***	0.68
	(0.64)	(0.47)	(1.06)	(1.50)	(2.00)	(2.15)	(1.88)	(1.74)	(2.01)	(2.35)	(3.63)	(1.37)

the equity premium, and show that the bound is decisively violated in the data. The key to this puzzle is a strong empirical regularity: ESR and VOL are related in a pronounced U-shaped way across stocks. Firms in both tails of the ESR distribution are substantially more volatile. Because higher volatility raises option prices, it compresses EORs and largely offsets the higher expected payoffs of options written on high-ESR stocks. Once this cross-sectional ESR–VOL relation is taken into account, a calibrated Black–Scholes model reproduces the weak link between expected stock and option returns.

Our findings have two main implications. First, for investors, equity options are a surprisingly ineffective vehicle for exploiting predictable stock returns. Embedded leverage does not sufficiently amplified expected returns when volatility co-moves with expected returns. Second, for the academic literature—particularly work arguing that informed traders prefer options to express views about future stock returns—our results show that publicly predictable differences in stock returns do not map one-to-one into option returns. Understanding derivative risk premia therefore requires modeling the joint cross-sectional structure of expected returns and volatility, rather than treating them in isolation.

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A Proofs

A.1 Proof of Proposition 1

For notational simplicity, we omit subscripts throughout the proof. The expected call return reads

$$\frac{E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]}{C_t} = \frac{E_t^{\mathbb{P}}[C_{t+1}]}{C_t}. \quad (11)$$

Lemma 1: The payoff function $E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]$ is strictly increasing and strictly convex in μ .¹²

$$\frac{\partial E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]}{\partial \mu} > 0 \quad (12)$$

$$\frac{\partial^2 E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]}{\partial \mu^2} > 0 \quad (13)$$

Proof Lemma 1 The expected call payoff is:

$$E_t^{\mathbb{P}}[C_{t+1}] = E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)] = e^{\nu} (S_t \mathcal{P}_1 - e^{-\nu} K \mathcal{P}_2) = \underbrace{e^{\nu}}_{1+\mu} S_t \mathcal{P}_1 - K \mathcal{P}_2,$$

where \mathcal{P}_1 and \mathcal{P}_2 are the probabilities of the option being in the money under the physical measure (“asset price measure”, i.e., when using the stock price as numeraire). Note that we use μ to denote the expected stock return, while in option pricing models, it usually denotes the expected log return, which we have expressed here as $\nu = \ln(1 + \mu)$.

Taking the first derivative wrt μ gives:

$$\frac{\partial E_t^{\mathbb{P}}[C_{t+1}]}{\partial \mu} = \frac{\partial E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]}{\partial \mu} \quad (14)$$

$$= (S_t \mathcal{P}_1 - e^{-\nu} K \mathcal{P}_2) + e^{\nu} \frac{\partial (S_t \mathcal{P}_1 - e^{-\nu} K \mathcal{P}_2)}{\partial \nu} \frac{\partial \nu}{\partial \mu} \quad (15)$$

$$= S_t \mathcal{P}_1 - e^{-\nu} K \mathcal{P}_2 + e^{\nu} (e^{-\nu} K \mathcal{P}_2) e^{-\nu} \quad (16)$$

$$= S_t \mathcal{P}_1 > 0 \quad \forall K > 0 \quad (17)$$

where the first step uses the chain rule, and the second step is in analogy to the call’s rho (partial derivative of option price wrt $\ln(1 + R_f)$)¹³.

¹²Note that the convexity result only holds for $K > 0$. For completeness, for $K = 0$, we have $E[R^C] = E[R^S]$, and hence Proposition 1 trivially holds with equality.

¹³For the general formulas for Greeks, see for example Bakshi, Cao, and Chen (1997), and Reiss and Wystup (2001).

The second derivative wrt μ is:

$$\frac{\partial^2 E_t^{\mathbb{P}}[C_{t+1}]}{\partial \mu^2} = \frac{\partial^2 E_t^{\mathbb{P}}[\max(0, S_{t+1} - K)]}{\partial \mu^2} \quad (18)$$

$$= S_t \frac{\partial \mathcal{P}_1}{\partial \mu} > 0 \quad \forall K > 0 \quad (19)$$

where the last expression is the probability that the call expires in the money, which always increases in μ .

Proof Proposition 1 Given that the call payoff is increasing and strictly convex in μ , we know that the ECOR in (11) is a strictly convex function of μ (since C_t is the same for all stocks). Then, by the properties of increasing and strictly convex functions and mean-preserving spreads, the slope of the line that connects $E[R_H^C]$ and $E[R_L^C]$ in a $\mu - E[R^C]$ graph is larger than the slope that connects $E[R_M^C]$ and $E[R_f^C]$:

$$\frac{E[R_H^C - R_L^C]}{\mu_H - \mu_L} > \frac{E[R_M^C - R_f^C]}{\mu_M - R_f} \quad (20)$$

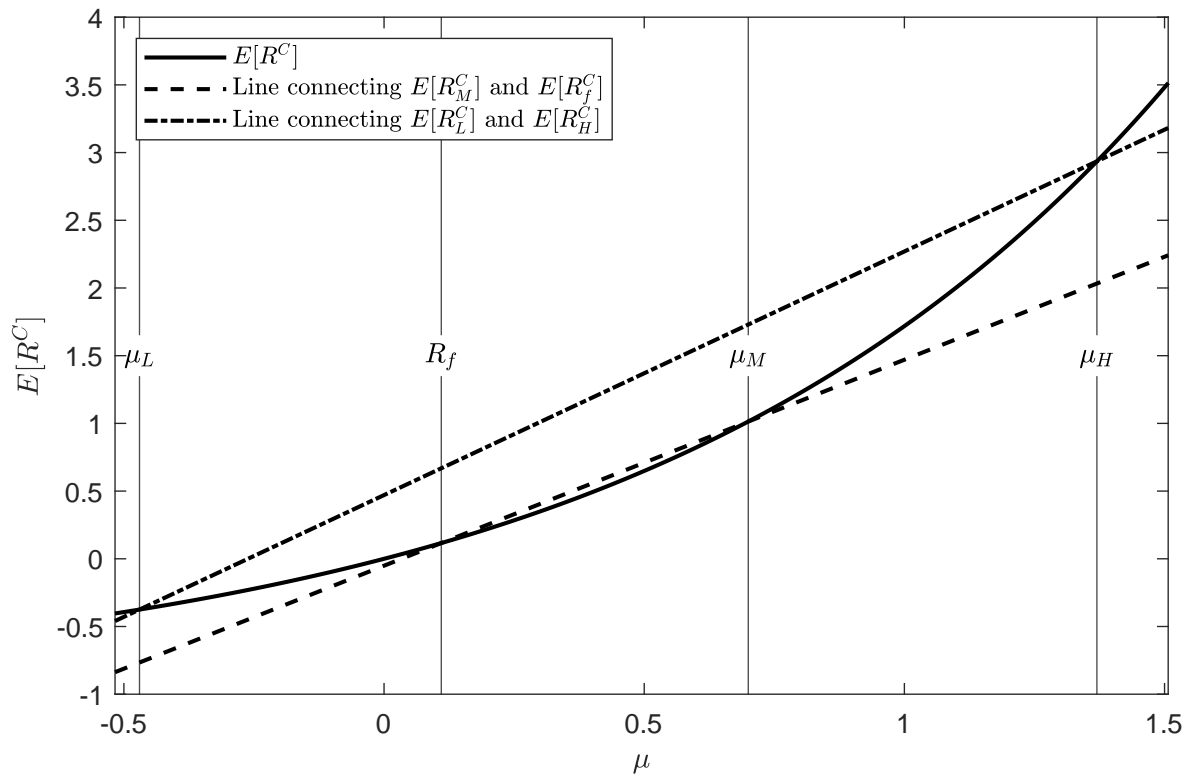
$$\rightarrow E[R_H^C - R_L^C] > \frac{\mu_H - \mu_L}{\mu_M - R_f} E[R_M^C - R_f^C] \quad (21)$$

Figure A.1 provides a graphical illustration.

Of course, the inequality might likely also hold for some values where $(\mu_H + \mu_L)/2$ is slightly below $(\mu_M + R_f)/2$, i.e., if the first is a mean preserving spread of the second, but shifted downwards. However, we cannot determine in a model-free way how much we can shift it, as that depends on the degree of convexity.

Figure A.1: Illustration Proposition 1 and Convexity

This figure provides a graphical illustration of the convexity result, and Proposition 1.



A.2 Proof Proposition 2 for Puts

We derive an upper bound for expected put returns.

Proposition 2 (Upper Bound on Expected Put Returns) *Under the same assumptions of Proposition 1, the following inequality holds:*

$$\mathbb{E}[R_H^P - R_L^P] < \mathbb{E}[R_M^P - R_f^P]. \quad (22)$$

Proof:

The steps for the proof are similar to those for Proposition 1 for calls.

The expected put return reads

$$\frac{E_t^{\mathbb{P}}[\max(0, K - S_{t+1})]}{P_t} = \frac{E_t^{\mathbb{P}}[P_{t+1}]}{P_t}. \quad (23)$$

Lemma 2: The payoff function $E_t^{\mathbb{P}}[\max(0, K - S_{t+1})]$ is strictly decreasing and strictly convex in μ :

$$\frac{\partial E_t^{\mathbb{P}}[\max(0, K - S_{t+1})]}{\partial \mu} < 0 \quad (24)$$

$$\frac{\partial^2 E_t^{\mathbb{P}}[\max(0, K - S_{t+1})]}{\partial \mu^2} > 0. \quad (25)$$

Proof Lemma 2 The expected put payoff is:

$$E_t^{\mathbb{P}}[P_{t+1}] = E_t^{\mathbb{P}}[\max(0, K - S_{t+1})] = e^\nu \left(e^{-\nu} K(1 - \mathcal{P}_2) - S_t(1 - \mathcal{P}_1) \right) = K(1 - \mathcal{P}_2) - e^\nu S_t(1 - \mathcal{P}_1),$$

where \mathcal{P}_1 and \mathcal{P}_2 are the probabilities of the option being in the money under the physical measure (“asset price measure”, i.e., when using the stock price as numeraire).

Taking the first derivative wrt μ gives:

$$\frac{\partial E_t^{\mathbb{P}}[P_{t+1}]}{\partial \mu} = \left(e^{-\nu} K(1 - \mathcal{P}_2) - S_t(1 - \mathcal{P}_1) \right) + e^\nu \frac{\partial (e^{-\nu} K(1 - \mathcal{P}_2) - S_t(1 - \mathcal{P}_1))}{\partial \nu} \frac{\partial \nu}{\partial \mu} \quad (26)$$

$$= e^{-\nu} K(1 - \mathcal{P}_2) - S_t(1 - \mathcal{P}_1) - e^{-\nu} K(1 - \mathcal{P}_2) \quad (27)$$

$$= -S_t(1 - \mathcal{P}_1) < 0 \quad \forall K > 0 \quad (28)$$

where the first step uses the chain rule, and the second step is in analogy to the put’s rho (partial derivative wrt $\ln(1 + R_f)$)¹⁴.

¹⁴For the formulas for Greeks, see for example Bakshi et al. (1997), and Reiss and Wystup (2001).

The second derivative wrt μ is:

$$\frac{\partial^2 E_t^{\mathbb{P}}[P_{t+1}]}{\partial \mu^2} = \frac{\partial - S_t(1 - \mathcal{P}_1)}{\partial \mu} > 0 \quad \forall K > 0 \quad (29)$$

where the last expression is the negative of the probability that the put expires in the money, which always increases in μ .

Proof Proposition 2 Given that the put payoff is decreasing in μ , it is straightforward to see that, given $\mu_H > \mu_M > R_f \geq \mu_L$, Proposition 2:

$$E[R_H^P - R_L^P] < E[R_M^P - R_f^P] \quad (30)$$

must hold. Figure A.2 provides a graphical illustration.

Two more comments:

1. Under the alternative assumptions that $\mu_H > \mu_M > R_f \geq \mu_L$, and $(\mu_H + \mu_L)/2 \leq (\mu_M + R_f)/2$, we could derive the following upper bound on EPOR that is analogous to Proposition 1:

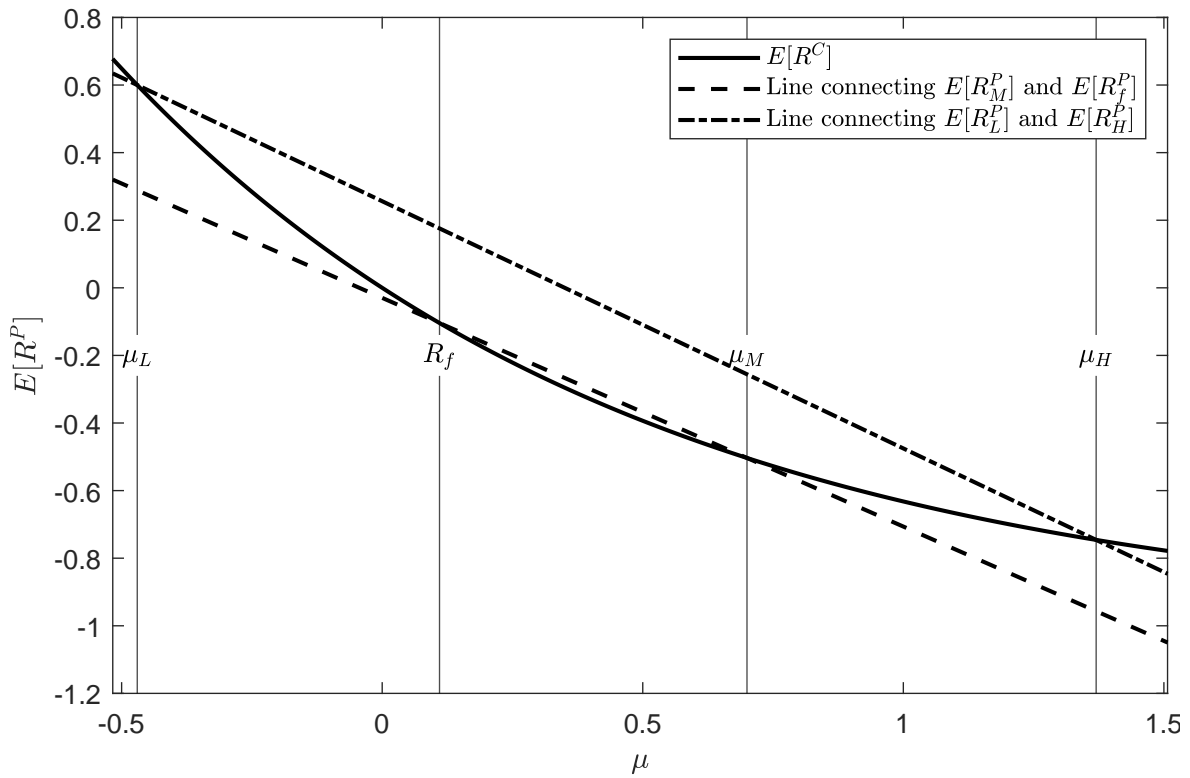
$$E[R_M^P - R_f^P] < \frac{\mu_H - \mu_L}{\mu_M - R_f} E[R_H^P - R_L^P]. \quad (31)$$

The steps are analogous to those for Proposition 1 above. However, that bound is only generally true if μ_H and μ_L are a mean preserving spread over μ_M and R_f , possibly shifted downwards. However, an empirical test of this bound has little power, since there are few stocks with returns far enough below R_f .

2. Of course, the bound in (31) might also hold if μ_H and μ_L are a mean preserving spread over μ_M and R_f , but shifted upwards, as it is the case in Figure A.2. Figure A.4 shows that this is indeed a good and tight approximation in standard models. However, we cannot guarantee that property to hold in general, but it is model and parameter specific.

Figure A.2: Illustration Proposition 2 and Convexity

The figure provides a graphical illustration of the convexity result, and Proposition 2.



A.3 Illustration of Bounds on Expected Option Returns

We quantitatively illustrate the bounds on expected option returns from Section 2.2. We calibrate the μ_L and μ_H using the returns of PF1 and PF10 from the ESR-sorted optionable stock portfolios, μ_M as the average (equal-weighted) return of all optionable stocks, and R_f using the average risk-free rate from OptionMetrics. All other model parameters are taken from Bakshi, Cao, and Zhong (2021). We rearrange the bounds in Equations (5), (22) and (34) in order to increase the readability of the plots. Note that in the case of puts, this flips the sign of the inequality, since $E[R_M^P - R_f^P]$ is negative. For illustration, we use several standard option pricing models, namely the models of Black and Scholes (1973), Heston (1993), and Bates (1996).

Figure A.3 shows that the lower bound on ECOR is fairly tight in all cases. Figure A.4 shows that the lower bound in (22) is rather loose (the sign of the inequality is flipped in the plot, since $E[R_M^P - R_f^P]$ is negative). In fact, the true return ratio is close to the spread in returns, as in the approximation in (34), although the latter is a tight lower bound.

Figure A.3: Illustration of Bound for Expected Call Returns

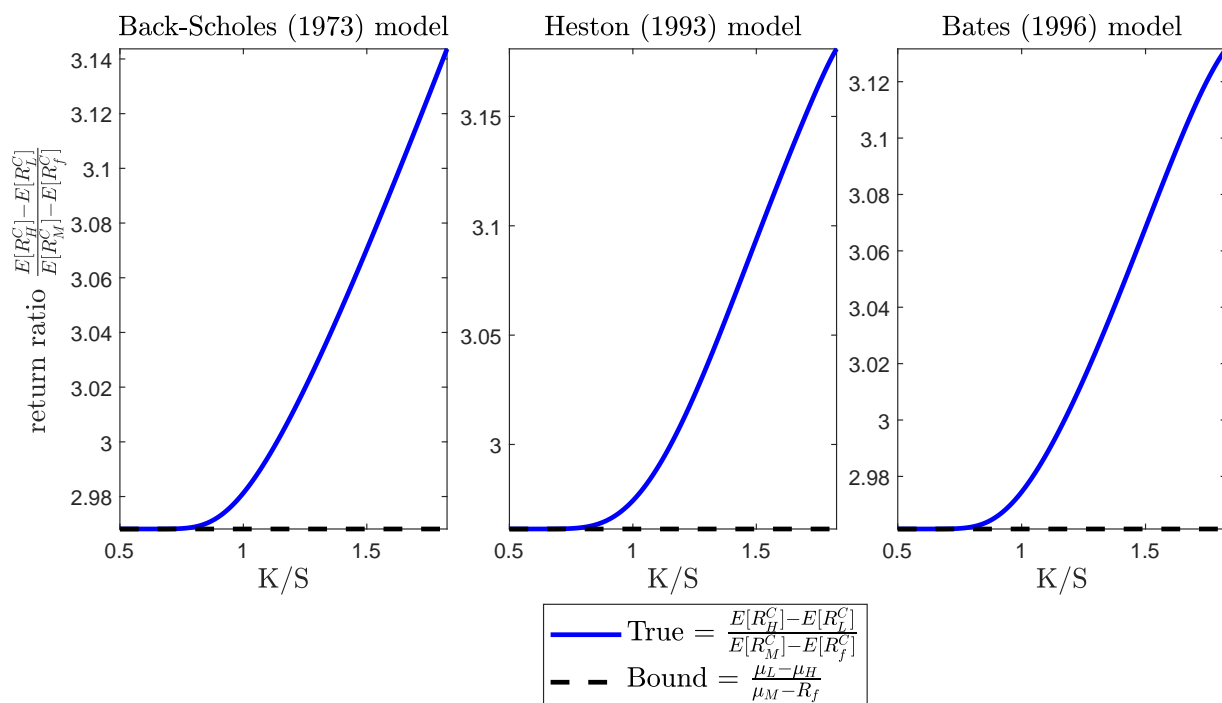
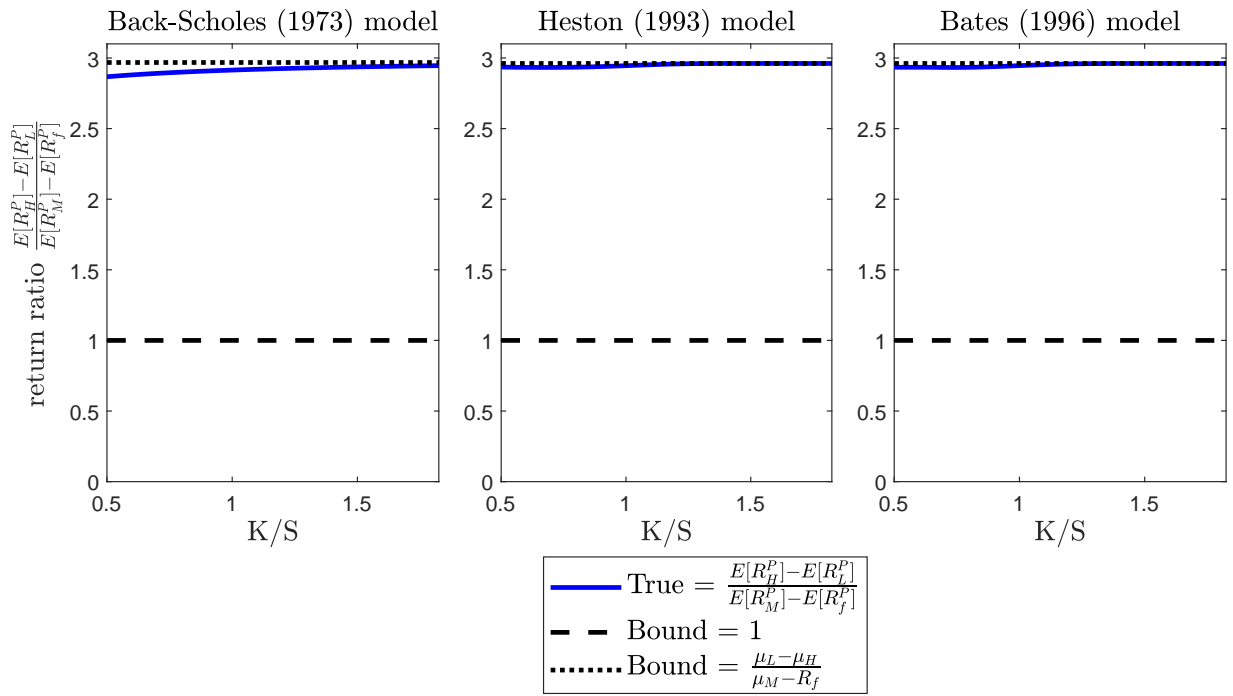


Figure A.4: Illustration of Bound for Expected Put Returns



Betting on Stocks with Options?*

Internet Appendix

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Abstract

This appendix provides variable definitions, additional empirical results, and robustness tests.

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System.

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IA1 Results for Puts

This section reproduces the main theoretical and empirical results for puts.

IA1.1 Relationship between ESR and EOR for Puts

The return function of hold-to-maturity put options can be expressed as:

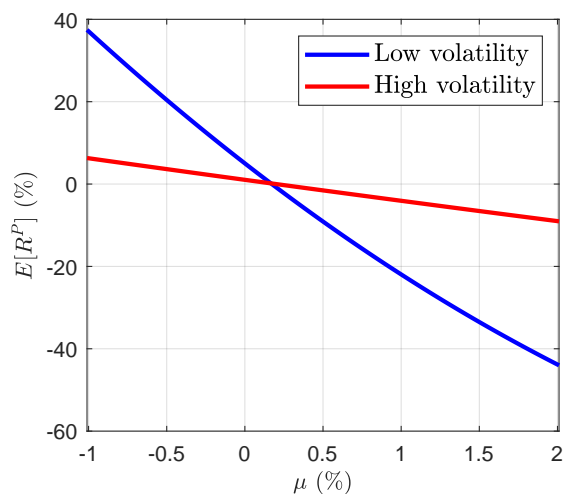
$$R_{i,t+1}^P = \frac{\max(0, K - S_{i,t+1})}{P_{i,t}} - (1 + R_{f,t}). \quad (32)$$

where $P_{i,t}$ denotes the price of the put on stock i with strike K at time t , and $R_{f,t}$ is the risk-free rate.

To build intuition and clarify the role of the assumptions, Figure IA.1 illustrates expected put returns using the Black-Scholes model. Holding volatility σ fixed, expected put returns are decreasing in μ . Moreover, expected put returns are convex functions of μ . The figure also highlights that volatility is a first-order determinant of expected option returns.

Figure IA.1: Expected Put Returns as a Function of μ and σ

This figure displays the expected put return in the Black-Scholes model as a function of monthly expected stock return μ , for low volatility ($\sigma = 15\%$ p.a.) and high volatility ($\sigma = 80\%$ p.a.) stocks. Time-to maturity is fixed at one month, the risk-free rate at 2% p.a., and $K/S = 1$.



IA1.2 Bounds for Puts

We derive an upper bound for expected put returns. Proposition 2 from Appendix A.2 shows that the following inequality holds:

$$\mathbb{E}[R_H^P - R_L^P] < \mathbb{E}[R_M^P - R_f^P]. \quad (33)$$

The result is intuitive, as expected put returns are monotonically decreasing in μ . In Appendix A.3 we show that the bound in (22) is typically very slack, while the following approximation is pretty exact:

$$\mathbb{E}[R_H^P - R_L^P] \approx \frac{\mu_H - \mu_L}{\mu_M - R_f} \mathbb{E}[R_M^P - R_f^P]. \quad (34)$$

In the data, $\frac{\mu_H - \mu_L}{\mu_M - R_f} \approx 2.63$, i.e., the bound in (22) is far from being tight. Hence, potential violations of the slack bound in (22) are even more informative than violations of the tight bounds for calls in (5).

IA1.3 Summary Statistics for Puts

Panels A and B of Table IA.1 present summary statistics of our ATM and OTM sample of put returns. There are only a few qualitative differences relative to our sample of calls in Table 1. First, as mentioned above, they are less traded than calls, and second, average returns and deltas are of course negative.

The testing sample contains two major stock market crashes, the financial crisis in 2008, and the Covid-19 pandemic crash in February–March 2020. These episodes substantially impact put returns.

We argue that especially the Covid-19 crash represents such a strong outlier in the sample that it likely renders the sample unrepresentative. This market crash is distinctive not because of its magnitude (which is only slightly larger than other crashes, such as in 2008), but because it was not preceded by elevated volatility levels. As a result, ex-ante option prices were unusually low. Specifically, the S&P 500 dropped by -31.9% in the 30-day period following February 19, 2020. According to Schreindorfer and Sichert (2025), this drop was so extreme relative to ex-ante volatility that it corresponds to an event occurring roughly once every 500,253 years. For single stocks, the average stock return normalized by their respective ex-ante ATM implied volatility was -3.66, compared to -1.68 in September–October 2008, the second-worst month in our sample. Therefore, we present some key results for puts both for the full sample and for the sample excluding the February–March 2020 period.

Table IA.1: Summary Statistics for Put Option Returns

This table reports descriptive statistics for monthly, hold-to-maturity put option returns in excess of the risk-free rate for the period from 1996 to 2022. Option returns are measured from the first trading day after a third Friday until the next third Friday. Panel A) reports returns (per month) and option characteristics for at-the-money put options ($N \times T = 386,806$), Panel B) reports returns for out-of-the-money put options ($N \times T = 271,745$). We consider put options with moneyness in the range of $K/S \in [0.8, 1.1]$. For each stock, we maintain the contract that has moneyness closest to 1 (ATM) and 0.9 (OTM), respectively. Time to maturity is the number of days left until option expiration. Option implied volatility is provided by OptionMetrics and expressed in %, p.a. Absolute delta follows the model of Black and Scholes (1973). “Skew.” denotes skewness and “Kurt.” denotes excess kurtosis.

	Mean	SD	10%	25%	50%	75%	90%	Skew.	Kurt.
<i>A. At-the-money puts</i>									
Option Return	-9.29	189.93	-100.34	-100.11	-100.01	26.81	184.11	9.61	316.93
Time-to-Maturity	27.92	3.38	25.00	25.00	26.00	32.00	33.00	0.60	-1.50
Moneyness	0.99	0.04	0.94	0.97	1.00	1.01	1.04	0.02	0.25
Implied Volatility	46.38	26.44	21.20	28.33	39.83	57.02	79.64	2.01	7.13
Delta	-0.45	0.13	-0.61	-0.52	-0.45	-0.37	-0.29	-0.24	0.64
<i>B. Out-of-the-money puts</i>									
Option Return	-11.22	541.15	-100.42	-100.18	-100.03	-100.01	70.69	27.43	1552.76
Time-to-Maturity	27.96	3.41	25.00	25.00	26.00	32.00	33.00	0.55	-1.56
Moneyness	0.88	0.04	0.82	0.84	0.88	0.92	0.94	-0.12	-1.17
Implied Volatility	49.24	27.53	22.76	30.37	42.44	60.52	83.95	1.94	6.53
Delta	-0.16	0.08	-0.28	-0.22	-0.15	-0.09	-0.06	-0.38	-0.70

IA1.4 ESR-Sorted Put Returns

Table IA.2 is the analogue to Table 3 in the main paper. Relative to calls, the H–L spread is somewhat larger but remains close to the put market return. Thus, option returns react far more weakly to predictable variation in expected stock returns than the behavior of the underlying stocks would suggest.

IA1.5 Test of Bound

Table IA.3 is the analogue to Table 4 in the main paper. For puts, the slack upper bound is not rejected, consistent with the theoretical restriction. However, realized ESR-sorted put return spreads are far below the tighter approximation in (34), which is nearly binding in calibrated models. This indicates that, while the model-free restriction is satisfied, the economic amplification of expected stock return differences through put returns is quantitatively weak.

The results also show that for puts, the Covid crash has a larger impact. Because implied

Table IA.2: **ESR-Sorted Put Returns**

This table presents average put option portfolio returns, sorted by expected stock return based on the neural network model described in Section 3.6 for the period from 2006 to 2022. Sharpe ratios are denoted in annual terms. Panel A) reports results for at-the-money put options. Panel B) reports results for out-of-the-money put options. Absolute t -statistics are reported in parentheses. All portfolios are equally-weighted. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
<i>A. At-the-money puts</i>												
Mean	-1.51 (0.28)	-5.23 (0.95)	-9.09 (1.57)	-10.65** (1.95)	-9.35 (1.53)	-12.65** (2.19)	-11.63** (2.24)	-12.19*** (2.46)	-10.72** (1.96)	-10.92** (2.24)	9.41*** (3.46)	-9.39* (1.78)
SD	79.78	87.95	98.47	89.80	104.11	91.35	87.30	81.16	86.42	78.36	32.70	86.58
SR	-0.07	-0.21	-0.32	-0.41	-0.31	-0.48	-0.46	-0.52	-0.43	-0.48	1.00	-0.38
<i>B. Out-of-the-money puts</i>												
Mean	-2.01 (0.22)	-4.70 (0.40)	-10.14 (0.76)	-6.88 (0.48)	-12.15 (0.84)	-16.16 (1.23)	-16.93 (1.35)	-18.55* (1.84)	-14.14 (1.28)	-17.43** (2.03)	15.42* (3.26)	11.90 (1.04)
SD	136.63	191.46	212.73	245.35	236.04	208.73	208.26	166.92	179.95	139.39	64.46	188.69
SR	-0.05	-0.09	-0.17	-0.10	-0.18	-0.27	-0.28	-0.39	-0.27	-0.43	0.83	0.22

volatilities were low prior to the crash, realized put payoffs in that month were exceptionally high relative to ex-ante prices. This increases average returns, but even more so increases return volatilities, and hence standard errors. As a result, violations of the tight bound become less significant when the crash is included, particularly for OTM puts. Once this episode is excluded, violations of the tight bound are again statistically strong.

IA1.6 The Role of VOL for Puts

Table IA.4 is the analogue to Table 6 in the main paper. The results show a significant hump-shaped relationship between VOL and ATM put returns, in contrast to the monotonically decreasing pattern for calls. The H-L spread is small and insignificant, as the hump shape means high- and low-VOL portfolios have similar average returns.

This contrast with calls reflects two channels. First, volatility has opposing effects on the equity and variance risk premium components of put returns — raising the variance risk premium but reducing equity risk exposure — so the net dampening effect is weaker than for calls, where both components move in the same direction (Schlag and Sichert, 2025). Second, the comovement between VOL and ESR partially offsets the VOL-put return relationship: high-VOL stocks tend to have low ESR, and while high VOL raises expected put returns, low ESR reduces them. For calls, both effects compress expected returns, amplifying rather than dampening the relationship.

Table IA.3: **Test of Put Option Return Bounds**

This table reports the results for tests of the slack bound in for put returns in (22), and the tight upper bound (34). In the last two columns, we exclude the period February 24–March 20, 2020. The null hypothesis H_0 is that the respective option return bound holds. We multiply all returns with -1 such that they are positive. All values are in percent.

	Full sample		Excl. Covid	
	ATM	OTM	ATM	OTM
$R_M^P - R_f^P$	8.00	10.01	9.01	14.81
$R_H^P - R_L^P$	9.41	15.42	9.19	15.80
Slack upper bound (UB)	8.00	10.01	9.01	14.81
$(R_H^P - R_L^P) - UB$	1.41	5.42	0.18	0.99
t -statistic	0.85	1.17	0.14	0.45
p -value (one-sided, %)	80.17	87.80	55.52	67.20
Tight lower bound (LB)	21.09	26.38	18.84	30.96
$(R_H^P - R_L^P) - LB$	-11.68	-10.96	-9.65	-15.16
t -statistic	-2.75	-0.82	-4.75	-4.53
p -value (one-sided, %)	0.32	20.52	0.00	0.00

Table IA.4: **Put Option Returns Sorted on VOL**

This table reports average ATM put option returns (per month, in %) for VOL sorted portfolios (measured using past 21 day returns) for the period from 2006 to 2022. t -statistics (in parentheses) are adjusted for heteroscedasticity and autocorrelation (Newey and West, 1987) with four lags. Sharpe ratios (SR) are annualized. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Low VOL	2	3	4	5	6	7	8	9	High VOL	H-L	SR
-10.01	-11.67**	-11.64**	-10.82**	-10.14**	-9.99**	-6.24	-6.42	-6.73	-10.19***	0.18	0.01
(-1.20)	(-1.93)	(-1.89)	(-1.95)	(-1.77)	(-1.91)	(-1.20)	(-1.25)	(-1.41)	(-2.53)	(0.03)	

IA1.7 Double Sorts on ESR and VOL

Table IA.5 is the analogue to Table 7 in the main paper. The stock-level results in Panel A–C are virtually identical to for calls (minor differences are due to differences in the stock-put relative to the stock-call matched sample). In contrast to calls, however, Panel D shows that ESR predictability in stocks does translate into significant put return spreads already in medium-volatility portfolios. This difference relative to calls is consistent with the weaker offsetting role of volatility for expected put returns discussed above.

Nevertheless, this does not overturn the broader conclusion. Even where ESR predicts put returns, the implied amplification relative to the stock strategy remains modest once option prices and return volatilities are taken into account. Thus, while puts exhibit less attenuation than calls, options more generally remain an inefficient vehicle for exploiting predictable variation in expected stock returns.

Table IA.5: Puts – Dependent Double Sorts on ESR and Volatility

This table reports results for dependent double sorts on ESR and volatility (historical stock volatility from daily returns of the past 21 trading days) for the period from 2006 to 2022. t -statistics (in parentheses) are adjusted for heteroscedasticity and autocorrelation (Newey and West, 1987) with four lags. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively. Sharpe ratios (SR) are annualized.

	Low	2	3	4	High	Low	2	3	4	High
<i>A. VOL (% , p.a.)</i>						<i>B. VOL – IV (% , p.a.)</i>				
Low ESR	19.02***	28.05***	36.17***	47.56***	86.20***	–6.38***	–6.47***	–7.32***	–7.63***	6.20***
2	19.06***	27.83***	35.82***	46.73***	79.15***	–5.57***	–4.59***	–4.20***	–4.05***	7.53***
3	19.38***	27.89***	35.75***	46.48***	75.80***	–5.94***	–4.70***	–3.73***	–2.07***	9.00***
4	19.82***	28.08***	35.86***	46.47***	74.27***	–6.43***	–5.43***	–4.05***	–1.63**	10.31***
High ESR	20.64***	28.42***	36.18***	46.90***	75.56***	–8.14***	–8.09***	–7.15***	–5.76***	7.79***
H–L	1.62*** (10.11)	0.37*** (8.58)	0.01 (0.17)	–0.66*** (–6.19)	–10.65*** (–8.70)	–1.76*** (–5.40)	–1.63*** (–4.49)	0.17 (0.39)	1.87*** (2.77)	1.59* (1.88)
<i>C. Stock Returns (%)</i>						<i>D. Put Option Returns (% , short position)</i>				
Low ESR	0.77	0.68	0.32	–0.07	–0.76	11.14**	8.77*	5.44	–1.02	2.85
2	0.70	0.84	0.82	0.52	–0.05	8.32	11.79**	9.45*	4.77	6.58
3	0.80	0.82	0.95	0.71	0.77	12.20**	11.56**	12.07**	8.74*	11.64**
4	0.71	0.90	0.96	0.78	0.57	10.31**	13.11***	11.60**	8.70*	10.50**
High ESR	0.73	0.95	1.28*	1.03	1.21*	12.22**	10.93**	11.82**	10.50**	10.82**
H–L	–0.05 (–0.26)	0.27 (1.47)	0.96*** (3.61)	1.10*** (3.20)	1.97*** (4.85)	1.08 (0.41)	2.16 (0.74)	6.37** (2.05)	11.52*** (4.30)	7.97*** (2.71)
SR	–0.07	0.40	1.03	1.00	1.43	0.11	0.21	0.59	1.22	0.78

IA1.8 Test of the Option Return Bound while Controlling for Volatility

Table IA.6 is the analogue to Table 8 in the main paper. We find that the slack upper bound implied by Proposition 2 is not rejected, while the tighter lower benchmark derived from calibrated models is violated. This pattern is consistent with the theoretical prediction that expected put returns should lie between these two bounds. Quantitatively, however, ESR-sorted put return spreads remain modest even in the high-volatility subsample, reinforcing the conclusion that predictable variation in expected stock returns translates only weakly into option returns once volatility is taken into account.

Table IA.6: Test of Option Return Bounds for Puts – Controlling for Volatility

This table is analogous to Table IA.3, but uses only the subset of high volatility stocks (top 20%). Within this subset, we perform a sort on ESR and test the tight upper bound for put returns in (22), and the slack bound in (34). The null hypothesis H_0 is that the respective option return bound holds. For puts, we multiply all returns with -1 such that they are positive. All values are in percent.

	ATM	OTM
<i>Put returns (% , short position)</i>		
$R_M^S - R_f$	0.35	0.29
$R_H^S - R_L^S$	1.97	2.19
$R_M^P - R_f$	7.20	12.01
$R_H^P - R_L^P$	7.97	13.04
Slack upper bound (UB)	7.20	12.01
$(R_H^P - R_L^P) - UB$	0.77	1.03
t -statistic	0.30	0.24
p -value (one-sided, %)	61.76	59.45
Tight lower bound (LB)	40.72	91.45
$(R_H^P - R_L^P) - LB$	-32.75	-78.42
t -statistic	-12.68	-18.25
p -value (one-sided, %)	0.00	0.00

IA1.9 EOR-Sorted Put Returns

Table IA.7 present the analogue to Table 10, i.e., portfolios returns of puts sorted based on EORs. EORs are based on the neural network model described in Section 3.7 and returns are for the out-of-sample period for the period from 2006 to 2022. Relative to calls, the H-L spread is slightly lower (30.75% per month vs. 32.22%), while the SR is substantially lower due to higher volatility (0.90 vs. 3.03 p.a.).

Table IA.7: **EOR Sorted Put Returns**

This table reports average put option portfolio returns, sorted by expected option return (EOR) based on the neural network model described in Section 3.7 for the period from 2006 to 2022. Stock and option features are used as predictors. Sharpe ratios are denoted in annual terms. Absolute t -statistics are reported in parentheses. All portfolios are equally-weighted. Returns are denoted in % per month. *, **, and *** indicate statistical significance at the 10%, 5% and 1% levels.

	Low	2	3	4	5	6	7	8	9	High	H-L	Market
Mean	-27.78** (-2.20)	-20.07** (-2.13)	-17.98*** (-2.44)	-14.78** (-2.26)	-10.65* (-1.69)	-11.05* (-1.84)	-9.75* (-1.69)	-7.71 (-1.38)	-6.72 (-1.22)	2.97 (0.41)	30.75*** (3.66)	-9.39* (-1.78)
SD	206.15	159.91	123.21	111.27	103.50	98.21	95.84	92.35	88.29	109.66	119.11	86.58
SR	-0.47	-0.44	-0.51	-0.46	-0.36	-0.39	-0.35	-0.29	-0.26	0.09	0.90	-0.38

IA2 Feature Description

IA2.1 Stock Features

We build stock-level features based on the cross-section of stock returns literature. Beside the 94 features in Gu et al. (2020), we also include beta, beta squared, and idiosyncratic volatility computed at the daily frequency, as well as higher-moment measures including skewness and kurtosis. Table IA.8 provides details of the features.

Table IA.8: Description of Stock Features

This table reports the list of stock features used in the paper, as well as the respective sources.

No.	feature	paper's author(s)	year, journal
1	size	Banz	1981, JFE
2	beta (weekly)	Fama & MacBeth	1973, JPE
3	beta (daily)	Fama & MacBeth	1973, JPE
4	idiosyncratic volatility (daily)	Ali, Hwang & Trombley	2003, JFE
5	beta squared (daily)	Fama & MacBeth	1973, JPE
6	beta squared (weekly)	Fama & MacBeth	1973, JPE
7	change in 6-month momentum	Gettleman & Marks	2006, WP
8	dollar trading volume	Chordia, Subrahmanyam & Anshuman	2001, JFE
9	idiosyncratic volatility (weekly)	Ali, Hwang & Trombley	2003, JFE
10	industry momentum	Moskowitz & Grinblatt	1999, JF
11	1-month momentum	Jegadeesh & Titman	1993, JF
12	6-month momentum	Jegadeesh & Titman	1993, JF
13	12-month momentum	Jegadeesh	1990, JF
14	36-month momentum	Jegadeesh & Titman	1993, JF
15	price delay	Hou & Moskowitz	2005, RFS
16	share turnover	Datar, Naik & Radcliffe	1998, JFM
17	absolute accruals	Bandyopadhyay, Huang & Wirjanto	2010, WP
18	working capital accruals	Sloan	1996, TAR
19	# years since first Compustat coverage	Jiang, Lee & Zhang	2005, RAS
20	asset growth	Cooper, Gulen & Schill	2008, JF
21	cash flow to debt	Ou & Penman	1989, JAE
22	cash productivity	Chandrashekar & Rao	2009, WP
23	cash flow to price	Desai, Rajgopal & Venkatachalam	2004, TAR
24	cash flow to price (industry-adjusted)	Asness, Porter & Stevens	2000, WP
25	change in asset turnover (industry-adjusted)	Soliman	2008, TAR
26	change in shares outstanding	Pontiff & Woodgate	2008, JF
27	change in employees (industry-adjusted)	Asness, Porter & Stevens	1994, WP
28	change in inventory	Thomas & Zhang	2002, RAS
29	change in profit margin (industry-adjusted)	Soliman	2008, TAR
30	convertible debt indicator	Soliman	2008, TAR

Table IA.8 — continued — Description of Stock Features

No.	feature	paper's author(s)	year, journal
31	current ratio	Ou & Penman	1989, JAE
32	depreciation PP&E	Holthausen & Larcker	1992, JAE
33	dividend initiation	Michaely, Thaler & Womack	1995, JF
34	dividend omission	Michaely, Thaler & Womack	1995, JF
35	dividend to price	Litzenberger & Ramaswamy	1982, JF
36	growth in common shareholder equity	Richardson, Sloan, Soliman & Tuna	2005, JAE
37	earnings to price	Basu	1977, JF
38	gross profitability	Novy-Marx	2013, JFE
39	growth in capital expenditures	Anderson & Garcia-Feijoo	2006, JF
40	growth in long-term net operating assets	Fairfield, Whisenant & Yohn	2003, TAR
41	industry sales concentration	Hou & Robinson	2006, JF
42	employee growth rate	Bazdresch, Belo & Lin	2014, JPE
43	capital expenditures and inventory	Chen & Zhang	2010, JF
44	leverage	Bhandari	1988, JF
45	growth in long-term debt	Richardson, Sloan, Soliman & Tuna	2005, JAE
46	size (industry-adjusted)	Asness, Porter & Stevens	2000, WP
47	operating profitability	Fama & French	2005, JFE
48	organizational capital	Eisfeldt & Papanikolaou	2013, JF
49	% change in capital expenditures (industry-adjusted)	Abarbanell & Bushee	1998, TAR
50	% change in current ratio	Ou & Penman	1989, JAE
51	% in depreciation	Holthausen & Larcker	1992, JAE
52	% change in gross margin - % change in sales	Abarbanell & Bushee	1998, TAR
53	% change in quick ratio	Ou & Penman	1989, JAE
54	% change in sales - % change in inventory	Abarbanell & Bushee	1998, TAR
55	% change in sales - % change in A/R	Abarbanell & Bushee	1998, TAR
56	% change in sales - % change in SG&A	Abarbanell & Bushee	1998, TAR
57	% change in sales-to-inventory	Ou & Penman	1989, JAE
58	percent accruals	Hafzalla, Lundholm & Van Winkle	2011, TAR
59	financial statements score	Piotroski	2000, JAR
60	quick ratio	Ou & Penman	1989, JAE
61	R&D increase	Eberhart, Maxwell & Siddique	2004, JF
62	R&D to market capitalization	Guo, Lev & Shi	2006, JBFA
63	R&D to sales	Guo, Lev & Shi	2006, JBFA
64	real estate holdings	Tuzel	2010, RFS
65	return on invested capital	Brown & Rowe	2007, WP
66	sales to cash	Ou & Penman	1989, JAE
67	sales to inventory	Ou & Penman	1989, JAE
68	sales to receivables	Ou & Penman	1989, JAE
69	secured debt	Valta	2016, JFQA
70	secured debt indicator	Valta	2016, JFQA

Table IA.8 — continued — Description of Stock Features

No.	feature	paper's author(s)	year, journal
71	sales growth	Lakonishok, Shleifer & Vishny	1994, JF
72	sin stocks	Hong & Kacperczyk	2009, JFE
73	sales to price	Barbee, Mukherji, & Raines	1996, FAJ
74	debt capacity/firm tangibility	Almeida & Campello	2007, RFS
75	tax income to book income	Lev & Nissim	2004, TAR
76	abnormal earnings announcement volume	Lerman, Livnat & Mendenhall	2007, WP
77	cash holdings	Palazzo	2012, JFE
78	change in tax expense	Thomas & Zhang	2011, JAR
79	corporate investment	Titman, Wei & Xie	2004, JFQA
80	earnings announcement return	Kishore, Brandt, Santa-Clara & Venkatachalam	2008, WP
81	number of earnings increase	Barth, Elliott & Finn	1999, JAR
82	return on assets	Balakrishnan, Bartov & Faurel	2010, JAE
83	earnings volatility	Francis, LaFond, Olsson & Schipper	2004, TAR
84	return on equity	Hou, Xue & Zhang	2015, RFS
85	revenue surprise	Kama	2009, JBFA
86	accrual volatility	Bandyopadhyay, Huang & Wirjanto	2010, WP
87	cash flow volatility	Huang	2009, JFE
88	financial statement score	Mohanram	2005, RAS
89	bid-ask spread	Amihud & Mendelson	1989, JF
90	illiquidity	Amihud	2002, JFM
91	maximum daily return	Bali, Cakici & Whitelaw	2011, JFE
92	return volatility	Ang, Hodrick, Xing & Zhang	2006, JF
93	volatility of liquidity (dollar trading volume)	Chordia, Subrahmanyam & Anshuman	2001, JFE
94	volatility of liquidity (share turnover)	Chordia, Subrahmanyam & Anshuman	2001, JFE
95	zero trading days	Liu	2006, JFE
96	book-to-market	Rosenberg, Reid & Lanstein	1985, JPM
97	book-to-market (industry-adjusted)	Asness, Porter & Stevens	2000, WP
98	skewness	Condard, Robert & Ghysels	2012, JF
99	kurtosis	Condard, Robert & Ghysels	2012, JF

IA2.2 Option Features

Following Bali et al. (2023), we construct 80 option-based features based on the cross-section of option and stock returns literature. These features capture information in the option market and can be defined at the stock, bucket (i.e., groups of options with the same option type and moneyness), and contract levels.

Table IA.9: Description of Option Features

This table reports the list of option features used in the paper, as well as the respective sources.

No.	feature	paper's author(s)	year, journal
1	implied volatility slope	Vasquez	2017, JFQA
2	risk-neutral skewness ($\tau = 30$)	Borochin, Chang & Wu	2020, JEF
3	risk-neutral skewness ($\tau = 91$)	Borochin, Chang & Wu	2020, JEF
4	risk-neutral skewness ($\tau = 182$)	Borochin, Chang & Wu	2020, JEF
5	risk-neutral skewness ($\tau = 273$)	Borochin, Chang & Wu	2020, JEF
6	risk-neutral skewness ($\tau = 365$)	Borochin, Chang & Wu	2020, JEF
7	risk-neutral kurtosis ($\tau = 30$)	Borochin, Chang & Wu	2020, JEF
8	risk-neutral kurtosis ($\tau = 91$)	Borochin, Chang & Wu	2020, JEF
9	risk-neutral kurtosis ($\tau = 182$)	Borochin, Chang & Wu	2020, JEF
10	risk-neutral kurtosis ($\tau = 273$)	Borochin, Chang & Wu	2020, JEF
11	risk-neutral kurtosis ($\tau = 365$)	Borochin, Chang & Wu	2020, JEF
12	option-implied variance asymmetry	Huang & Li	2019, JBF
13	option implied tail loss	Vilkov & Xiao	2012, WP
14	stock vs. option volume	Roll, Schwartz & Subrahmanyam	2010, JFE
15	log of stock vs. option volume	Roll, Schwartz & Subrahmanyam	2010, JFE
16	dollar stock vs. option volume	Roll, Schwartz & Subrahmanyam	2010, JFE
17	log of dollar stock vs. option volume	Roll, Schwartz & Subrahmanyam	2010, JFE
18	modified stock vs. option volume	Johnson & So	2012, JFE
19	put-call ratio	Blau, Nguyen & Whitby	2014, JBF
20	contribution of market frictions to expected returns	Hiraki & Skiadopoulos	2020, WP
21	option demand pressure	Cao, Vasquez, Xiao & Zhan	2019, WP
22	proportional bid-ask spread	Cao & Wei	2010, JFM
23	dollar trading volume	Cao & Wei	2010, JFM
24	absolute illiquidity	Cao & Wei	2010, JFM
25	percentage illiquidity	Cao & Wei	2010, JFM
26	trading volume	Cao & Wei	2010, JFM
27	number of traded options		
28	total open interest		
29	volatility uncertainty	Cao & Wei	2010, JFM
30	atm implied volatility volatility	Baltussen, van Bakkum & van der Grient	2018, JFQA
31	variance spread (realized - implied)	Bali & Hovakimian	2009, MS
32	variance spread (implied / realized)		
33	near-the-money call minus put iv	Bali & Hovakimian	2009, MS
34	atm call minus put iv from surface		
35	change in atm call iv	An, Ang, Bali & Cakici	2014, JF

Table IA.9 — continued — Description of Option Features

No.	feature	paper's author(s)	year, journal
36	change in atm put iv	An, Ang, Bali & Cakici	2014, JF
37	change in atm put minus call iv	An, Ang, Bali & Cakici	2014, JF
38	iv skew (otm put - atm call)	Xing, Zhang & Zhao	2010, JFQA
39	weighted put-call spread	Cremers & Weinbaum	2010, JFQA
40	change in weighted put-call spread	Cremers & Weinbaum	2010, JFQA
41	put-call parity violations	Ofek, Richardson & Whitelaw	2004, JFE
42	implied shorting fees in options	Muravyev, Pearson & Pollet	2021, JF
43	implied volatility duration	Schlag, Thimme & Weber	2020, JFE
44	at-the-money implied volatility		
45	illiquidity	Bao, Pan & Wang	2011, JF
46	Rolls daily measure of illiquidity	Roll	1984, JF
47	illiquidity based on zero returns	Lesmond, Ogden & Trzcinka	1999, RFS
48	modified zero return illiquidity (fht)	Fong, Holden & Trzcinka	2017, RF
49	amihud measure of illiquidity	Amihud	2002, JFM
50	extended rolls measure	Goyenko, Holden & Trzcinka	2009, JFE
51	extended fht measure	Fong, Holden & Trzcinka	2017, RF
52	std. deviation of amihud measure	Amihud	2002, JFM
53	Pástor and Stambaugh's liquidity measure	Pástor & Stambaugh	2003, JPE
54	historical volatility		
55	historical skewness		
56	historical kurtosis		
57	disposition effect	Bergsma, Fodor & Tedford	2020, JBF
58	open interest vs. stock volume		
59	volume within option bucket		
60	dollar volume within option bucket		
61	relative volume share within option bucket		
62	option turnover (volume / open interest)		
63	implied volatility rank vs. last year	Heston & Li	2020, WP
64	call indicator		
65	put indicator		
66	expiration flag (within month)		
67	time-to-maturity		
68	moneyness		
69	standardized moneyness		
70	implied volatility	Black & Scholes	1973, JPE
71	delta	Black & Scholes	1973, JPE
72	gamma	Black & Scholes	1973, JPE
73	theta	Black & Scholes	1973, JPE
74	vega	Black & Scholes	1973, JPE
75	volga	Black & Scholes	1973, JPE
76	embedded leverage	Karakaya	2014, WP
77	open interest		
78	dollar open interest		
79	mid price		
80	bid-ask spread		

IA3 Additional Empirical Results

IA3.1 Summary Statistics for Subsamples

Table IA.10: Sample Split Training and Testing Sample

This table reports descriptive statistics for monthly, hold-until-maturity option returns analogously to Table 1. We split the sample into the initial training and validation period from 1996 to 2005 in Panel I, and the out-of-sample testing period from 2006 to 2022 in Panel II.

	Mean	SD	10%	25%	50%	75%	90%	Skew.	Kurt.
<i>I. Initial training and validation period 1996 to 2005</i>									
<i>A. At-the-money calls</i>									
Option Return	7.13	213.02	-100.48	-100.42	-100.11	47.41	240.28	4.64	47.05
Time-to-Maturity	28.41	3.35	26.00	26.00	26.00	33.00	33.00	0.59	-1.61
Moneyness	1.03	0.05	0.96	1.00	1.03	1.06	1.08	-0.51	-0.36
Implied Volatility	51.19	25.74	24.35	32.46	45.29	64.08	86.17	1.32	2.46
Delta	0.44	0.16	0.24	0.34	0.44	0.55	0.65	0.16	-0.13
<i>B. Out-of-the-money calls</i>									
Option Return	-0.41	288.66	-100.50	-100.46	-100.20	-100.09	252.22	5.69	54.61
Time-to-Maturity	28.48	3.38	26.00	26.00	26.00	33.00	33.00	0.54	-1.67
Moneyness	1.10	0.04	1.06	1.07	1.10	1.13	1.16	0.63	-0.40
Implied Volatility	58.45	26.64	29.92	38.59	52.77	72.73	94.48	1.18	1.90
Delta	0.28	0.10	0.14	0.20	0.28	0.36	0.42	0.01	-0.77
<i>C. At-the-money puts</i>									
Option Return	-12.60	150.92	-100.48	-100.30	-96.24	33.11	160.01	5.00	72.33
Time-to-Maturity	28.36	3.34	26.00	26.00	26.00	33.00	33.00	0.62	-1.58
Moneyness	1.01	0.05	0.94	0.98	1.01	1.05	1.08	-0.18	-0.81
Implied Volatility	51.69	26.38	24.59	32.54	45.34	64.56	87.60	1.37	2.62
Delta	-0.51	0.17	-0.74	-0.62	-0.50	-0.39	-0.30	-0.27	-0.37
<i>D. Out-of-the-money puts</i>									
Option Return	-19.70	270.13	-100.49	-100.44	-100.17	-100.09	173.19	8.98	159.16
Time-to-Maturity	28.48	3.38	26.00	26.00	26.00	33.00	33.00	0.55	-1.67
Moneyness	0.90	0.03	0.85	0.88	0.91	0.93	0.94	-0.80	-0.04
Implied Volatility	56.57	27.68	27.54	36.04	50.10	70.81	94.48	1.26	2.11
Delta	-0.22	0.08	-0.32	-0.28	-0.22	-0.16	-0.11	0.12	-0.85
<i>E. Optionable stocks</i>									
Stock Return	0.33	17.09	-18.43	-8.14	0.25	8.33	18.29	0.77	8.65

Table IA.10 — continued — Sample Split Training and Testing Sample

	Mean	SD	10%	25%	50%	75%	90%	Skew.	Kurt.
<i>II. Out-of-sample testing period 2006 to 2022</i>									
<i>A. At-the-money calls</i>									
Option Return	7.98	289.30	-100.22	-100.09	-100.02	10.75	241.23	19.00	1727.14
Time-to-Maturity	27.82	3.39	25.00	25.00	26.00	32.00	33.00	0.60	-1.50
Moneyness	1.04	0.04	0.98	1.01	1.05	1.08	1.09	-0.82	0.07
Implied Volatility	44.21	24.96	20.29	27.22	38.30	54.18	74.96	2.08	8.02
Delta	0.37	0.17	0.15	0.24	0.36	0.49	0.61	0.44	-0.26
<i>B. Out-of-the-money calls</i>									
Option Return	3.78	369.09	-100.26	-100.14	-100.02	-100.01	240.54	16.87	1073.44
Time-to-Maturity	27.86	3.41	25.00	25.00	26.00	32.00	33.00	0.55	-1.57
Moneyness	1.10	0.03	1.06	1.08	1.09	1.11	1.14	0.86	0.61
Implied Volatility	49.40	26.00	24.29	31.64	43.28	60.03	81.52	2.03	7.56
Delta	0.23	0.10	0.10	0.15	0.23	0.31	0.38	0.33	-0.50
<i>C. At-the-money puts</i>									
Option Return	-8.37	175.17	-100.19	-100.02	-84.56	35.24	156.99	4.77	72.18
Time-to-Maturity	27.80	3.38	25.00	25.00	26.00	32.00	33.00	0.61	-1.49
Moneyness	1.02	0.05	0.95	0.98	1.02	1.05	1.08	-0.22	-0.74
Implied Volatility	44.90	26.27	20.52	27.34	38.39	54.70	76.78	2.22	8.80
Delta	-0.54	0.18	-0.79	-0.67	-0.53	-0.40	-0.30	-0.10	-0.58
<i>D. Out-of-the-money puts</i>									
Option Return	-10.27	372.83	-100.23	-100.11	-100.02	-100.01	168.48	11.15	428.87
Time-to-Maturity	27.84	3.40	25.00	25.00	26.00	32.00	33.00	0.57	-1.55
Moneyness	0.91	0.03	0.87	0.90	0.92	0.94	0.95	-1.25	0.99
Implied Volatility	47.50	27.21	21.96	29.28	40.82	57.90	80.56	2.16	8.13
Delta	-0.22	0.07	-0.32	-0.28	-0.22	-0.16	-0.12	0.09	-0.67
<i>E. Optionable stocks</i>									
Stock Return	0.59	16.04	-15.56	-6.55	0.60	7.31	15.72	2.70	83.50

IA3.2 ESR-VOL Relationship for All CRSP Stocks

Figure IA.2 present the analogue to Figure 4 of the main paper for the full sample of all CRSP stocks. The results are from a replication of the NN3 in Gu et al. (2020), and the out-of-sample predictions are for 1996-2022.

Figure IA.2: **Dependence between ESR and VOL for All CRSP Stocks**

This figure shows the average number of stocks per month in each portfolio for a 10x10 independent double sort on ESR and stock volatility (VOL; historical volatility over the past 21 trading days) for the period from 1996-2022. The average number of total stocks/month is 6,689. The values are for out-of-sample predictions for all stocks in the CRSP sample.

Average number of firms per portfolio

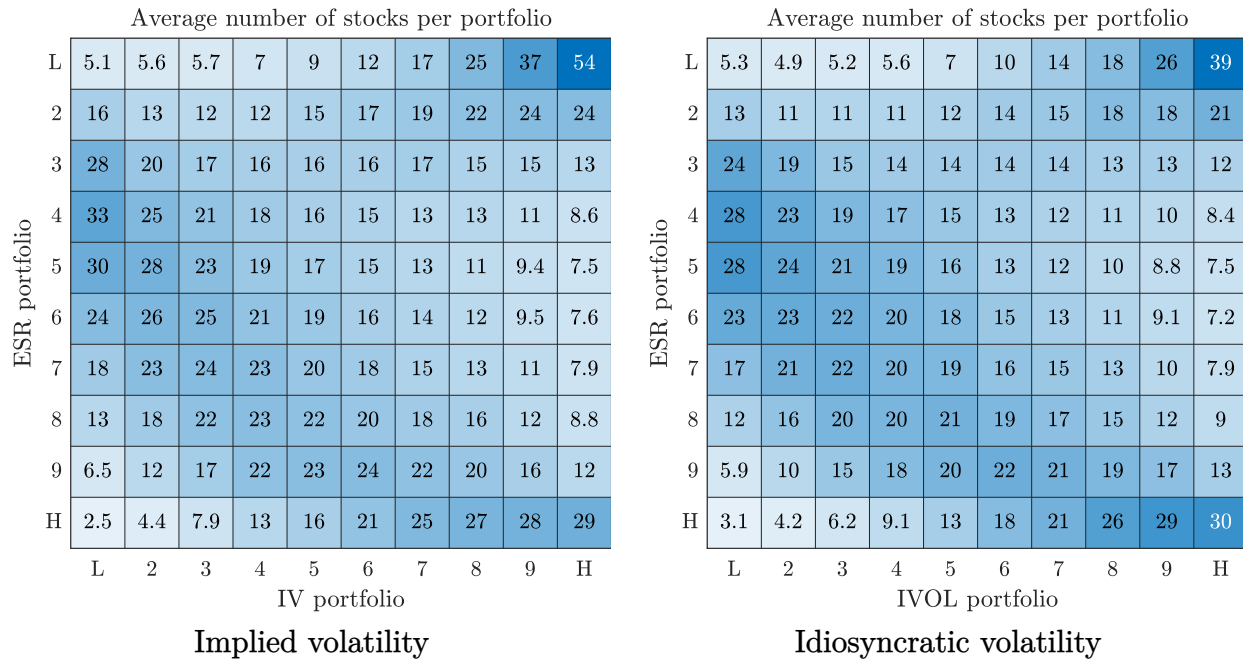
L	16	18	23	31	43	61	85	113	139	144
2	45	49	51	57	64	75	83	88	85	72
3	86	71	67	66	67	69	68	65	57	46
4	107	84	74	68	66	63	58	52	44	34
5	112	91	80	73	67	61	54	46	38	29
6	109	98	87	80	71	62	53	45	36	27
7	83	94	94	87	78	68	58	47	37	28
8	66	83	90	91	86	75	63	52	40	31
9	38	61	73	80	82	80	77	70	63	54
H	16	24	29	34	43	53	68	90	128	202
	L	2	3	4	5	6	7	8	9	H

VOL portfolio

IA3.3 Double Sorts on Alternative Volatility Measures

Figure IA.3: **Robustness: Number of Stocks for Independent Double Sort on ESR and Alternative Volatility Measures.**

This figure shows the average number of stocks per month in each portfolio for an 10x10 independent double sort on ESR and implied volatility/idiosyncratic volatility for the period from 2006-2022. The average number of total stocks/month is 1,747.



IA3.4 Option Moneyness Across ESR Deciles

Table IA.11: **Summary Statistics of Moneyness by ESR**

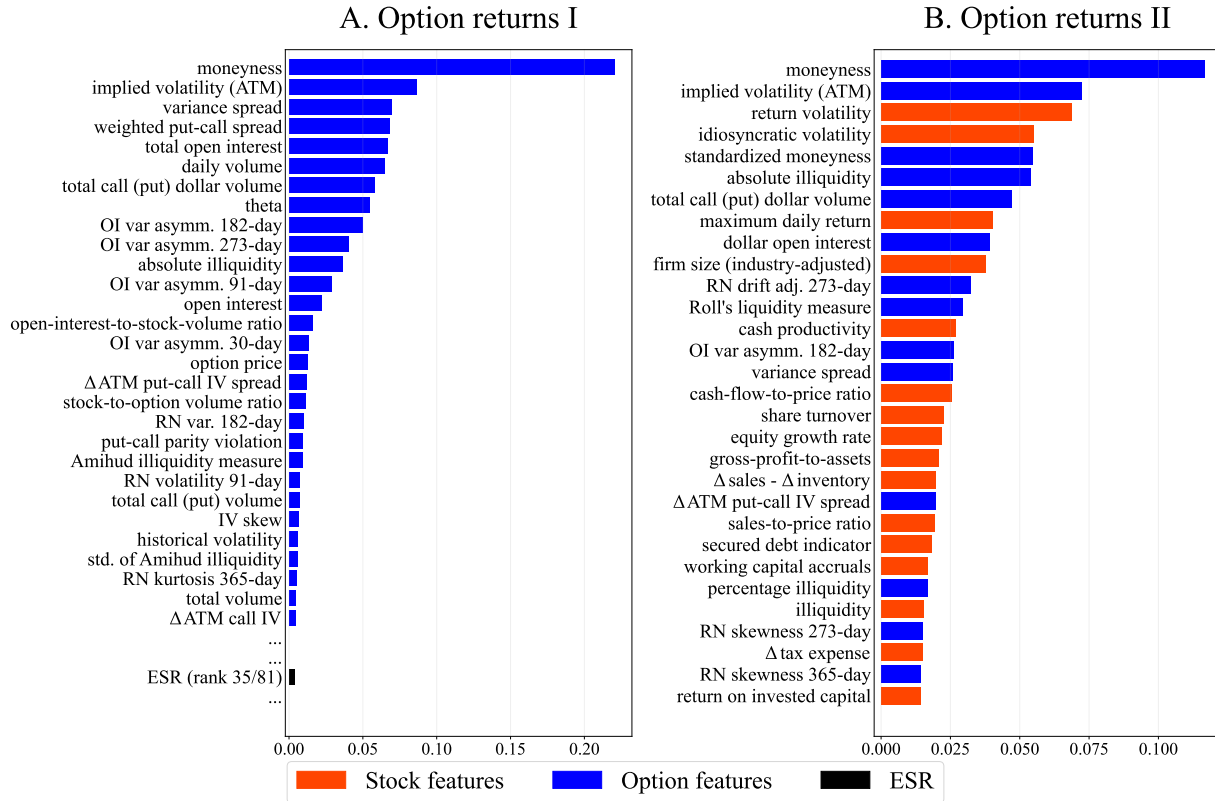
This table reports the average moneyness across ESR deciles for the period from 2006 to 2022.

ESR Deciles	ATM		OTM	
	Call	Put	Call	Put
1	1.0052	0.9929	1.1659	0.8481
2	1.0064	0.9928	1.1576	0.8549
3	1.0066	0.9929	1.1541	0.8581
4	1.0063	0.9925	1.1493	0.8609
5	1.0061	0.9925	1.1455	0.8631
6	1.0061	0.9929	1.1437	0.8641
7	1.0065	0.9932	1.1419	0.8647
8	1.0066	0.9932	1.1425	0.8641
9	1.0067	0.9927	1.1462	0.8625
10	1.0076	0.9934	1.1667	0.8537

IA3.5 Feature Importance with Alternative List of Variables

Figure IA.4: Feature Importance for Option Return Prediction—Alternative Specifications

This figure shows the top 30 features and ESR ranked by feature importance for the out-of-sample period from 2006 to 2022, measured as the average decrease in out-of-sample R-squared. Following Gu et al. (2020), we determine the influence of feature j based on the reduction in out-of-sample R-Squared from setting all values of predictor j to their median, while holding the remaining model estimates fixed. Panel A displays the feature importance when predicting call option returns using option features and our expected stock return measure. Panel B displays the resulting feature importance when predicting call option returns using both option and stock features. Variable importance within each model is normalized to sum to 1.



IA3.6 Double Sorts of Bias Adjusted Option Returns

Table IA.12: **Bias Adjusted Option Returns–Dependent Double Sorts on ESR and Volatility**

This table is analogous to Table 7, but report bias-adjusted option returns (following Duarte et al. (2024), and the corresponding stock returns. Stock returns are weighted with the same weights as option returns. t -statistics (in parentheses) are adjusted for heteroscedasticity and autocorrelation (Newey and West, 1987) with four lags. Sharpe ratios (SR) are annualized. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	Low	2	3	4	High	Low	2	3	4	High
<i>A. Stock Returns (%)</i>						<i>B. Call Option Returns (%)</i>				
Low ESR	0.76	0.58	0.34	0.05	−0.87	14.78***	8.63**	7.54*	5.80	−6.57
2	0.70	0.83	0.95	0.43	0.18	18.99***	15.95***	13.09***	4.53	−0.01
3	0.85	0.77	0.86	0.75	0.57	21.05***	11.86***	9.88**	3.13	0.31
4	0.75	0.86	0.96	0.68	0.63	18.60***	10.98***	13.58***	4.33	−1.14
High ESR	0.79	1.07	1.17*	1.03	1.20*	18.03***	11.75***	13.40***	4.91	4.51
H–L	0.03	0.49**	0.83***	0.98***	2.07***	3.25	3.12	5.86	−0.89	11.08***
	(0.19)	(2.40)	(2.91)	(2.80)	(4.82)	(0.62)	(0.77)	(1.64)	(−0.23)	(3.26)
SR	0.05	0.66	0.81	0.84	1.38	0.15	0.20	0.40	−0.06	0.83

IA4 Unlevering Returns

We apply the [Merton \(1974\)](#) model to unlever stock returns. Asset returns are changes in the market value of firms. Under the assumption that the total value of a firm follows GBM, the Merton model assumes that the equity of the firm is a call option on the underlying value of the firm, with a strike price equal to the face value of the firm’s debt. The equity can be priced by

$$E = V\mathcal{N}(d_1) - e^{-r_f T} B\mathcal{N}(d_2), \quad (35)$$

where

$$d_1 = \frac{\ln(V/B) + (r_f + 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}}, \quad (36)$$

$$d_2 = d_1 - \sigma_v\sqrt{T} \quad (37)$$

and E is the market value of equity; V is the market value of the firm; B is the face value of debt; T is debt’s time-to-maturity; and r_f is the instantaneous risk-free rate. The model relates asset volatility σ_v and equity volatility σ_e by

$$\sigma_e = \left(\frac{V}{E}\right) \mathcal{N}(d_1)\sigma_v. \quad (38)$$

For each firm in each month, we use Equations (35) and (38) to estimate σ_v , V , and its asset returns (growth rates of V). Following [Bharath and Shumway \(2008\)](#), [Gilchrist and Zakrajšek \(2012\)](#), and [Chang, d’Avernas, and Eisfeldt \(2024\)](#), we approximate the face value of the debt maturing in one year as the sum of the firm’s current liabilities and one-half of its long-term liabilities. We implement an iterative procedure to estimate the volatility and market value of firm assets: First, we guess an initial value of $\tilde{\sigma}_v = \sigma_e[E/(E+B)]$ and insert it into Equation (35) to infer the market value of each firm \hat{V} every day for the previous month. Second, we calculate the implied return on assets each day and use the return series to generate a new estimate $\tilde{\sigma}_v$. Third, we iterate the steps until $\tilde{\sigma}_v$ converges so that the absolute difference in adjacent $\tilde{\sigma}_v$ ’s is less than 10^{-3} and denote the last $\tilde{\sigma}_v$ as $\hat{\sigma}_v$, our proxy for σ_v . Finally, we insert $\hat{\sigma}_v$ into Equation (35) to estimate the firms market value for each month \hat{V} . Repeating this procedure produces a time series of \hat{V} , from which we compute asset returns.

On average, our asset-level dataset contains 1,550 firms per month. Compared with the stock- and option-level datasets, we lose some observations on average during the unlevering process because either some variables cannot be reliably estimated or the iterative procedure fails to converge.

We then implement training and estimation procedures similar to those described in Section 3.6, but using asset returns as signals to estimate the relationships between asset

returns and the same set of features. This process ultimately yields a panel of out-of-sample estimates of expected asset returns (EAR) spanning the period from 2006 to 2022.