Disentangling Credit Spreads and Equity Volatility*

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Abstract

In this paper, I provide a structural approach to quantify the forces that govern the joint dynamics of corporate bond credit spreads and equity volatility. I build a dynamic model and estimate a wide array of fundamental shocks using a large firm-level database on credit spreads, equity prices, accounting statements, and bond recovery ratios in the U.S. from 1973 to 2014. A structural decomposition reveals that the joint dynamics of credit spreads and equity volatility is driven by fluctuations in firms' asset values and aggregate asset volatility. I find that aggregate asset volatility captures the informational content of credit spreads for predicting economic activity. All together, my results suggest that aggregate asset volatility is key for the transmission channel that links the fundamental drivers of financial indicators to the real economy.

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1 Introduction

Indicators of financial distress and uncertainty, such as corporate bond credit spreads and equity volatility, are powerful predictors of real economic activity.[1] Understanding what are the fundamental drivers of these financial indicators is critical to comprehend the linkages between financial markets and the real economy.

Yet, it remains difficult to assess empirically which shocks are important in accounting for the dynamics of these financial indicators. The high degree of comovement complicates the identification of fundamental shocks (see Stock and Watson, 2012, and Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek, 2016). To address this challenge, I propose a dynamic structural model with shocks to firms' asset values, bankruptcy costs, firms' aggregate and idiosyncratic asset volatility, and the market price of risk. I structurally estimate the shocks with a large firm-level database on credit spreads, equity prices, accounting statements, and bond recovery ratios in the U.S. from 1973 to 2014. The model accurately accounts for the historical levels and dynamics, both over time and in the cross-section, of five financial indicators: (i) default risk, (ii) corporate bond credit spreads, (iii) aggregate and (iv) idiosyncratic equity volatility, and (v) corporate bond bid-ask spreads. A structural decomposition yields that shocks to firms' asset values and aggregate asset volatility are key to account for the joint dynamics of these financial indicators. Moreover, fluctuations in firms' aggregate asset volatility strongly predict future economic activity.

My structural model primarily builds on Chen, Cui, He, and Milbradt (2016). There are two types of shocks in this economy: small and frequent shocks to firms' asset values and large but infrequent shocks to macroeconomic conditions. Fluctuations in macroeconomic conditions include shocks to bankruptcy costs, firms' aggregate and idiosyncratic asset volatility, and the market price of risk. Firms' assets generate cash flows and are financed through equity and debt. Firms' asset, equity, and debt are priced by a common stochastic discount factor. The optimal capital structure of firms is based on the trade-off between tax benefits of debt and

[1]See Philippon (2009); Bloom (2009); Stock and Watson (2012); Gilchrist and Zakrajšek (2012), Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016); Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016); and many others.

deadweight losses of default. Firms decide when to default based on their cash flow level and macroeconomic conditions. The secondary market for corporate bonds is subject to over-the-counter liquidity frictions.

In the model, over-the-counter market illiquidity—and consequently *bid-ask spreads*—arise endogenously as in Chen, Cui, He, and Milbradt (2016). Investors face uninsurable idiosyncratic liquidity shocks, which drive up their costs of holding corporate bonds. To sell their bonds, investors must search for dealers to intermediate transactions with other investors. In the meantime, they incur the cost of having to hold on to the bond. This cost is affected by the bond price. Therefore, the liquidity discount of corporate bonds and bid-ask spreads set by dealers fluctuates with bond prices. Hence, shocks that impact bond prices also affect bid-ask spreads. Interestingly, model-implied fluctuations in bid-ask spreads arising from changes in bond prices successfully match the empirical measurements of Bao, Pan, and Wang (2011) without the need of additional shocks to liquidity frictions parameters as in Chen, Cui, He, and Milbradt (2016).

The model provides a structural mapping between the exogenous shocks and the endogenous financial indicators. While the financial indicators functionally depend on all shocks, some relationships are stronger than others. For example, bankruptcy costs are borne by creditors and not by equity holders. Thus, *equity volatility* is not impacted much by shocks to bankruptcy costs, but is very sensitive to shocks to firms' asset values and firms' asset volatility. The *default risk* indicator embodies the probability that firms' asset values hit the boundary at which equity holders decide to default. Thus, default risk is sensitive to fluctuations in firms' asset values, firms' aggregate and idiosyncratic asset volatility, and the market price of risk. *Credit spreads* compensate for the cost of bearing exposure to corporate credit risk, which fluctuates with changes in default risk or bankruptcy costs. Shocks to the market price of risk change the compensation required by investors for bearing aggregate risk beyond expected losses.

Given the structure of the model, I identify the fundamental shocks with a large firm-level panel dataset of U.S. public firms' monthly observations of equity prices and volatilities, accounting statements, and bond recovery ratios from 1973 to 2014.

I fit the levels of firms' asset values each month to match observations on firms' leverage, measured as the book value of debt relative to the market value of equity. Thus, I uncover realized shocks to firms' asset values from observations on the market value of firms' equity relative to the level of their debt. As the model implies a tight link between firms' asset volatility, firms' asset values, and equity volatility, I can retrieve monthly model-implied values for shocks to firms' aggregate and idiosyncratic asset volatility for each firm in my dataset. I measure time-varying bankruptcy costs with bond recovery ratios from Moody's corporate default study. The stochastic discount factor is calibrated to match the average equity premium. Finally, parameters driving over-the-counter liquidity frictions are calibrated to target Edwards, Harris, and Piwowar's (2007) cross-sectional measurement of average bid-ask spreads from 2003 to 2005. With these measurements, the model generates levels and fluctuations for each financial indicator that match accurately their empirical counterparts, not only over the period from 1973 to 2014—including the unprecedented spike during the 2007–08 financial crisis—but also in the cross-section. In particular, the match between model-implied and historical credit spreads substantiate the model's assumptions, as data on credit spreads is not used during the estimation of shocks and calibration of parameters.

Two results arise from a structural decomposition of economic channels and shocks to macroeconomic conditions. First, holding constant observed default probabilities, the pricing of the risk of shocks to macroeconomic conditions, i.e., the risk aversion of the representative agent, accounts for 45% (32%) of investment-grade (speculative-grade) credit spreads' levels from 1973 to 2014.[2] Thus, the compensation demanded by investors for bearing exposure to corporate credit risk—beyond expected losses—is crucial to account for credit spreads' levels. The components of pure default risk and liquidity frictions account for 27% and 30% of investment-grade (52% and 16% of speculative-grade) credit spreads' levels, respectively. Second, during the financial crisis, a large negative shock to firms' asset values and a large in-

[2]A firm that is speculative-grade has a rating lower than Baa from Moody's Investors Service, a rating lower than BBB from Standard & Poor's or both. Firms with ratings of Baa, BBB or higher are termed investment-grade.

crease in aggregate asset volatility were both key determinants of changes in default risk, credit spreads, aggregate equity volatility, and bid-ask spreads. From January 2007 to January 2009, fluctuations in firms' aggregate asset volatility were responsible for about 45% (41%) of the total spike of investment-grade (speculative-grade) credit spreads explained by the model.

These findings shed light on economic mechanisms at play during the 2008-09 financial crisis. During that period, the large spike in aggregate firms' asset volatility increased the probability of default in two ways. First, holding asset values constant, it increased the probability that a large negative shock pushes a firm into bankruptcy. Second, firms' asset values fell because aggregate volatility is priced adversely by the representative investor. A quantitatively smaller surge in firms' idiosyncratic asset volatility, which is not priced by the representative investor, had a mild impact on default risk. Overall, the increase in firms' aggregate asset volatility, jointly with a large negative shock to firms' asset values, raised aggregate credit risk. In turn, this inflated the compensation demanded by investors for bearing more aggregate risk and led to unprecedentedly high *credit spreads.* In addition, the increase in firms' aggregate asset volatility and a large negative shock to firms' asset values raised aggregate *equity volatility.* Thus, fluctuations in firms' aggregate asset volatility and firms' asset values are powerful drivers of financial indicators—because it greatly influences the quantity of credit risk that is priced by the representative investor.

When predicting real economic activity, fluctuations in firms' aggregate asset volatility estimated from the model are as powerful and contain the same information as credit spreads themselves. That is, using credit spreads to predict real GDP growth four quarters ahead, as in Gilchrist and Zakrajšek (2012), or using the estimated time series of firms' aggregate asset volatility yields similar standardized coefficients and adjusted R-squared. All together, my results suggest that fluctuations in firms' aggregate asset volatility are key for the transmission channel that links the fundamental drivers of financial indicators to the real economy. These results are consistent, for example, with the notion that an increase in firms' aggregate volatility induces a flight-to-quality across financial markets and depresses future investments.

Related Literature In line with the work of Hackbarth, Miao, and Morellec (2006); Almeida and Philippon (2007); David (2008); Chen, Collin-Dufresne, and Goldstein (2009); and Bhamra, Kuehn, and Strebulaev (2010), I show that macroeconomic conditions and time variations in the market price of risk have rich implications for firms' credit spreads. I contribute to this literature by structurally estimating the time series of shocks that drive credit spreads and other financial indicators using a large firm-level panel dataset of U.S. public firms. Chen, Cui, He, and Milbradt (2016) explore how the interactions between default and liquidity affect corporate bond pricing. I find that shocks to firms' asset values and aggregate asset volatility generate variation in bid-ask spreads consistent with empirical observations.

A recent theoretical and empirical research aimed at understanding the 2008–2009 financial crisis has pointed to financial and uncertainty shocks as main drivers of business cycles. Stock and Watson (2012) and Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) emphasize the difficulty to empirically distinguish these two types of shocks, because increases in equity volatility—a widely used proxy for macroeconomic uncertainty—are frequently associated with spikes in credit spreads—a widely used proxy for financial turmoil. In parallel, Bloom (2009) emphasizes the importance of aggregate uncertainty shocks to explain sharp recessions. Atkeson, Eisfeldt, and Weill (2013) and Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) study the macro dynamics of firms' aggregate and idiosyncratic equity volatility and their association with macroeconomic fluctuations. Using a structural approach, I find that shocks to firms' asset value and aggregate asset volatility are the main drivers of the dynamics of credit spreads and aggregate equity volatility.

Finally, a large literature, spurred by Harvey (1988) and Estrella and Hardouvelis (1991) and furthered by Friedman and Kuttner (1993); Gertler and Lown (1999); Mody and Taylor (2004); and King, Levin, and Perli (2007) links measures of credit risk and real activity. More recently, Gilchrist, Yankov, and Zakrajšek (2009); Gilchrist and Zakrajšek (2012); and Faust, Gilchrist, Wright, and Zakrajšek (2013) find that corporate bond credit spreads have considerable predictive power for economic activity, and significantly exceed that of widely used default-risk indicators. I complement this literature by showing that the predictive content of corporate bond credit spreads is captured by firms' aggregate asset volatility.

The remainder of the paper is organized as follows. Section 2 describes the model. In Section 3, I provide empirical regularities that motivate the model's structure and estimation strategy. Section 4 estimates the shocks and calibrates the parameters. Section 5 describes the model's capability to replicate levels and dynamics of financial indicators. Section 6 provides a structural decomposition of economic channels and shocks to macroeconomic conditions. Section 7 examines the predictive power of fluctuations in firms' aggregate asset volatility for economic activity. Section 8 concludes.

2 The Model

The model provides a structural mapping between the exogenous shocks and the endogenous financial indicators: default risk, corporate bond credit spread, aggregate and idiosyncratic equity volatility, and bid-ask spreads. Thus, the model yields predictions for all financial indicators that can be compared to their historical counterparts (see Section 5). In Section 4, I detail the estimation of the exogenous shocks given firm-level observations of equity prices and volatilities, accounting statements, and bond recovery ratios. In Section 6, I provide a structural decomposition of economic channels and shocks to macroeconomic conditions that drive the financial indicators.

I introduce secondary over-the-counter market search frictions (as in Duffie, Gârleanu, and Pedersen, 2005) into a structural credit risk model with macroeconomic fluctuations (as in Chen, 2010). My model is similar to Chen, Cui, He, and Milbradt (2016), except that I assume different shocks to state-dependent parameters. Most importantly, I estimate these shocks using a large panel dataset. Chen, Cui, He, and Milbradt (2016) use their model to derive the implications on credit spreads of shocks to over-the-counter liquidity frictions. I find that shocks to firms' asset values and aggregate asset volatility generate fluctuations in bid-ask spreads consistent with empirical observations.

2.1 Shocks and Technology

Shocks There are two types of shocks in this economy: small and frequent shocks to firms' asset values and large but infrequent shocks to macroeconomic conditions. Shocks to macroeconomic conditions include shocks to bankruptcy costs, firms' aggregate and idiosyncratic asset volatility, and the market price of risk. Specifically, the small and frequent shocks are represented by diffusions: namely, a standard Brownian motion Z_t^A generates aggregate shocks common to all firms, while a standard Brownian motion Z_t^I provides idiosyncratic shocks, on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$. The large and infrequent shocks are represented by a continuous time Markov chain: the aggregate state s_t follows an S-state time-homogeneous Markov chain, and takes values in the set $\{1,\ldots,S\}$. The physical transition density between state s and state s' is denoted by $\zeta_{\mathcal{P}}^{ss'}$. Equivalently, this Markov chain can be expressed as a sum of Poisson processes,

$$ds_t = \sum_{s_t \neq s_{t-}} (s_t - s_{t^-}) dN_t^{(s_{t^-},s_t)},$$

where $N_t^{(s,s')}$ are independent Poisson processes with intensity parameters $\zeta_{\mathcal{P}}^{ss'}$. Because shocks to macroeconomic conditions are state-dependent, they comove together with the Markov states. However their correlation structure is not restricted by the Markov chain and is estimated in Section 4.

Stochastic Discount Factor I assume an exogenous stochastic discount factor that follows a Markov-modulated jump-diffusion process:

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta(s_t)dZ_t^A + \sum_{s_t \neq s_{t-}} \left(e^{\kappa(s_{t^-},s_t)} - 1\right) dM_t^{(s_{t^-},s_t)} \tag{1}$$

where $r(s)$ is the risk-free rate, $\kappa(s,s')$ is the relative jump size of the discount factor when the Markov chain switches from state s to s', and M_t is a matrix of compensated processes such that

$$dM_t^{(s,s')} = dN_t^{(s,s')} - \zeta_{\mathcal{P}}^{ss'}.$$

The risk price of aggregate shocks is given by $\eta(s)$. Changes in the state of the economy cause jumps in the discount factor. The relative jump sizes $\kappa(s,s')$ of the stochastic discount factor are the risk prices for these shocks. Transition intensities are adjusted by the size of the corresponding jump in the stochastic discounts factor $\kappa(s,s')$ such that

$$\zeta_{\mathcal{Q}}^{ss'} = e^{\kappa(s,s')}\zeta_{\mathcal{P}}^{ss'}$$

under the risk-neutral measure $\mathcal{Q}$.

Firm's Cash Flows Let y_t be an individual firm's cash flow level, which follows the process

$$\frac{dy_t}{y_t} = \mu_{Y,F}dt + \sigma_{Y,A}(s_t)dZ_t^A + \sigma_{Y,I}(s_t)dZ_t^I, \tag{2}$$

where $\sigma_{Y,A}(s)$ and $\sigma_{Y,I}(s)$ are the firm's cash flow aggregate and idiosyncratic volatility, respectively, which vary with the state variable s. Given the stochastic discount factor Λ_t, the cash flow process under the risk-neutral measure $\mathcal{Q}$ becomes

$$\frac{dy_t}{y_t} = \mu_{Y,\mathcal{Q}}(s_t)dt + \sigma_{Y,T}(s_t)dZ_t^{\mathcal{Q}},$$

where $Z_t^{\mathcal{Q}}$ is a standard Brownian motion under $\mathcal{Q}$. The state-dependent risk-neutral growth rate and total volatility are given by

$$\begin{aligned}
\mu_{Y,\mathcal{Q}}(s_t) &= \mu_{Y,F} - \sigma_{Y,A}\eta(s_t), \\
\sigma_{Y,T}(s_t) &= \sqrt{(\sigma_{Y,A}(s_t))^2 + (\sigma_{Y,I}(s_t))^2}.
\end{aligned}$$

Firm's Asset Value Given the current cash flow level y_t and the state of the economy s_t, the value of the firm's assets is

$$V_t = v(s_t)y_t,$$

where the price-earnings ratio $v(\cdot)$ is given by a vector $\mathbf{v} = [v(1), \ldots, v(S)]^\intercal$ solving

$$\mathbf{v} = \left(\mathbf{r} - \boldsymbol{\mu}_{Y,\mathcal{Q}} - \boldsymbol{\zeta}_{\mathcal{Q}}\right)^{-1} \mathbf{1}. \tag{3}$$

In equation (3), $\mathbf{r}$ is an $S \times S$ diagonal matrix with its i-th diagonal element given by $r(i)$, $\boldsymbol{\mu}_{Y,\mathcal{Q}}$ is an $S \times S$ diagonal matrix with its i-th diagonal element given by $\mu_{\mathcal{Q}}(i)$, the vector $\mathbf{1}$ is an $S \times 1$ vector of ones, and $\boldsymbol{\zeta}_{\mathcal{Q}}$ is the generator of the Markov chain under the risk-neutral measure:

$$\begin{aligned}
[\boldsymbol{\zeta}_{\mathcal{Q}}]_{ss'} &= \zeta_{\mathcal{Q}}^{ss'}, \quad s \neq s' \\
[\boldsymbol{\zeta}_{\mathcal{Q}}]_{ss} &= -\sum_{s \neq s'} \zeta_{\mathcal{Q}}^{ss'}.
\end{aligned}$$

Therefore, the value of the firm's assets inherits the drift and the volatility of the cash flow process and follows a Markov-modulated jump-diffusion under the physical measure $\mathcal{P}$ according to:

$$\frac{dV_t}{V_t} = \mu_{Y,F}dt + \sigma_{Y,A}(s_t)dZ_t^A + \sigma_{Y,I}(s_t)dZ_t^I + \sum_{s_t \neq s_{t-}} \left(v(s_t)/v(s_{t^-}) - 1\right) dN_t^{(s_{t^-},s_t)}, \tag{4}$$

where $v(s')/v(s)$ represents the jump in asset value from state s to state s'. Thus, I will refer to $\sigma_{Y,A}(s)$ and $\sigma_{Y,I}(s)$ as the firms' aggregate and idiosyncratic asset volatility, respectively.

Financing and Default Firms can issue two types of financial assets: debt and equity. Firms make financing and default decisions with the objective of maximizing equity holders' value. Because interest expenses are tax deductible, firms lever up with debt to exploit the tax shield (e.g., Leland, 1994). As the amount of debt increases, so does the probability of default, which raises the expected default losses. Thus, firms will lever up to a point at which the net marginal benefit of debt is zero.

In each period, a levered firm first uses its asset returns to make interest payments, then pays taxes, and distributes the rest to equity holders as dividends. The firm is able to issue equity to cover the firm's interest expenses when internally generated returns are not sufficient. The firm defaults when equity holders are no longer willing to inject more capital. Equity holders use the stochastic discount factor given in equation (1) to price the firm's continuation value.

The firm has a unit measure of bonds in place that are identical except for their time to maturity, with the individual bond coupon and face value being c and p. Equity holders commit to keeping the aggregate coupon and outstanding face value constant before default, and thus issue new bonds of the same average maturity as the bonds that are maturing. Each bond matures with intensity m, and the maturity event is independent and identically distributed across individual bonds. Thus, by the law of large numbers, over $[t, t + dt)$ the firm retires a fraction mdt of its bonds. This implies an expected average debt maturity of $1/m$.

At the time of default, the absolute priority rule applies. Specifically, equity holders receive nothing at default, whereas debt holders recover only a fraction $\alpha(s)$ of the value of the firm's assets due to bankruptcy costs. Thus, in the event of default in state s, bond holders receive

$$\alpha(s)v(s)d(s),$$

where $d(s)$ is the asset return level at which equity holders decide to default and $v(s)$ is found in equation (3).

2.2 Liquidity Frictions

Liquidity frictions potentially account for a significant fraction of credit spreads. Adding over-the-counter liquidity frictions to the model yields predictions for bid-ask spreads, an important empirical measure of distress in market liquidity. All trades must be intermediated through dealers. Bond investors use the stochastic discount factor given in equation (1) to price bonds. They can hold either zero or one unit of the bond. They start in the H state without any holding cost when purchasing corporate bonds in the primary market. As time passes, H-type bond holders are hit by idiosyncratic liquidity shocks with intensity ξ^H. These liquidity shocks lead them to become L-types, who bear a positive holding cost per unit of time. L-type bond holders then look for a dealer to intermediate a transaction with an H-type bond holder. L-type investors leave the market forever after successfully selling the bond. Following He and Milbradt (2014), the secondary market is a sellers' market. That is, the flow of H-type buyers contacting dealers is assumed to be greater than the flow of L-type sellers contacting dealers.

Holding Costs and Equilibrium Prices Chen, Cui, He, and Milbradt (2016) provide microfoundations for the functional form of holding costs. The core idea is that when an agent is hit by a liquidity shock, he will need to raise an amount of cash that is large relative to his financial asset holdings. This assumption implies that the agent will borrow at a high uncollateralized rate, in addition to selling all of his liquid assets. The agent can reduce the financing cost of uncollateralized borrowing by using the bond as collateral. Intuitively, a more risky collateral asset will face a larger haircut, which lowers its marginal value for an investor hit by liquidity shocks. This interaction between holding costs and bond prices create an amplification mechanism between credit risk and liquidity frictions: Higher default risk increases holding costs and reduces the bond price; A lower bond price increases the cost of rolling over maturing debt, and therefore increases credit risk.

As in Chen, Cui, He, and Milbradt (2016), I specify holding costs hc that depend on prevailing bond prices as follows:

$$hc(y, s) = \chi(B(s) + N - P(y, s)) \tag{5}$$

where N, χ are positive constants and $P(y, s)$ is the endogenous market price of the bond as a function of the log asset return y. $B(s)$ is the value of a bond that delivers the same interest payments but without the risk of default or illiquidity shocks. That is, $B(s)$ is the s-th value of the vector $\mathbf{B}$ given by

$$\mathbf{B} = \left(\mathbf{r} + \operatorname{diag}(m) - \boldsymbol{\zeta}_{\mathcal{Q}}\right)^{-1}(c\mathbf{1} + mp\mathbf{1}).$$

Thus, holding costs depend linearly on the bond price with an intercept of $\chi(B(s) + N)$ and a slope of χ. When the bond becomes riskier, its price decreases. In turn, holding costs increases. This effect feeds back into the bond price and leads to an amplification effect.

As in Duffie, Gârleanu, and Pedersen (2005), Nash-bargaining weights are assumed to be constant across all dealer-investor pairs, β for the investor and $1-\beta$ for the dealer. The observed bond prices are assumed to be mid-prices between the bid and ask prices in the secondary market, i.e.,

$$P(y,s) = \frac{(1+\beta)D^H(y,s) + (1-\beta)D^L(y,s)}{2}, \tag{6}$$

where D^H (D^L) is the bond value of an H-type (L-type) bond investor. It is assumed that the L-type is absorbing, i.e., those L-type investors leave the market forever after successfully selling the bond. However, an L-type bond holder meets a dealer with intensity λ and sells the bond for $\beta D^H(y,s) + (1-\beta)D^L(y,s)$. Thus the L-type intensity-modulated surplus when meeting the dealer can be rewritten as

$$\lambda\beta(D^H(y,s) - D^L(y,s)).$$

2.3 Debt and Equity Valuation

When taking all the elements cited above together, the risky debt valuation $D^H_s(y)$ in state $s \in \{1,\ldots,S\}$ must satisfy

$$\begin{aligned} r(s)D^H(y,s) = {} & \mu_{Y,\mathcal{Q}}(s)\frac{\partial D^H_s(y,s)}{\partial \log(y)} + 0.5\sigma^2_{Y,T}(s)\frac{\partial^2 D^H(y,s)}{\partial \log(y)^2} + c + m(p - D^H(y,s)) \\ & + \sum_{s'\neq s}\zeta^{ss'}_{\mathcal{Q}}(D^H(y,s') - D^H(y,s)) + \xi^H(D^L(y,s) - D^H(y,s)), \end{aligned}$$

where $\zeta_{ss'}^{\mathcal{Q}}$ is the transition intensity from state s to state s', ξ^H is the transition from type H to type L, and m is the intensity of bonds maturing, upon which the bond holders receive the principal value of the bond p. Similarly, the risky debt valuation $D^L(y,s)$ in state $s \in \{1,\ldots,S\}$ must satisfy

$$\begin{aligned} r(s)D^L(y,s) = {} & \mu_{Y,\mathcal{Q}}(s)\frac{\partial D^L(y,s)}{\partial \log(y)} + 0.5\sigma_{Y,T}^2(s)\frac{\partial^2 D^L(y,s)}{\partial \log(y)^2} + c + m(p - D^L(y,s)) \\ & + \sum_{s' \neq s} \zeta_{\mathcal{Q}}^{ss'}(D^L(y,s') - D^L(y,s)) + \lambda\beta(D^H(y,s) - D^L(y,s)) \\ & - \chi(B(s) + N - P(y,s)), \end{aligned}$$

where $\lambda\beta(D^H(y,s) - D^L(y,s))$ is the intensity-modulated surplus when meeting the dealer. The equity valuation $E(y,s)$ in state $s \in \{1,\ldots,S\}$ must satisfy

$$\begin{aligned} r(s)E(y,s) = {} & \mu_{Y,\mathcal{Q}}(s)\frac{\partial E(y,s)}{\partial \log(y)} + 0.5\sigma_{Y,T}^2(s)\frac{\partial^2 E(y,s)}{\partial \log(y)^2} + y - (1-\tau)c + m(D^H(y,s) - p) \\ & + \sum_{s' \neq s} \zeta_{\mathcal{Q}}^{ss}(E(y,s') - E(y,s)), \end{aligned} \tag{7}$$

where c is the coupon, τ the tax benefits of debt. With intensity m, the firm refinances maturing bonds at market vale $D^H(y,s)$. Finding equity and bond prices

$$\left\{E(y,s), D^H(y,s), D^L(y,s)\right\}_{y\in\mathbb{R}, s\in S}$$

requires to solve a system of second order ordinary differential equations with endogenous boundaries by the method of undertermined coefficients. Default boundaries $d(s)$ are solved numerically such that the following smooth-pasting conditions are satisfied:

$$\left.\frac{\partial E_s(y)}{\partial y}\right|_{y=d(s)} = 0 \quad \text{for all } s.$$

The full solution of this problem, including all the smooth pasting conditions, is given in Appendix B.

3 Common Macroeconomic Fluctuations

Collin-Dufresne, Goldstein, and Martin (2001) first established that credit spreads, after controlling for standard indicators of firms' default risk, are mostly driven by a single common factor. Gilchrist and Zakrajšek (2012) find that this common component of credit spreads is a powerful predictor of economic activity. Similarly, the cross-sectional distribution of firms' equity volatilities is approximately log-normal, and a large portion of the dynamics of this cross-sectional distribution is also accounted for by a single principal component (Atkeson, Eisfeldt, and Weill, 2013; Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2016). Bloom (2009) studies the extent to which these dynamics in firms' equity volatilities predict GDP growth. In the following section, I show that a single principal component drives the joint dynamics of credit spreads, aggregate and idiosyncratic equity volatility, and firms' leverage.

These empirical regularities motivate the use of fluctuations in macroeconomic conditions—i.e., a factor common to all firms—to explain fluctuations in credit spreads, leverage, and equity volatility over time and in the cross-section. During the estimation of shocks, I exploit the comovements of these variables and show that shocks to firms' asset values and firms' aggregate asset volatility—key fundamentals common to credit spreads, leverage, and aggregate equity volatility—can explain these joint macroeconomic dynamics.

Data For a sample of U.S. nonfinancial firms covered by the S&P's Compustat database and the Center for Research in Security Prices (CRSP), I obtained month-end secondary market option adjusted credit spreads of their outstanding senior unsecured bonds from the Lehman/Warga and Merrill Lynch databases. I matched these corporate securities with their issuers' quarterly income and balance sheet data from Compustat and daily data on equity valuations from CRSP, which yielded a matched sample of 300,887 monthly credit spreads observations from 2,355 firms for the period between January 1973 and October 2014. I use similar restrictions as Gilchrist and Zakrajšek (2012) to ensure that the results are not driven by a small number of extreme observations.[3]

I construct monthly volatility of firm-level equity returns. Total equity volatility, which is estimated using data from the CRSP daily stock file from 1973 to 2014, is defined as the standard deviation of a stock's daily returns from the last 63 days.[4] Idiosyncratic returns are constructed by estimating a factor model using all observations for that firm. The factor model takes the form:

$$r_t^i - r_t^f = \gamma_0^i + \mathbf{F}_t \boldsymbol{\gamma}^i + \varepsilon_t^i,$$

where r_t^i is the equity return from day $t-1$ to t, including dividends of firm i, and r_t^f is the 1-month treasury bill rate. I specify $\mathbf{F}_t$ as a 4-factor model—namely, the Fama and French (1992) 3-factor model, augmented with the momentum risk factor proposed by Carhart (1997). The aggregate equity volatility $v_{E,A}^i(t)$ and idiosyncratic equity volatility $v_{E,I}^i(t)$ of firm i in month t is then given by:

$$v_{E,A}^i(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} \left(\mathbf{F}_k \widehat{\boldsymbol{\gamma}}^i\right)^2}$$

$$v_{E,I}^i(t) = \sqrt{\frac{1}{K_t} \sum_{k=L_t-63}^{L_t} \left(\widehat{\varepsilon}_k^i\right)^2},$$

where L_t is the last day in month t. In short, idiosyncratic equity volatility is the volatility of residuals after a 4-factor model regression.

Following Strebulaev and Yang (2013), the market leverage ratio of firm i at time

[3] I eliminated all observations with credit spreads below 5 basis points and greater than 3,000 basis points. In addition, I drop very small corporate issues (equity market value of less than \$1 million) and all observations with a remaining term to maturity of less than 6 months or more than 20 years. Some firms tend to have many different corporate bond securities outstanding. To avoid overweighting firms that issue a lot of different securities, when different prices were available for the same firm, I kept only the security with time to maturity closest to 8 years (sample average). Financial, utility, and public administration firms are also excluded from the sample. Restricting to unique credit spreads' monthly observations for each firm eliminates 45% of the dataset; other restrictions affect less than 5% of the rest.

[4] It is standard to assume that financial markets have 21 open days on average in a month, and therefore quarters consist of 63 days. The results are robust to alternative windows.

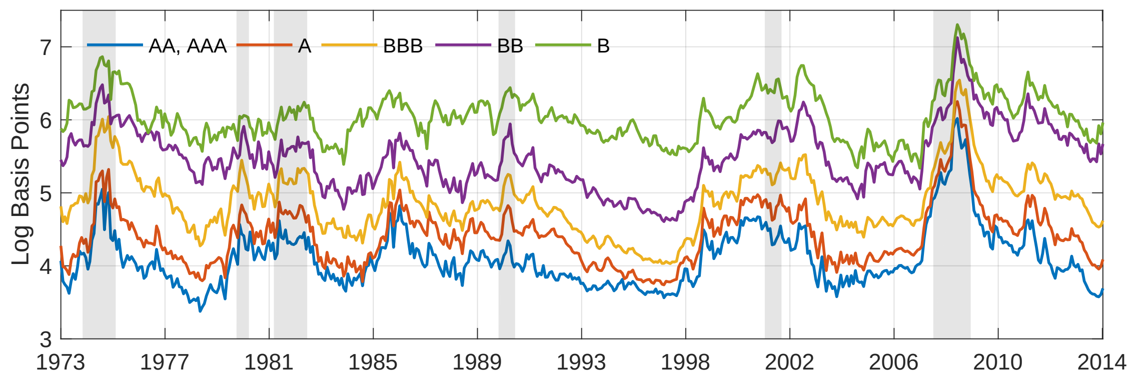

Figure 1: Average Log Corporate Bond Credit Spread by Rating Class This figure shows the arithmetic average of log bond credit spreads by rating class. See main text for details on the dataset.

t is defined as:

$$\text{lev}_{it} = \frac{\text{DLTT}_{it} + \text{DLC}_{it}}{\text{DLTT}_{it} + \text{DLC}_{it} + \text{CSHO}_{it} \times \text{PRCC}_{it}},$$

where DLTT and DLC are the Compustat long-term debt and debt in current liabilities, CSHO is the number of shares outstanding, and PRCC is the stock price from CRSP. This measure is often used in the empirical literature (e.g., Strebulaev and Yang, 2013, or Chen, Cui, He, and Milbradt, 2016). One alternative is to use total liabilities (e.g., Rajan and Zingales, 1995). However, a nontrivial portion of nondebt liabilities (such as accounts payable) can reflect day-to-day business arrangements instead of financing considerations. Another alternative is to use the sum of short-term liabilities and half of long-term liabilities (e.g., Gilchrist and Zakrajšek, 2012, or Moody's KMV framework) to capture the notion that short-term debt requires a repayment of the principal relatively soon, whereas long-term debt requires the firm to meet only the coupon payments. This adjustment is not pertinent to my structural approach in which bond maturity is explicitly modeled.

Common Factor Figure 1 shows the arithmetic average log credit spread by rating class. The first principal component of these time series explains 85% of the total

	log(cs)	lev	$\log(\sigma_E)$	$\log(\sigma_{E,A})$	$\log(\sigma_{E,I})$
log(cs)	1.00	0.72	0.75	0.74	0.72
lev	0.72	1.00	0.53	0.53	0.47
$\log(\sigma_E)$	0.75	0.53	1.00	0.94	0.98
$\log(\sigma_{E,A})$	0.74	0.53	0.94	1.00	0.87
$\log(\sigma_{E,I})$	0.72	0.47	0.98	0.87	1.00

Table 1: First Principal Components Correlation Matrix This table displays the correlation between the first principal components of each variable, averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC where log(cs) is log credit spread, lev is market leverage, $\log(\sigma_E)$ is log total equity volatility, $\log(\sigma_{E,A})$ is log aggregate equity volatility, and $\log(\sigma_{E,I})$ is log idiosyncratic equity volatility. See Appendix G for more details.

variation. Another way to capture the common variation is to regress log credit spreads on credit rating (21 rating classes from Aaa to C) and a time fixed effect. This yields an adjusted R-squared of 72%. The common factor structure also holds for aggregate and idiosyncratic equity volatility and market leverage. The first principal component of log aggregate equity volatility, log idiosyncratic equity volatility, and market leverage accounts for 94%, 93%, and 90% of the total variation, respectively.

As shown in Figure 2, the first principal component of log credit spread is highly correlated with the first principal components of average market leverage, aggregate equity volatility, and idiosyncratic equity volatility (see Table 1). The first principal component of credit spreads, leverage, and total equity volatility by rating classes (15 time series in total) explain 78% of total variation. Therefore, a single factor can account for the joint macroeconomic dynamics of firms' credit spreads, leverage, and total equity volatility. For further analysis and details on the common factor structure of credit spreads, leverage, and equity volatilities, I refer the reader to Appendices G and E. For discussion of the link between the credit spread common factor and the excess bond premium of Gilchrist and Zakrajšek (2012), see Appendix E.

As a result of these strong comovements, I can efficiently use Markov states to identify common changes in macroeconomic conditions, reflected by large shocks to bankruptcy costs, firms' aggregate and idiosyncratic asset volatility, and the market price of risk.

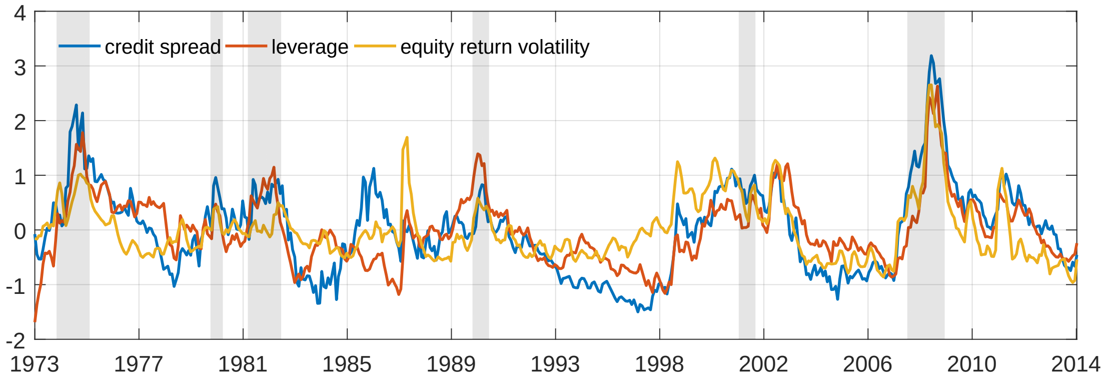

Figure 2: First Principal Components These series represents the first principal components of each variable averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC. See Appendix G for more details.

4 Estimation and Calibration

This section presents the estimation of shocks and calibration of parameters of the model.

4.1 Estimation

In this subsection, I present the estimation of two types of shocks: small and frequent shocks to firms' asset values and large but infrequent shocks to macroeconomic conditions. Shocks to macroeconomic conditions include shocks to bankruptcy costs $\alpha(s)$, firms' aggregate and idiosyncratic asset volatility $\sigma_{Y,A}(s)$ and $\sigma_{Y,I}(s)$, and the market price of risk $\eta(s)$. These shocks depend on the macroeconomic state that follows a Markov chain. All these objects are estimated using the large panel dataset (see Section 3) of firm-level observations of equity prices, aggregate and idiosyncratic equity volatilities, corporate debt book values, and bond recovery ratios. The model provides a mapping between observables and model variables (see Table 2). While I present each part of the estimation procedure separately, I iterate through each step until convergence.

observations	model variables
leverage	firms' asset value $V(y,s)$
aggregate equity volatility	aggregate asset volatility $\sigma_{Y,A}(s)$
idiosyncratic equity volatility	idiosyncratic asset volatility $\sigma_{Y,I}(s)$
bond recovery ratio	bankruptcy costs $\alpha(s)$

Table 2: Mapping Between Observations and Model Variables

4.1.1 Shocks to Firms' Asset Values

As is typical in structural corporate bond pricing models, model-implied variables are highly convex in market leverage (default probability, credit spread, bid-ask spread). This convexity implies that the variables' averages are higher than those of variables at average market leverage. Therefore, I follow David (2008) in computing model-implied aggregate moments. I match the market leverage of each firm's observation to its model counterpart. Then I can compute model-implied moments according to the empirical distribution of leverage.

The model gives a direct mapping between market leverage lev and log cash flows y conditional on being in state s according to

$$lev = \frac{p}{p + E(y,s)},$$

where p (principal) is the book value of outstanding debt. Therefore, for every firm-level monthly observation of leverage in my dataset, I recover the model-implied level of log cash flows y. Thus, once scaled by the book value of debt, model-implied and historical values of equity are matched perfectly. Time series of model-implied levels of log cash flows y implicitly measure firm-level shocks to asset values dZ_t^A and dZ_t^I from equation (4). In Section 6, I discuss whether model-implied aggregate shocks to firms' asset values dZ_t^A are consistent with the model's assumptions (independent and identically distributed according to a standard normal distribution).

4.1.2 Shocks to Macroeconomic Conditions

Aggregate and Idiosyncratic Asset Volatility The stochastic processes for firm's asset volatilities are estimated for two types of firms: investment- and speculative-grade firms. This separation is useful to generate predictions for the financial indicators in the cross-section. Estimating the stochastic process for each firm is unfeasible. Therefore, type-specific volatilities drive the dynamics of each firm of that type, and the model is solved for each firm's type, not for each firm. A firm that is speculative-grade has a rating lower than Baa from Moody's Investors Service, a rating lower than BBB from Standard & Poor's, or both. Firms with ratings of Baa, BBB or higher are termed investment-grade. The relationship between firm i's aggregate asset volatility $\sigma^i_{Y,A}(s)$ and its type j aggregate asset volatility $\sigma^j_{Y,A}(s)$ is given by:

$$\log\left(\sigma^i_{Y,A}(s)\right) = \log\left(\sigma^j_{Y,A}(s)\right) + \varepsilon_i \tag{8}$$

where ε_i is independent and identically distributed measurement noise. The same holds for idiosyncratic volatility.

While in the dataset a firm might move across rating classes, the model assumes that an investment-grade firm never becomes speculative-grade, and vice versa. Introducing this feature is relatively unfeasible with Markov states, as it would exponentially increase the amount of states required to solve the model.[5] However, in the estimation procedure, if an investment-grade firm in month t is downgraded to speculative-grade in month $t+1$, its observations contribute to the representative investment-grade firm in month t and to the representative speculative-grade firm in month $t+1$.

Using Ito's Lemma, the model implies the following relationships:

$$\sigma_{E,A}(y,s)E(y,s) = E'(y,s)\sigma_{Y,A}(s),$$
$$\sigma_{E,I}(y,s)E(y,s) = E'(y,s)\sigma_{Y,I}(s),$$

[5] Currently, a firm of type $j \in J$ can transition to S states in the next period. With this feature, a firm of type $j \in J$ could transition to $J \times S$ states in the next period.

where $E_s(y,s)$ is the solution to the system of ODEs in equation (7), and $\sigma_{E,A}(y|s)$ and $\sigma_{E,I}(y|s)$ are the aggregate and idiosyncratic equity volatility of a firm with log asset return y in state s. Thus, I can estimate firm i's asset volatilities $\widehat{\sigma}^i_{Y,A}(t|s_t)$ and $\widehat{\sigma}^i_{Y,I}(t|s_t)$ given observed values for equity volatilities $\sigma^i_{E,A}(t)$ and $\sigma^i_{E,I}(t)$ according to

$$\widehat{\sigma}^i_{Y,A}(t|s_t) = \frac{E^j(y_{it},s_t)\sigma^i_{E,A}(t)}{E^{j,\prime}(y_{it},s_t)},$$
$$\widehat{\sigma}^i_{Y,I}(t|s_t) = \frac{E^j(y_{it},s_t)\sigma^i_{E,I}(t)}{E^{j,\prime}(y_{it},s_t)},$$

for every firm i, type j, time t, and state s.[6] Section 3 details how observations of aggregate and idiosyncratic equity volatility are constructed. From equation (8) it follows that $\widehat{\sigma}^j_{Y,A}(t|s)$ and $\widehat{\sigma}^j_{Y,I}(t|s)$ can be estimated according to

$$\log\left(\widehat{\sigma}^j_{Y,A}(t|s)\right) = \frac{1}{N^j_t}\sum_{i\in I(j,t)} \log\left(\widehat{\sigma}^i_{Y,A}(t|s)\right), \tag{9}$$
$$\log\left(\widehat{\sigma}^j_{Y,I}(t|s)\right) = \frac{1}{N^j_t}\sum_{i\in I(j,t)} \log\left(\widehat{\sigma}^i_{Y,I}(t|s)\right), \tag{10}$$

for every type j, time t, and state s, where N^j_t is the amount of firms of type j at time t, and $I(j,t)$ is the set of firms of type j at time t.

Volatility States The above estimates are conditional on being in a state s at each time t. Given the insights from Section 3 that equity volatilities, credit spreads, and leverage comove, I use model-implied observations of asset volatilities to iden-

[6]Note that the estimate of aggregate equity volatility from CRSP at time t must be adjusted for the size of the jump in equity value from the transition between state s_{t-1} to state s_t. More precisely,

$$\sigma^i_{E,A}(t) = \sqrt{\left(v^i_{E,A}(t)\right)^2 - \left(E_{s_t}(y)/E_{s_{t-1}}(y) - 1\right)^2/21},$$

where $v^i_{E,A}(t)$ is the equity return aggregate volatility of firm i at time t described in Section 3. This adjustment turns out to be relatively insignificant.

tify fluctuations in macroeconomic states. This method identifies movements in the macroeconomic state, but does not impose anything on the correlations between state-dependent variables. Markovian states are estimated using the Baum-Welch algorithm for hidden Markov models:

1. Initiate with values for the Markov chain $\mathcal{M} = \left\{\sigma^j_{Y,A}(s), \sigma^j_{Y,I}(s), \boldsymbol{\zeta}_{\mathcal{P}}\right\}$.

2. Solve the structural model and estimate $\mathcal{Y} = \left\{\widehat{\sigma}^j_{Y,A}(t|s), \widehat{\sigma}^j_{Y,I}(t|s)\right\}$.

3. Identify the state s_t at each time t by maximizing the likelihood of being in state s at time t, given the whole dataset $\mathcal{Y}$ (see Appendix C).

4. Get new estimates of aggregate and idiosyncratic asset volatilities

$$\log\left(\sigma^j_{Y,A}(s)\right) = \frac{\sum_{t=1}^{T} \log\left(\widehat{\sigma}^j_{Y,A}(t|s)\right) \mathbf{1}\left\{s_t = s\right\}}{\sum_{t=1}^{T} \mathbf{1}\left\{s_t = s\right\}}$$
$$\log\left(\sigma^j_{Y,I}(s)\right) = \frac{\sum_{t=1}^{T} \log\left(\widehat{\sigma}^j_{Y,I}(t|s)\right) \mathbf{1}\left\{s_t = s\right\}}{\sum_{t=1}^{T} \mathbf{1}\left\{s_t = s\right\}}.$$

5. Update transition intensities $\zeta^{ss'}_{\mathcal{P}}$ with the empirical discrete transition probabilities $\pi^{ss'}$ given by

$$\pi^{ss'} = \frac{\sum_{t=1}^{T-1} \mathbf{1}\left\{s_t = s\right\} \mathbf{1}\left\{s_{t+1} = s'\right\}}{\sum_{t=1}^{T} \mathbf{1}\left\{s_t = s\right\}}$$

   for all s, s'. See Appendix D for more details.

6. Iterate on 2-5 until convergence.

This estimation procedure is dependent on initial guesses for $\mathcal{M}$ and the number of states. However, it is easy to check how well the estimation procedure approximates the continuous time series of estimated volatilities with the Markov chain by looking at the difference between $\left\{\widehat{\sigma}^j_{Y,A}(t|s_t), \widehat{\sigma}^j_{Y,I}(t|s_t)\right\}_{t=1}^{T}$ and $\left\{\sigma^j_{Y,A}(s_t), \sigma^j_{Y,I}(s_t)\right\}_{t=1}^{T}$ in Figures 3 and 4. Increasing the number of states improves the fit, but at the cost of more transition probabilities to estimate. Likelihood plateaus at 8 states.

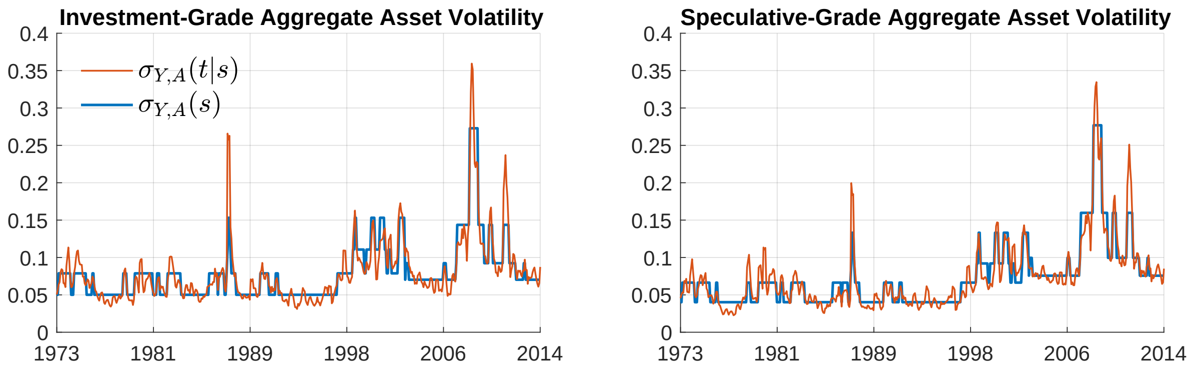

Figure 3: Aggregate Asset Volatility These graphs show the Markov estimates of aggregate asset volatility $\sigma^j_{Y,A}$ and model-implied aggregate asset volatility $\widehat{\sigma}^j_{Y,A}$.

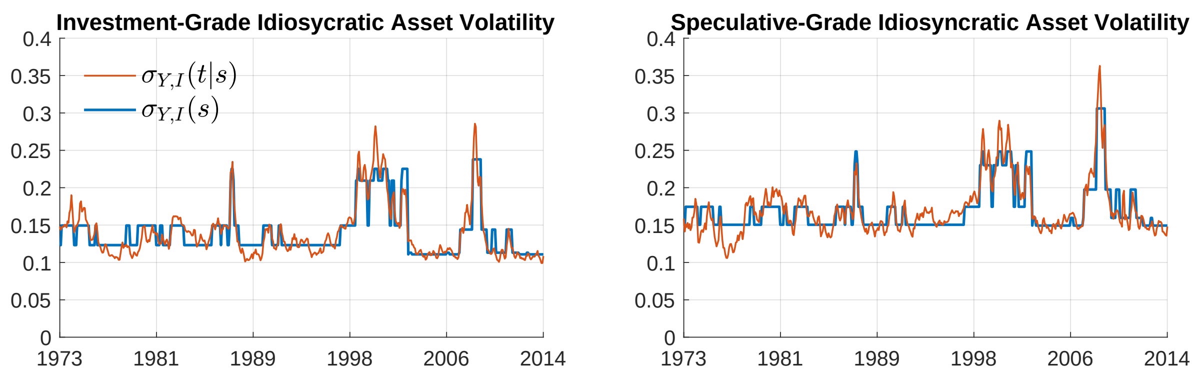

Figure 4: Idiosyncratic Asset Volatility These graphs show the Markov estimates of idiosyncratic asset volatility $\sigma^j_{Y,A}$ and model-implied aggregate asset volatility $\widehat{\sigma}^j_{Y,A}$.

Modeling large shocks with a continuous-time Markov chain not only provides closed-form solutions for equity and bond prices, but also eases the exercise of estimating the five-dimensional shock distribution. This exercise is similar to Tauchen's (1986) method, and therefore akin to discretizing a continuous process of macroeconomic fluctuations rather than estimating macroeconomic regimes.

Stochastic Discount Factor With S states, the stochastic discount factor in equation (1) requires $2S + (S^2 - S)/2$ parameters for $r(s)$, $\eta(s)$, and $\kappa(s, s')$. To alleviate the parametrization, I impose restrictions on the stochastic discount factor similar to Chen (2010). First, the representative agent has stochastic differential utility, as developed by Duffie and Epstein (1992). I define the utility of the marginal agent over his consumption process C as

$$U_t = \mathbb{E}_t \left(\int_t^\infty f\left(C_s, U_s\right) ds \right).$$

Following Epstein and Zin (1989), the function $f(c, u)$ is a normalized aggregator of consumption and continuation value in each period defined as

$$f(c,u) = \frac{\rho}{1-1/\psi} \frac{c^{1-1/\psi} - ((1-\gamma)u)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)u)^{\frac{1-1/\psi}{1-\gamma}-1}}$$

where ρ is the rate of time preference, γ is the coefficient of relative risk aversion, and ψ determines the elasticity of intertemporal substitution.

Second, aggregate output follows

$$\frac{dY_t}{Y_t} = \mu_Y(s_t)dt + \sigma_Y(s_t)dZ_t^A,$$

which equals to the consumption process of the representative agent.

Third, aggregate output volatility $\sigma_Y(s)$ and the common factor in firms' aggregate asset volatility $\sigma_A(s)$ are related according to

$$\sigma_Y(s) = \bar{\sigma}_Y + \varphi\left(\sigma_A(s) - \bar{\sigma}_A\right), \tag{11}$$

where $\bar{\sigma}_Y$ and $\bar{\sigma}_A$ are long-run averages. The common factor in firms' aggregate asset volatility $\sigma_A(s)$ is defined as:

$$\log\left(\sigma_{Y,A}^j(s)\right) = \theta^j + \log\left(\sigma_A(s)\right), \tag{12}$$

for every state s and firm type j where θ^1 is normalized to 0. Atkeson, Eisfeldt, and Weill (2013) provide evidence that the distribution of firm-level aggregate asset volatility is log-normal with time-dependent mean. Section 3 and Appendix G present evidence that the average of log volatilities by rating class is indeed driven

state	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	$\mathbb{E}(\cdot)$
$\mu_Y(s)$	2.48	1.70	1.66	2.04	2.50	1.22	0	-2.56	1.72
$\sigma_Y(s)$	1.71	2.18	2.73	3.84	2.57	3.07	4.33	5.70	2.93
$\eta(s)$	0.13	0.16	0.21	0.29	0.19	0.23	0.33	0.43	0.19
$\exp(\kappa(3,s))$	0.86	0.92	1.00	1.13	1.92	2.14	2.58	3.34	1.31

Table 3: Pricing Kernel State-Dependent Parameters This table shows pricing kernel growth $\mu_C(s)$, volatility $\sigma_C(s)$, price of aggregate shocks $\eta(s)$, and jump premium from the median state (state 3) to the other states $\exp(\kappa(3,s))$.

by a strong common factor. The coefficients θ^j are found by minimizing

$$\sum_{j=1}^{J}\sum_{t=1}^{T}\left(\theta^j + \log\left(\sigma_A(s_t)\right) - \log\left(\sigma^j_{Y,A}(s_t)\right)\right)^2.$$

I use real gross domestic product per capita from NIPA to estimate state-dependent growth rates $\mu_Y(s)$. The sensitivity of aggregate asset volatility to variation in aggregate output volatility, φ, is set to target the average equity premium (see Table 6). I follow Bansal and Yaron (2004) for the estimate of long-term systemic volatility ($\bar{\sigma}_Y = 0.0293$).

Table 3 summarizes important state-dependent parameters of the Markov chain model related to the pricing kernel. This specification yields countercyclical risk prices and sizable jump-risk premia. For example, the risk-neutral probability of switching from the medium state (state 3) to the financial crisis state (state 8) is about 3 times higher than its actual probability.

Bond Recovery Ratios Economic conditions greatly affect the cost of default. A large literature on fire sales, starting with Shleifer and Vishny (1992), argues that liquidation of assets is particularly costly when many firms are in distress. Therefore, instead of assuming bankruptcy costs as a constant fraction of the value of assets at default, I estimate state-dependent bankruptcy costs. I use Moody's annual average defaulted corporate bond recoveries series, which spans 1983 to 2015. As I identify the aggregate state in each month, I estimate the recovery rate $\alpha(s)$ for each state s

according to

$$\alpha(s) = \frac{\sum_{t=1}^{T} rec_t \times vb(s_t)\mathbf{1}\{s_t = s\}}{\sum_{t=1}^{T} \mathbf{1}\{s_t = s\}},$$

where rec_t is Moody's defaulted corporate bond recovery ratio at time t and $vb(s)$ is the value of the unlevered firm at the endogenous bankruptcy level divided by the bond principal value p in state s. Moody measures recovery ratios using post-default trading prices. Therefore, assuming that post-default prices are the bid prices at which investors are selling, we have that

$$\alpha(s) = \alpha^L(s) + \beta\left(\alpha^H(s) - \alpha^L(s)\right),$$

where $\alpha^H(s)$ and $\alpha^L(s)$ are the recovery rate in state s for bond investors of H- and L-type, respectively. The estimated average recovery ratio over the whole sample $\mathbb{E}[\alpha(s)]$ is about 45%, in line with the estimate of bankruptcy recovery by Chen (2010). The lowest $\alpha(s)$ is equal to 33% and corresponds to the state of the 2008–2009 financial crisis. Note that this recovery ratio is different from the ultimate recovery of the bond after resolution of bankruptcy. With an average resolution period of 1.37 years, according to the Moody's default and recovery database, and an excess return of 23% on a portfolio of all defaulted bonds over 1987–2011 estimated by He and Milbradt (2014), the average ultimate recovery rate is about 70%. Estimating separate recovery ratios for each firm's type does not yield significant differences.

4.2 Calibration

I follow Bansal and Yaron (2004) for the parameters of risk aversion and intertemporal substitution, with $\gamma = 7.5$ and $\psi = 1.5$. With a discount rate ρ equal to 0.02, I get an average risk-free interest rate of 2%. Note that this is close to the lower bond for the firm's asset value v_s in (3) to exist, as the risk-free discount rate must be high enough relative to the drift of firms' asset value μ_Y of investment-grade firms. The calibration of μ_Y is discussed below.

γ	risk aversion	7.5	Bansal and Yaron (2004)
ψ	intertemporal substitution	1.5	Bansal and Yaron (2004)
ρ	time discount rate	0.02	risk-free interest rate
$\bar{\sigma}_Y$	long-run volatility	0.0293	Bansal and Yaron (2004)
φ	sensitivity of σ_A(s) to $\sigma_Y(s)$	6	average equity premium
τ	tax shield	0.12	Graham (2000)
$1/m$	average debt maturity	8	average maturity
β	bargaining power	0.03	Feldhütter (2012)
λ	meeting intensity	50	Chen, Cui, He, and Milbradt (2016)
ξ	liquidity shock intensity	0.7	Chen, Cui, He, and Milbradt (2016)
N	holding cost intercept	14.75	Edwards, Harris, and Piwowar (2007)
χ	holding cost slope	0.055	Edwards, Harris, and Piwowar (2007)

Table 4: Calibrated Parameters

I use the tax rate estimates of Graham (2000), which take into account the fact that the tax benefits of debt at the corporate level are partially offset by the individual tax disadvantages of interest income. The size of the debt is normalized with the bond's principal $p = 100$. I set the maturing intensity, m, to match the empirical average debt maturity (8 years).

Edwards, Harris, and Piwowar (2007) report that in normal times (2003–2005), the transaction cost for defaulted bonds of median-sized trades is about 200 basis points. Since the transaction cost can be constructed as the intermediary's profit $(1-\beta)(D^H(y,s) - D^L(y,s))$ over the mid-price $P(y,s)$ from (6), we have

$$2\% = \frac{2(1-\beta)(\alpha^H(s) - \alpha^L(s))}{\alpha^H(s) + \beta(\alpha^H(s) - \alpha^L(s)) + \alpha^L(s)}. \tag{13}$$

Given the estimation of $\alpha(s) = \alpha^L(s) + \beta\left(\alpha^H(s) - \alpha^L(s)\right)$ in Section 4, this pins down $\alpha^H(s)$ and $\alpha^L(s)$.

I fix the meeting intensity, λ, as in Chen, Cui, He, and Milbradt (2016), so that it takes a bond holder on average 1 week to find an intermediary and divest of all bond holdings. They also report a value-weighted turnover of corporate bonds during NBER expansion periods about 0.7 times per year ($\xi = 0.7$). For the bargaining power allocation between dealers and investors, β, I follow Feldhütter (2012).

Edwards, Harris, and Piwowar (2007) use the Trade Reporting and Compliance Engine bond price database from January 2003 to January 2005 to estimate reported bid-ask spreads for different firm ratings and trading sizes. I calibrate χ and N to match the average implied bid-ask spreads and their measurement of bid-ask spreads for investment- and speculative-grade bonds for a transaction of median size during that period. See Figure 10 for model-implied bid-ask spreads from 2002 to 2009.

The coupon payment c is set such that, on average, corporate bonds are issued at par value. That is, the coupon payment c of firms of type j satisfies the following condition:

$$\frac{1}{T}\sum_{t=1}^{T} D_j^H(\bar{y}_j, s_t) = p$$

where $D_j^H(\bar{y}_j, s)$ is the value of a bond of type j for the H-type investor in state s, $\bar{y}_j$ is the asset return level corresponding to the average leverage level of firms of type j, and s_t is the state identified during the estimation procedure.

The drift of firms' asset values under the physical measure essentially affects the overall match between default rates, credit spreads, and leverage. While important for the quantitative performance of a credit risk model, no consistent measurement has been agreed upon or consensus reached in the literature by which a wide range of values is used.[7] Therefore, I calibrate asset return drift by firm type to target Moody's historical 8-year average cumulative default rate (see Figure 5). Huang and Huang (2012) reveal that structural models typically imply a much steeper term structure of cumulative default rates than reflected in the data. Extensions of the model, such as introducing jumps in firms' asset values, are likely to help in that dimension, but accurately matching the term structure of cumulative default rates is beyond the scope of this paper.

[7] He and Milbradt (2014): 0.018; Bhamra, Kuehn, and Strebulaev (2010): -0.04 and 0.08, with 2 aggregate states; Chen (2010): from -0.10 to 0.11, with 9 aggregate states; Leland (2006): 0.045.

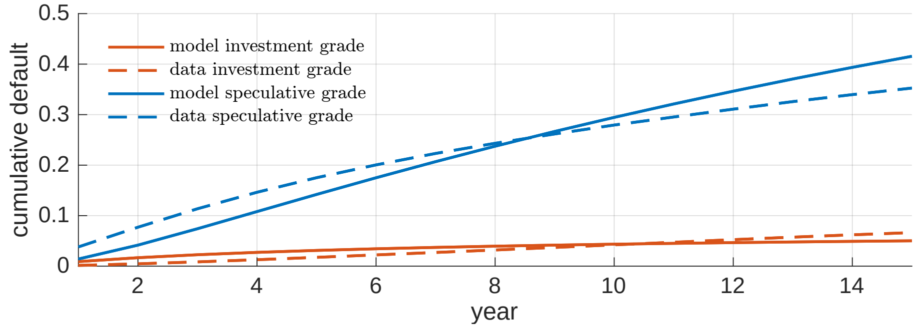

Figure 5: Average Data and Model Cumulative Default Rates from 1973 to 2014 Cumulative default rates are taken from Moody's 2015 Annual Default Study. Model default rates are the average of the model-implied default rate computed for every firm in Compustat at different horizon from 1973 to 2014. Model default probability for each firm depends on the firm's asset return drift, asset volatility, leverage, and default boundary in every month.

5 Results

This section details the model's quantitative performance to match default risk, credit spreads, aggregate and idiosyncratic equity volatility, and bid-ask spreads, and provide implications for equity and debt premiums. Given model-implied log cash flows, bankruptcy costs, firms' aggregate and idiosyncratic asset volatility, and market price of risk estimated in Section 4, each financial indicator is first computed at the firm level, then averaged each month.

Levels and fluctuations in credit spreads and default rates, as well as fluctuations in bid-ask spreads, are not targeted during the estimation of shocks and calibration of parameters. Therefore, the excellent match between model-implied and historical default rates, credit spreads, and bid-ask spreads, over time and in the cross-section, is an important external validation of the model's assumptions.

Default Risk Figure 6 shows model-implied aggregate default rates and Moody's issuer-weighted historical average default rates. Model-implied default rates are estimated by taking the average of model-implied expected default rates within 1 year for every firm-level observation in the first quarter of every year. Model-implied fluctuations in default rates are consistent with Moody's observations, but the level is

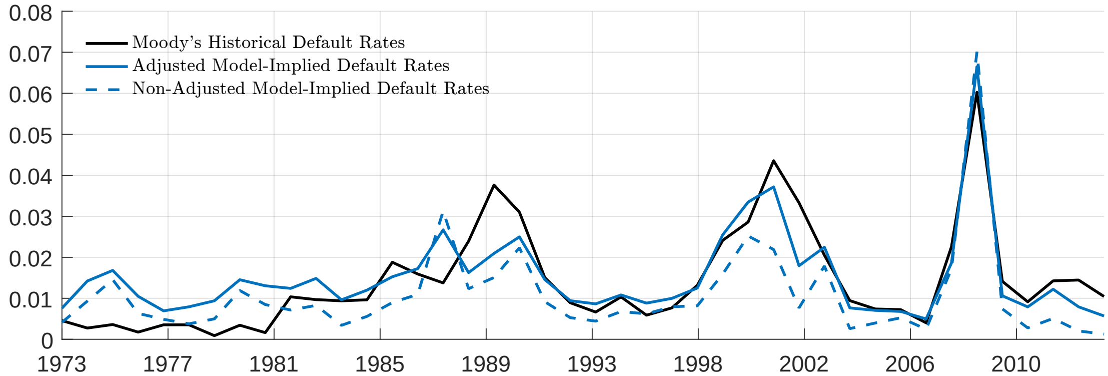

Figure 6: Model-Implied and Historical Default Rates This figure shows Moody's Moody's issuer-weighted historical average default rates for all rating categories, the non-adjusted model-implied default rates, and the adjusted model-implied default rates.

somewhat off—a known shortcoming of structural models of credit risk. Indeed, the term structure of model-implied default rates yields significantly lower short-term default rates than empirical observations (see Figure 5). However the model-implied default rate within 8 years is accurately targeted. Thus, I use the following adjustment:

$$\widehat{\pi}_t^{t+1} = 1 - \left(1 - \pi_t^{t+8}\right)^{1/8}, \tag{14}$$

where π_t^{t+j} is the average expected default rate of firms at time t within the next j years. Models with constant Poisson default probability are better at matching the historical default term structure in the short term. Thus, because the term structure of model-implied default rate matches the historical rate at 8 years, I use the model-implied default rate within 8 years to get the corresponding constant Poisson default probability and construct adjusted model-implied default rates within 1 year. With this adjustment, the model predicts levels and fluctuations of default rates consistent with Moody's estimates.

Credit Spreads Figure 7 shows average credit spreads for investment- and speculative-grade firms compared to their empirical counterparts. The model suc-

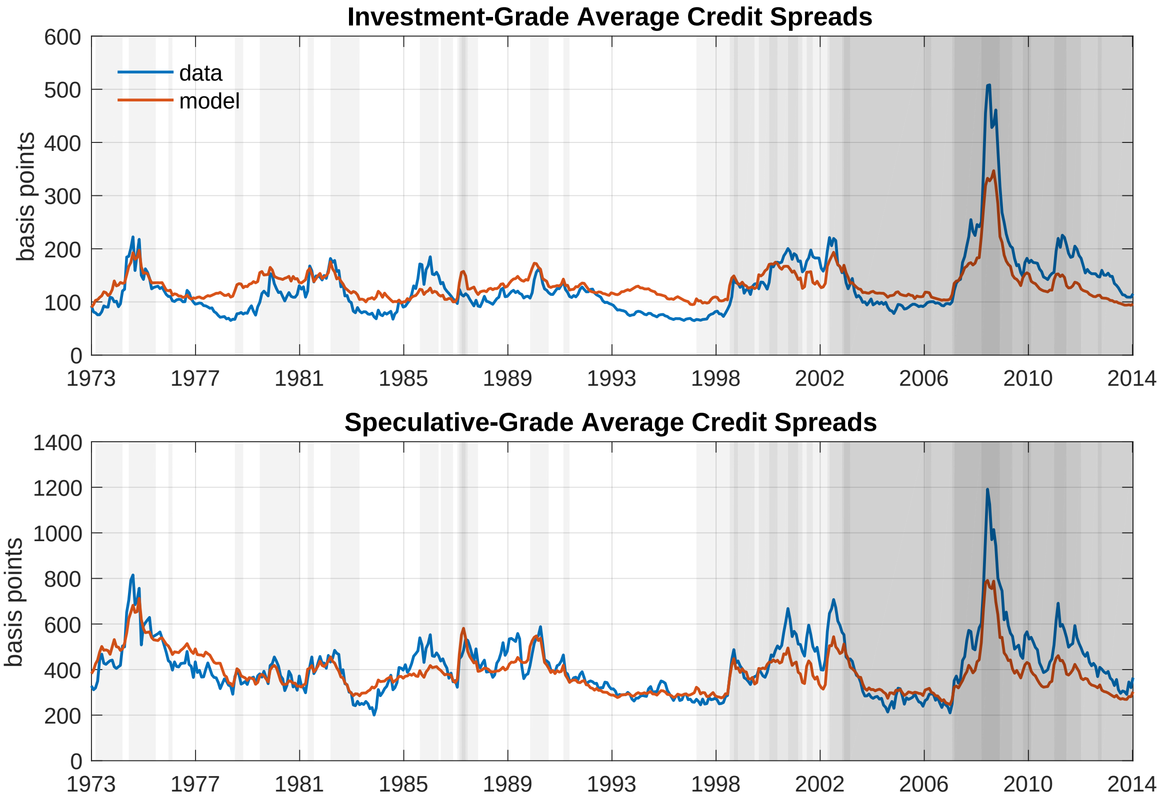

Figure 7: Average Credit Spreads by Rating Group These time series represent model-implied credit spreads computed for every firm and averaged each month by rating category. Shaded areas distinguish the 8 different Markov states estimated in Section 4.

	$R2(cs_t, \widehat{cs}_t)$	$R2(\Delta cs_t, \Delta\widehat{cs}_t)$
investment-grade firms	0.61	0.70
speculative-grade firms	0.71	0.74

Table 5: Goodness of Fit for Credit Spreads in Levels and First Differences The definition of the goodness of fit measure $R2(\cdot,\cdot)$ is given in the main text. The variable cs_t is the average of credit spreads within a rating class, while $\widehat{cs}_t$ corresponds to the model prediction. When taking first differences, the series are first averaged over each year.

ceeds in reproducing not only the level, but also the fluctuations of credit spreads over time and in the cross-section. Table 5 provides measures of goodness of fit in levels and first differences between the model and the data. The goodness of fit in

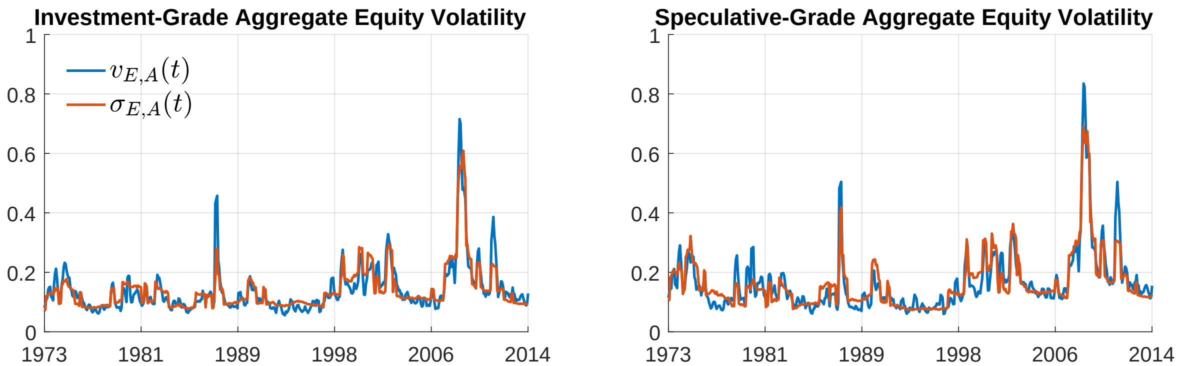

Figure 8: Aggregate Equity Volatility These graphs show the historical monthly average aggregate equity volatility $v_{E,A}$ and the model-implied aggregate equity volatility $\sigma_{E,A}$.

levels is defined as:

$$R2(cs_t, \widehat{cs}_t) = 1 - \frac{\sum_{t=1}^{T} \left(cs_t - \widehat{cs}_t\right)^2}{\sum_{t=1}^{T} (cs - \bar{cs})_t^2}$$

where cs_t is the empirical monthly average credit spread and $\widehat{cs}_t$ is the model-implied monthly average credit spread. For the goodness of fit in first differences, I average first the series over each year, that is:

$$R2(\Delta cs_t, \Delta\widehat{cs}_t) = 1 - \frac{\sum_{t=1}^{T} \left(\Delta \frac{1}{12} \sum_{i=1}^{12} cs_{it} - \Delta \frac{1}{12} \sum_{i=1}^{12} \widehat{cs}_{it}\right)^2}{\sum_{t=1}^{T} \left(\Delta \frac{1}{12} \sum_{i=1}^{12} cs_{it} - \bar{cs}\right)^2},$$

where $\Delta x_t = x_t - x_{t-1}$ and cs_{it} is the observation of month i in year t. Because the number of Markov states limits the frequency of changes in macroeconomic fluctuations, the model structure cannot address fluctuations of credit spreads at high frequency. Looking at differences of yearly credit spreads series helps alleviate this issue.[8] In Section 6, I decompose the importance of each shocks to account for the levels and dynamics of credit spreads.

[8]Not taking yearly averages first yields measures of goodness of fit in first differences of 0.07 and 0.19 for investment- and speculative-grade firms respectively.

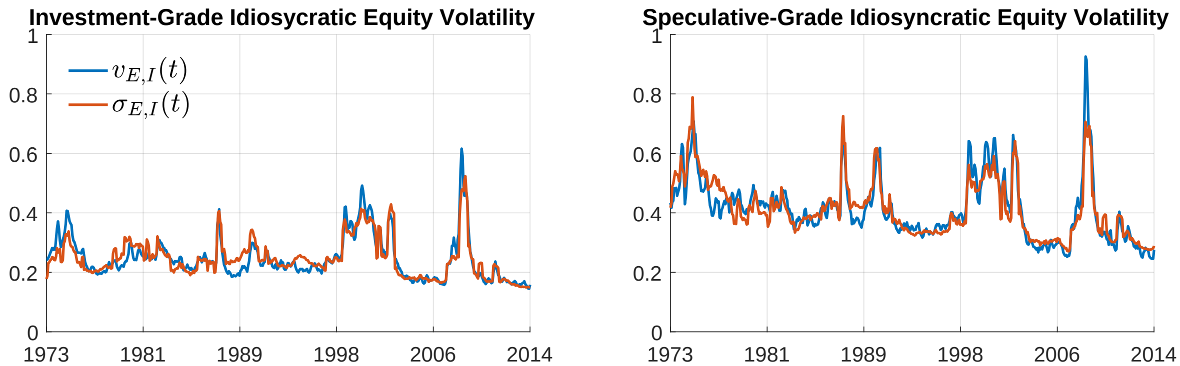

Figure 9: Idiosyncratic Equity Volatility These graphs show the historical monthly average idiosyncratic equity volatility $v_{E,I}$ and the model-implied idiosyncratic equity volatility $\sigma_{E,I}$.

Aggregate and Idiosyncratic Equity Volatility Figures 8 and 9 show the historical model-implied time series for aggregate and idiosyncratic equity volatility averaged within their respective rating class. Because shocks to firms' asset values and macroeconomic conditions are estimated using aggregate and idiosyncratic equity volatility, the match between the series is not surprising. Rather, it confirms that the Markov chain efficiently approximates the continuous volatility processes.

Bid-Ask Spreads Since holding costs increase as the value of the bond declines, average model-implied bid-ask spreads vary with macroeconomic conditions that affect bond prices. Figure 10 shows the average bid-ask spreads for investment- and speculative-grade firms. The variation in model-implied bid-ask spreads is consistent with Bao, Pan, and Wang's (2011) measurement of corporate bonds illiquidity.[9] Therefore, to replicate measured changes in implied bid-ask spreads, additional shocks to parameters that drive over-the-counter liquidity frictions are not neces-

[9] Bao, Pan, and Wang (2011) assume the following to measure the illiquidity of corporate bonds. Let p_t denote the log price of a bond at time t. They assume that p_t consists of two components:

$$p_t = f_t + u_t,$$

where f_t represents its fundamental value and u_t comes from the impact of illiquidity, which is transitory. They extract the transitory component in the price p_t with the measure γ_t given by

$$\gamma_t = -\text{Cov}(\Delta p_t, \Delta p_{t+1}).$$

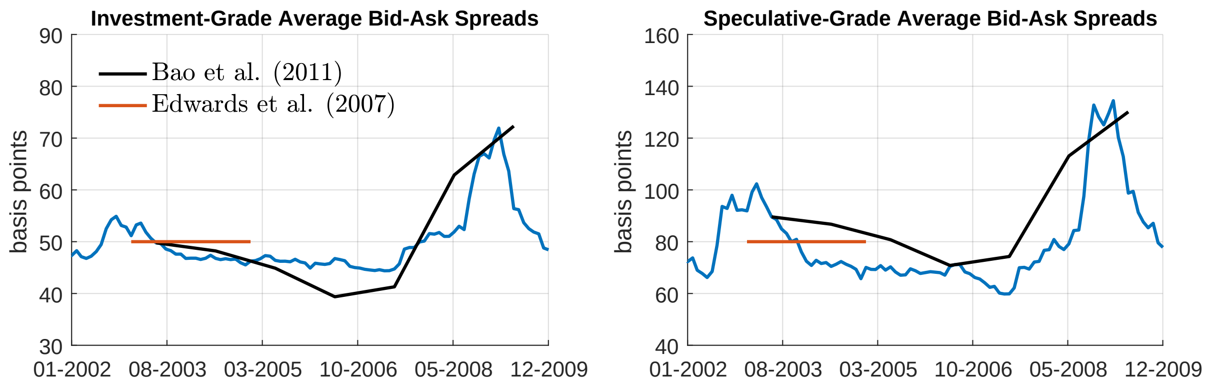

Figure 10: Average Bid-Ask Spreads by Rating Group Model average bid-ask spreads are the average of the model-implied bid-ask spreads computed for every firm in Compustat by rating category. The black line corresponds to the bid-ask spreads implied by Bao, Pan, and Wang (2011) measurement of corporate bonds illiquidity from 2003 to 2005. The relative measure is scaled to match the level of the first datapoint in June 2003. The red line shows the average level of bid-ask spreads estimated by Edwards, Harris, and Piwowar (2007) for investment- and speculative-grade bonds' transactions of median size.

sary. This helps resolve what Bao, Pan, and Wang (2011) call an intriguing result: Their aggregate illiquidity measure is closely connected to the VIX index. Aggregate equity volatility, of which the VIX is a proxy, is mainly driven by shocks to firms' asset values and aggregate asset volatility. These shocks greatly impact bond prices in two ways. First, shocks to firms' asset values and aggregate asset volatility affect the probability of default. Second, holding default risk constant, shocks to aggregate asset volatility change the compensation required by bond holders, because aggregate volatility is adversely priced by the representative investor. Thus, the sources of fluctuations in aggregate equity volatility impact bond prices and holding costs (cfr. equation 5), which trigger similar fluctuations in bid-ask spreads.

Equity and Debt Premiums Table 6 shows the model-implied expected equity and debt premiums. Firm-level monthly model-implied premiums are first averaged every month within each rating class, then over the whole sample. To measure equity excess returns, I look at the performance of Vanguard's index fund on the Standard

Following Roll (1984), the implied bid-ask spread, in the simple case in which transitory price movements arise from bid-ask bounce, is given by $2\sqrt{\gamma}$.

	equity premium			bond premium
	model	model	data	model
	1973–2014	1987–2014	1987–2014	1973–2014
investment-grade firms	524 bps	628 bps	622 bps	37 bps
speculative-grade firms	548 bps	655 bps		90 bps

Table 6: Equity Premium Firm-level monthly model-implied equity and bond premiums are first averaged every month within each rating class, then over the whole sample. The empirical measurement of equity excess returns corresponds to the cumulative annualized returns on Vanguard 500 Index Fund Investor Class (VFINX) relative to Vanguard Long-Term Treasury Fund Investor Shares (VUSTX).

& Poor's 500 Index. Since 1987, investors have earned annualized excess returns of 622 basis points on Vanguard 500 Index Fund Investor Class (VFINX) relative to Vanguard Long-Term Treasury Fund Investor Shares (VUSTX). Over the same period, the model predicts average equity premiums of 628 and 655 basis points for investment- and speculative-grade firms, respectively. The model predicts average bond premiums of 37 and 90 basis points for investment- and speculative-grade firms, respectively.

6 Structural Decomposition

In this section, I selectively shut down various features of the model to derive counterfactuals. I simulate the model under alternative specifications to provide a decomposition of economic channels and shocks to macroeconomic conditions that drive the financial indicators.

6.1 Economic Channels

A number of articles have studied the determinants of corporate bond credit spreads.[10] These papers aim to empirically identify what portion of credit spreads is directly

[10]See Duffie and Singleton (1997); Duffee (1999); Elton, Gruber, Agrawal, and Mann (2001); Collin-Dufresne, Goldstein, and Martin (2001); Longstaff, Mithal, and Neis (2005); Huang and Huang (2012); and many others.

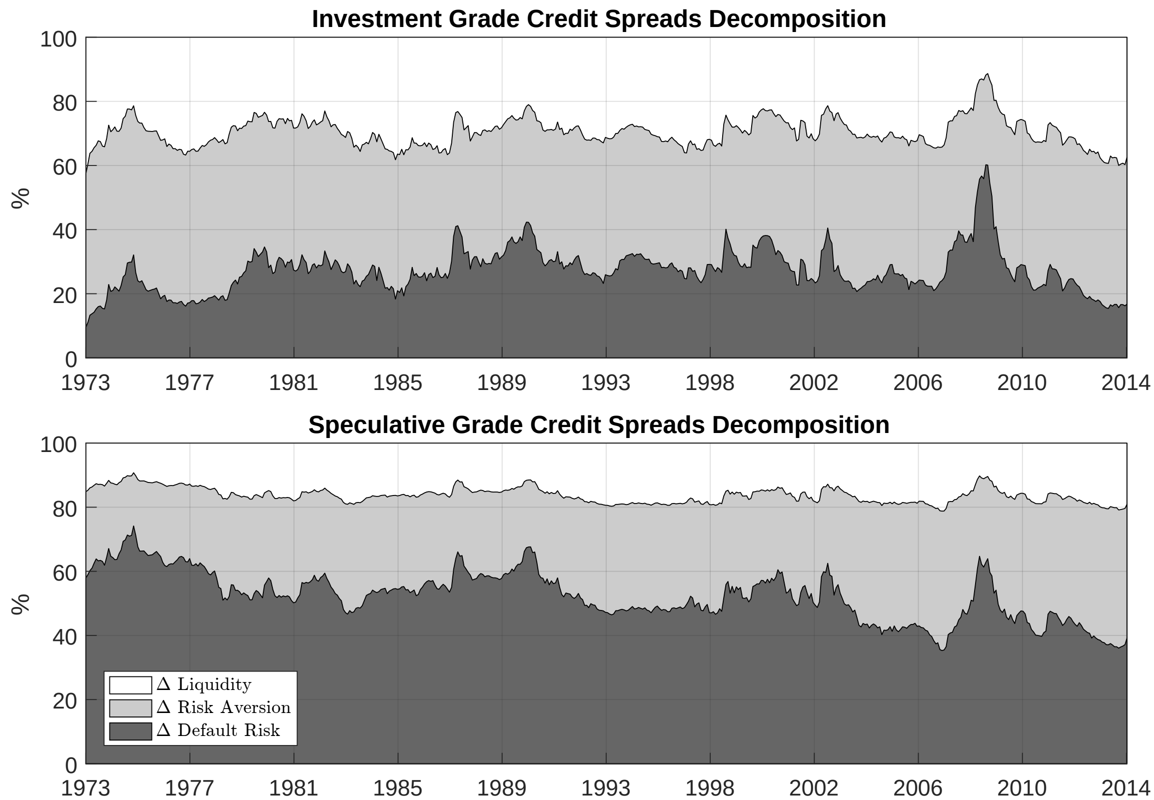

Figure 11: Credit Spreads Decomposition The shaded area Δ Default Risk corresponds to cs_{def}/cs. The shaded area Δ Risk Aversion corresponds to $(cs_\gamma - cs_{def})/cs$. The shaded area Δ Liquidity corresponds to $100 - cs_\gamma$.

attributable to default risk or nondefault factors such as a risk premium or liquidity. In the context of the structural model, I consider three potential sources of credit spreads: default risk, risk aversion, and liquidity. The default risk spread cs_{def} is the spread from the solution of the model with a risk-neutral representative agent and without over-the-counter liquidity frictions. The risk aversion spread cs_γ is the spread from the solution of the model with a risk averse agent and without over-the-counter liquidity frictions. The liquidity friction spread is given by $cs_{liq} = cs - cs_\gamma$ where cs is the credit spreads predicted by the full model. Thus, the default risk spread cs_{def} captures default risk; the risk aversion spread cs_γ captures default risk and risk aversion; and the full model captures default risk, risk aversion, and over-

	investment-grade	speculative-grade
Δcs_{liq}	30	16
Δcs_{γ}	43	32
Δcs_{def}	27	52

Table 7: Average Percentage Explained by Each Component Δcs_{def} is the default risk component, Δcs_{γ} is the risk aversion component, and Δcs_{liq} is the liquidity component.

the-counter liquidity frictions. The components' contribution to credit spreads are then derived by taking differences:

$$\Delta cs_{def} = \frac{cs_{def}}{cs}, \qquad \Delta cs_{\gamma} = \frac{cs_{\gamma} - cs_{def}}{cs}, \qquad \Delta cs_{liq} = 1 - \frac{cs_{\gamma}}{cs}.$$

For each alternative version of the model, I re-estimated the shocks from Section 4. Default probabilities depend mostly on leverage and firms' asset volatilities, which do not change significantly across versions of the model. Figure 11 shows the decomposition of credit spreads into default risk, risk aversion, and liquidity.

As illustrated in Table 7, liquidity frictions are more important for investment-grade firms than speculative-grade firms. The composition of investment-grade credit spreads is fairly stable over time, while the risk aversion component of speculative-grade credit spreads increased from 27% in 1973 to 42% in 2014. The growing contribution of risk aversion is consistent with Gilchrist and Zakrajšek's (2012) finding that the predictive content of the excess bond premium for economic activity over the 1985–2010 period is greater than that obtained for the full sample period. For both types of firms, the spike in credit spreads during the 2008–09 financial crisis was characterized by a surge in default risk, but not by the disruption in corporate bond liquidity.

Longstaff, Mithal, and Neis (2005) calculate that 53% (84%) of BB-rated (A-rated) bond credit spreads can be explained by credit risk between March 2001 and October 2002. They derive their estimates using credit default swap premiums, which include expected default losses plus the credit risk premium. Over the same period, I find that 72% (84%) of investment-grade (speculative-grade) bond credit spreads can be explained with the spread cs_{γ} that captures default risk and risk aversion.

Chen, Cui, He, and Milbradt (2016) find that the fraction of credit spreads that can be explained without liquidity frictions starts at only 20% for Aaa/Aa-rated bonds, and monotonically increases to about 67% for Ba-rated bonds for the period starting in January 1994 and ending in June 2012. In contrast, I find that 71% (83%) of credit spreads can be accounted for without liquidity friction for investment-grade (speculative-grade) bond credit spreads over the same period. An important factor in these differences resides in the measurement and inclusion of time-varying firms' asset volatilities to explain the levels of credit spreads and bid-ask spreads instead of time-varying parameters that drive over-the-counter liquidity frictions.

6.2 Aggregate Shocks to Firms' Asset Value

Before presenting the shocks decomposition, it is important to understand that without shocks to macroeconomic conditions, the model generates aggregate shocks to firms' asset values inconsistent with the model's assumptions. This can be shown by constructing a discrete approximation to aggregate shocks to firms' asset volatility dZ_t^A from equation (4). For a representative firm of type j, I retrieve model-implied levels of log cash flows y_t^j at time t following:

$$y_t^j = \frac{1}{N_t^j} \sum_{i \in I(j,t)} y_t^i,$$

where N_t^j is the amount of firms of type j at time t and $I(j,t)$ is the set of firms of type j at time t. The approximation to dZ_t^A is then given by:

$$\Delta Z_t^A = \frac{1}{J} \sum_{j=1}^{J} \frac{y_t^j - y_{t-1}^j - \mu_Y^j + 0.5\sigma_{Y,A}^j(s_t)}{\sigma_{Y,A}^j(s_t)},$$

where $\sigma_{Y,A}^j(s)$ is the aggregate asset volatility of firms of type j in state s, and μ_Y^j is the drift of asset values of firms of type j (see equations 2 and 4). According to the model, ΔZ_t^A should be identically and independently distributed according to a standard normal distribution. In Figure 12, two graphs represent histograms of

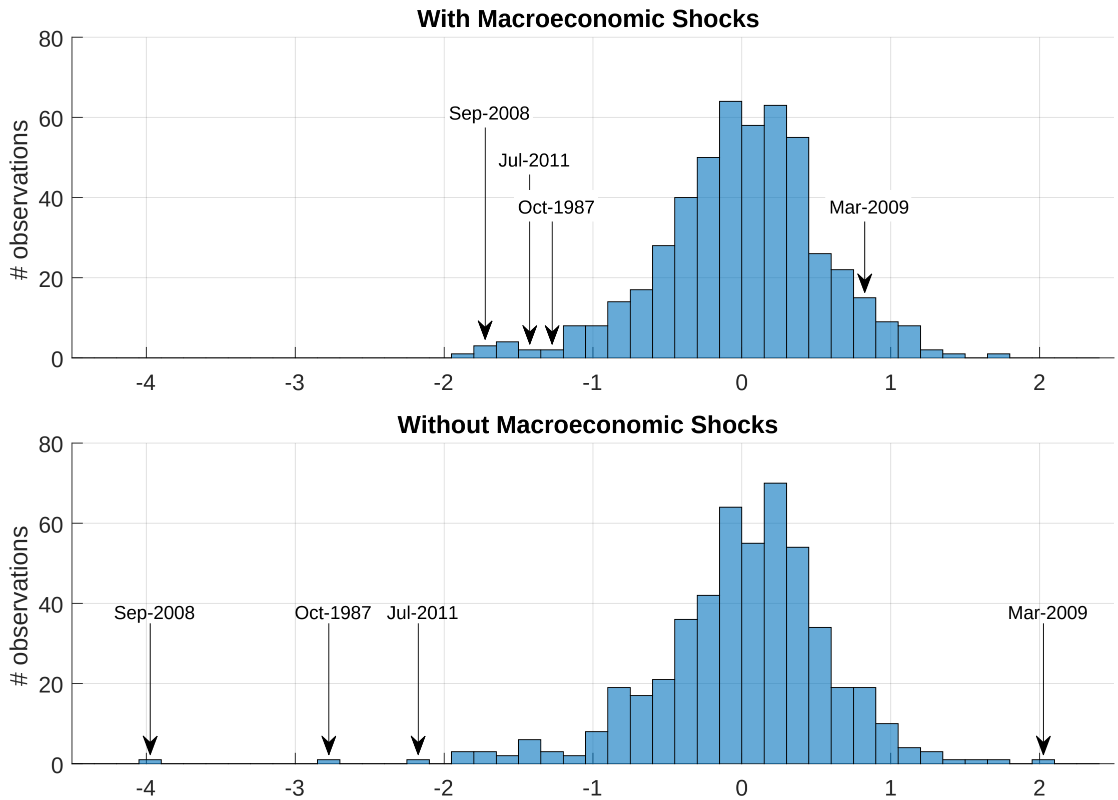

Figure 12: Histograms of Estimated Aggregate Shocks to Firms' Asset Values ΔZ_t^A This figure presents two histograms of estimated aggregate shocks to firms' asset values ΔZ_t^A to approximate dZ_t^A from equation (4). The "Without Macroeconomic Shocks" case sets $\alpha(s), \sigma_A(s), \sigma_I(s), \mu_C(s)$, and $\sigma_C(s)$ to their sample average, i.e., the aggregate shocks to firms' asset values dZ_t^A are estimated without shocks to default losses, firms' aggregate and idiosyncratic asset volatility, or the market price of risk. Annotations show the position of the aggregate shocks to firms' asset values in October 1987, September 2008, March 2009, and July 2011.

ΔZ_t^A estimated by two versions of the model: one version of the model with shocks to macroeconomic conditions (baseline) and another one without. The model without shocks to macroeconomic conditions assumes that default losses, firms' aggregate and idiosyncratic asset volatility, and the market price of risk are held constant at their sample average. The histogram for the model without macroeconomic shocks yields values that are highly improbable for key periods of macroeconomic fluctuations: October 1987, September 2008, March 2009, and July 2011. For example, the aggregate shock to asset values in September 2008 had a less than 0.0001 probability of occurring according to the model without macroeconomic shocks. The model cannot successfully account for large aggregate shocks to firms' equity values without shocks to macroeconomic conditions or assumptions about extremely large shocks to firms' asset values. Thus, shocks to macroeconomic conditions are necessary for the model to be consistent with observations of leverage over time.

6.3 Shocks to Macroeconomic Conditions

A recent theoretical and empirical research aimed at understanding the 2008–2009 financial crisis has pointed to financial and uncertainty shocks as main drivers of economic fluctuations. Stock and Watson (2012) and Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) emphasize the difficulty of empirically distinguishing these two types of shocks, because increases in aggregate equity volatility—a widely used proxy for macroeconomic uncertainty—are frequently associated with spikes in credit spreads—a widely used proxy for financial turmoil. In this subsection, I show that the joint dynamics of aggregate equity volatility and credit spreads is driven by shocks to firms' asset values and firms' aggregate asset volatility.

To illustrate the shocks decomposition strategy, I highlight the determinants of changes in credit spreads and equity volatility during the 2008–09 financial crisis in Figures 13 and 14. The cases $\alpha(s) = \overline{\alpha}$, $\sigma_I(s) = \overline{\sigma}_I$, $\sigma_A(s) = \overline{\sigma}_A$, and $\eta(s) = \overline{\eta}$, correspond to credit spreads' predictions of the model without shocks to default losses, firms' idiosyncratic asset volatility, firms' aggregate asset volatility, and the market price of risk, respectively; that is, their respective value is set at their sample average. The last case, $s = \overline{s}$, shows credit spreads' predictions without shocks to macroeconomic conditions, only fluctuations in firms' asset values. Notice that shutting down shocks have two effects on credit spreads' predictions. First, it diminishes the average level of credit spreads because the possibility of a potential increase in credit risk is attenuated. Second, it reduces the increase from 2007 to 2009 because realized risk during the crisis is lowered.

Figure 13 exhibits the results of this decomposition for credit spreads during

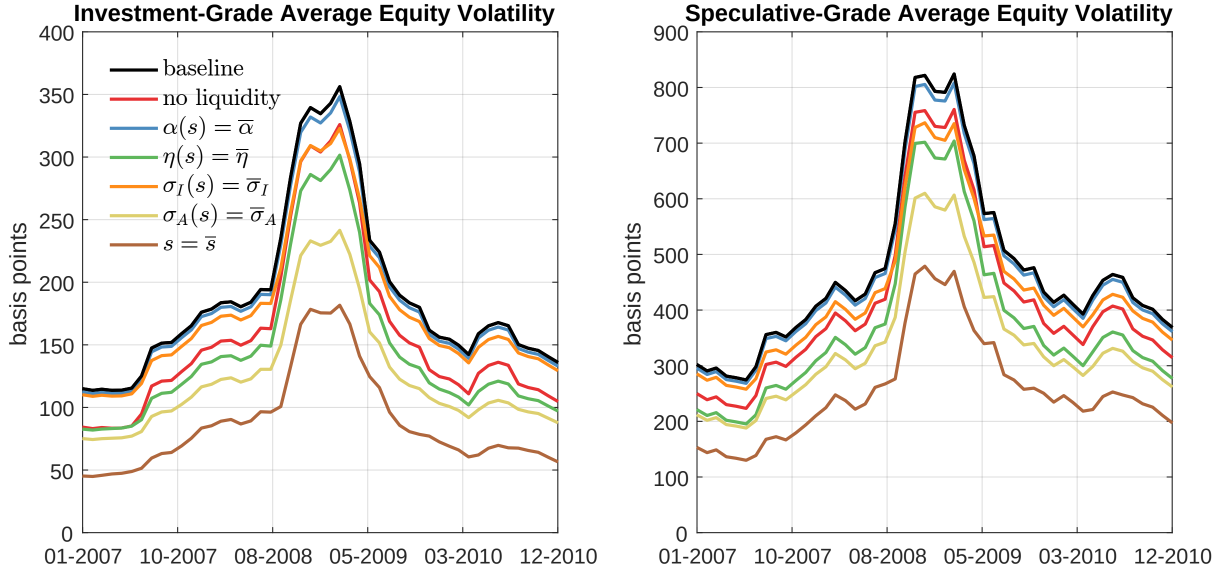

Figure 13: Shocks Decomposition for Credit Spreads during the Financial Crisis Baseline corresponds to credit spreads' predictions of the full model. $\alpha(s) = \overline{\alpha}$, $\sigma_I(s) = \overline{\sigma}_I$, $\sigma_A(s) = \overline{\sigma}_A$, and $\eta(s) = \overline{\eta}$ correspond to the cases without shocks to default losses, firms' idiosyncratic and aggregate asset volatility, and the market price of risk, respectively. $s = \overline{s}$ shows credit spreads' prediction without shocks to macroeconomic conditions, only fluctuations in firms' asset values.

the Great Recession. The blue line hints that fluctuations in default losses are too small to account for large credit spreads' variations. Liquidity frictions account for about 15% of the spike in credit spreads. The increase in firms' aggregate asset volatility dominates the other shocks. While firms' idiosyncratic asset volatility also increased, it had little impact on credit spreads. Two mechanisms are involved when the idiosyncratic asset volatility of a firm fluctuates. First, shareholders' equity in a levered company can be seen as a call option granted by creditors to shareholders on the firm's assets. Therefore, as the firm's idiosyncratic asset volatility increases, conditional on the firm's asset level, so does the value of the call option. In fact, as the firm's idiosyncratic asset volatility increases, shareholders are willing to sustain lower asset returns before declaring bankruptcy. Second, when the firm's idiosyncratic asset volatility increases, the probability of hitting a given bankruptcy level also increases. The latter effect attenuates the former, such that default risk is not impacted much by shocks to the firm's idiosyncratic volatility. However, the firm's aggregate asset volatility is adversely priced by the representative investor. Thus, when the firm's aggregate volatility increases, the value of equity decreases, shareholders declare bankruptcy earlier, and default risk increases. Furthermore, the representative investor requires a higher compensation for bearing more aggregate risk, which impacts credit spreads even more. This effect also explains the importance of shocks to the market price of risk, which interacts with firms' aggregate asset volatility.

A second disparity explains the difference between the impact on credit spreads of shocks to firms' aggregate asset volatility and shocks to firms' idiosyncratic asset volatility. The jump in firms' aggregate asset volatility, as measured in Section 4, was quantitatively unprecedented[11] during the Great Recession. The estimate of investment-grade (speculative-grade) firms' aggregate volatility increased from 0.07 (0.08) in June 2007 to 0.27 (0.29) in December 2009. In comparison, the estimate of investment-grade (speculative-grade) firms' idiosyncratic volatility increased from 0.11 (0.15) in June 2007 to 0.24 (0.32) in December 2009.

As shown in Figure 13, shocks to firms' aggregate asset predominate to explain the spike in equity volatility during the Great Recession. The leverage effect only explains about a third of the increase in volatility. This result is reminiscent of Schwert's (1989) finding that leverage alone cannot explain the historical movements in equity volatility during the Great Depression.

Table 8 shows the ratios of RMSE(no shocks x) to RMSE(no shocks), a measure of the relative importance of each shock to explain fluctuations in the financial indicators. RMSE(no shocks x) is root mean squared errors between the full model and the model without shocks to x, where x is either firms' asset values, default losses, aggregate asset volatility, idiosyncratic asset volatility, or the market price of risk. The "no liquidity" case corresponds to the model predictions without liquidity frictions. RMSE(no shocks) is root mean squared errors between the full model and

[11]The surge in total equity volatility during the Great Recession is only comparable to the surge in total equity volatility during the Great Depression according to the measurement of Atkeson, Eisfeldt, and Weill (2013).

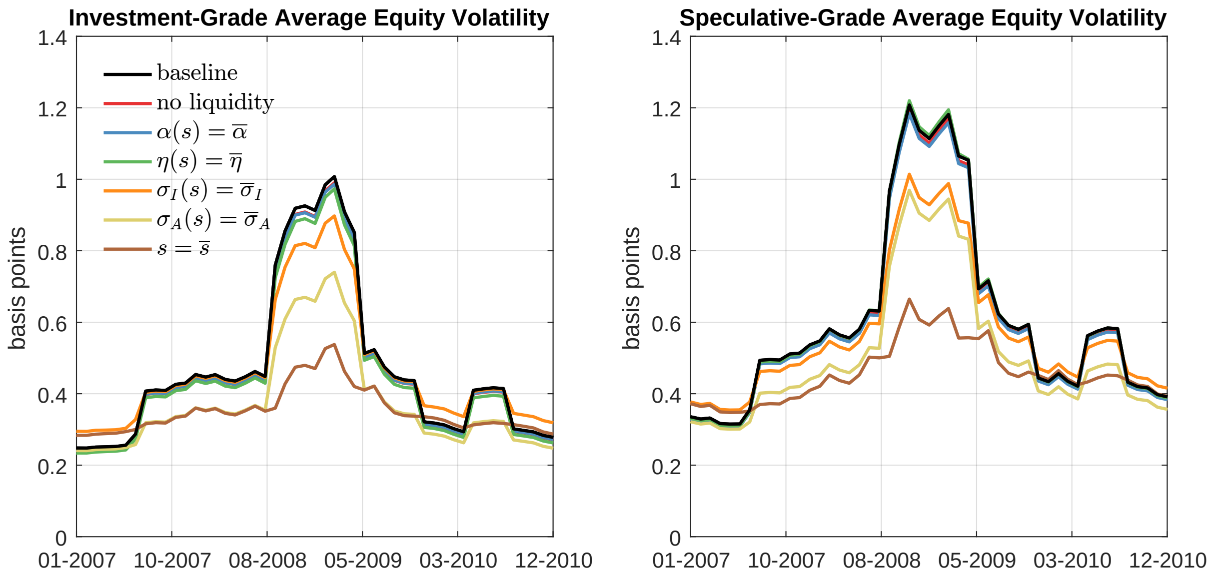

Figure 14: Shocks Decomposition for Equity Volatility during the Financial Crisis Baseline corresponds to credit spreads' predictions of the full model. $\alpha(s) = \overline{\alpha}$, $\sigma_I(s) = \overline{\sigma}_I$, $\sigma_A(s) = \overline{\sigma}_A$, and $\eta(s) = \overline{\eta}$ correspond to the cases without shocks to default losses, firms' idiosyncratic and aggregate asset volatility, and the market price of risk, respectively. $s = \overline{s}$ shows credit spreads' prediction without shocks to macroeconomic conditions, only fluctuations in firms' asset values.

the model without any macroeconomic shocks.

Overall, this relative measure shows that during the Great Recession, fluctuations in default risk, aggregate equity volatility, and credit spreads are mostly accounted for by shocks to firms' asset values and firms' aggregate asset volatility. Bid-ask spreads are driven by shocks to the market price of risk, firms' aggregate asset volatility, and firms' asset values. The same conclusions hold for the period from 1973 to 2014 (see Appendix G, Table 18).

		$\sigma_{E,A}$ inv	$\sigma_{E,A}$ spe	$\sigma_{E,I}$ inv	$\sigma_{E,I}$ spe	credit inv	credit spe	bid-ask inv	bid-ask spe	default all
SHOCKS	no liquidity	2	0	4	1	31	26	**100**	**100**	3
	$\alpha(s) = \overline{\alpha}$	0	0	1	0	2	1	0	0	1
	$\eta(s) = \overline{\eta}_C$	7	3	16	7	41	44	13	24	5
	$\sigma_A(s) = \overline{\sigma}_A$	**101**	**103**	18	25	**55**	**55**	17	**26**	**51**
	$\sigma_I(s) = \overline{\sigma}_I$	6	7	**113**	**118**	13	16	4	6	36
	$y_t = \overline{y}$	**55**	**33**	**117**	**74**	**62**	**47**	**20**	15	**63**

(Column group heading: FINANCIAL INDICATORS)

Table 8: Shock Decomposition of Financial Indicators during the Financial Crisis This table shows the ratios of root mean squared errors (RMSE) of different versions of the model multiplied by 100. The numerator corresponds to the RMSE between the full model and the model without one of the shocks. The denominator corresponds to the RMSE between the full model and the model without any shocks. The sample period is restricted to be between January 2007 and January 2011. No liquidity corresponds to credit spreads' predictions without any liquidity frictions. $\alpha(s) = \overline{\alpha}$, $\eta(s) = \overline{\eta}$, $\sigma_A(s) = \overline{\sigma}_A$, $\sigma_I(s) = \overline{\sigma}_I$, and $y_t = \overline{y}$ correspond to the cases without shocks to default losses, market price of risk, aggregate asset volatility, idiosyncratic asset volatility, or firms' asset values, respectively. The financial indicators $\sigma_{E,A}$, $\sigma_{E,A}$, credit, bid-ask, and default correspond to aggregate equity volatility, idiosyncratic equity volatility, credit spreads, bid-ask spreads and default risk. The category inv (spe) corresponds to investment-grade (speculative-grade) firms. Bold numbers highlight the two most important shocks for each financial indicator. The same table for the whole sample period is available in Appendix G, Table 18.

7 Forecasting Economic Activity

To assess the predictive ability of different factors for economic activity, I use the following univariate forecasting specification:

$$\Delta^h Y_{t+h} = \alpha + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + \gamma_1 FED_t + \gamma_2 INFL_t + \gamma_3 TSLO_t + \gamma_4 X_t + \varepsilon^Y_{t+h}, \quad (15)$$

where

$$\Delta^h Y_{t+h} \equiv \frac{c}{h+1} \ln\left(\frac{Y_{t+h}}{Y_{t-1}}\right),$$

$h \geq 0$ is the forecast horizon, and c is a scaling constant that depends on the frequency of the data (i.e., $c = 1,200$ for monthly data and $c = 400$ for quarterly data). In the

Real GDP Growth 4-Quarters Forecast Horizon

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
FED	0.260**	-0.045	-0.096***	0.039	-0.110	-0.119	0.016
TSLO	0.491***	0.405***	0.938***	0.412***	0.349***	0.331***	0.396***
GZ		-0.510***				-0.263**	
EBP			-0.336***				-0.144*
sig_{INV}				-0.356***			
sig_{SPE}					-0.508***	-0.339***	-0.437***
Adj. R^2	0.17	0.32	0.26	0.26	0.34	0.36	0.35

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 9: 164 quarterly observations from 1973:Q1 to 2014:Q4. The dependent variable is $\Delta^4 Y_{t+4}$, where Y_t denotes real GDP in quarter t and h is the forecast horizon. In addition to the specified financial indicator in month t, each specification also includes a constant and p lags of ΔY_t, where p is determined by the AIC. Entries in the table denote the standardized estimates of the OLS coefficients associated with each financial indicator. FED is the federal funds rate, TSLO the slope of the Treasury yield curve, GZ the GZ credit spread, EBP the excess bond premium, sig_{INV} the aggregate volatility estimate for investment-grade firms, and sig_{SPE} the aggregate volatility estimate for speculative-grade firms.

forecasting regression (15), FED_t denotes the federal funds rate; $INFL_t$ denotes the inflation rate from the personal consumption expenditure price index; $TSLO_t$ denotes the slope of the Treasury yield curve, defined as the difference between the three-month constant-maturity Treasury yield and the 10-year constant-maturity yield; X_t denotes a measure of economic uncertainty; and ε^Y_{t+h} is the forecast error. Thus, I examine the marginal information content of different measures of economic uncertainty conditional on key indicators of the stance on monetary policy. The forecasting regressions are estimated by ordinary least squares, with the lag length p of each specification determined by the Akaike Information Criterion. The structure of the error term ε^Y_{t+h} induced by overlapping observations is taken into account by computing standard errors according to Hansen and Hodrick (1980).

Gilchrist and Zakrajšek (2012) show that the predictive ability of corporate bond credit spreads for future economic activity significantly exceeds that of the other widely used default-risk indicators such as the standard Baa-Aaa corporate bond credit spread and the paper-bill spread. Therefore, I compare the performance of firms' aggregate asset volatility with the GZ credit spread, which is the monthly average of corporate bond credit spreads, and the excess bond premium, which captures the cyclical changes in the relationship between measured default risk and credit spreads (see Gilchrist and Zakrajšek, 2012, for details).

A model-free approximation to the estimated aggregate volatility time series from Section 4 is given by:

$$\log\left(\mathrm{sig}_t^j\right) = \frac{1}{N_{jt}} \sum_{i \in I(j,t)} \left[\log(1 - \mathrm{lev}_{it}) + \log\left(v_{E,A}^i(t)\right)\right],$$

where N_{jt} is the amount of firms of type j at time t and $I(j,t)$ is the set of firms of type j at time t. Market leverage lev_{it} and equity return aggregate volatility $v_{E,A}^i(t)$ are described in Section 3. This approximation is more convenient to construct from a panel dataset of equity and leverage, captures accurately the levels and fluctuations in the estimated aggregate asset volatility, and does not significantly alter the forecasting results. Derivation of this approximation is provided in Appendix F. Thus, recovering an accurate measure of aggregate asset volatility does not depend on departures from Modigliani and Miller's (1958) assumptions. This provides an important guidance for future research. The predictions of a mechanism that relates fluctuations in aggregate asset volatility to future economic activity should satisfy this regularity.

The second and third columns of Table 9 replicate the forecasting regressions of Gilchrist and Zakrajšek (2012) for the GZ credit spread and the excess bond premium, which have considerable predictive power for economic activity over the 1973–2014 period.

As indicated in the fourth and fifth columns of Table 9, firms' aggregate asset volatility is both economically and statistically a highly significant predictor of output growth at the year-ahead forecast horizon over the full sample period. Firms' aggregate asset volatility of speculative-grade firms is economically a stronger predictor of output growth than firms' aggregate asset volatility of investment-grade firms. The coefficient estimate implies that an increase in the investment-grade (speculative-grade) firms' aggregate asset volatility of 10 volatility points in quarter t leads to a drop in real GDP growth of more than 2 (2.5) percentage points over the subsequent 4 quarters.

The sixth and seventh columns show that the GZ credit spread and the excess bond premium do not contain significant independent explanatory power on top of firms' aggregate asset volatility to predict output growth. In other words, the informational content of corporate bond credit spreads for predicting output growth is captured by fluctuations in firms' aggregate asset volatility. For completeness, I replicated the forecast regressions of Gilchrist and Zakrajšek (2012) with payroll employment, unemployment rate, and industrial production at 3- and 12-month forecast horizons. Overall, firms' aggregate asset volatility does not outperform the GZ credit spread for these components but captures at least 60% of the credit spreads' forecasting power for all of these indicators of real economic activity.[12] Aggregate asset volatility significantly outperforms aggregate equity volatility to predict economic activity.

8 Conclusion

In this paper, I show that a structural model estimated with a large firm-level database can successfully account for the joint dynamics of important financial indicators of uncertainty and financial distress. I uncover that fluctuations in firms' asset values and firms' aggregate asset volatility are the main drivers of these financial indicators. According to the structure of the model, it occurs because firms' aggregate

[12]The percentage of credit spreads' forecasting power captured by aggregate volatility is defined as:

$$\frac{R^2_{SIG} - R^2_0}{R^2_{GZ} - R^2_0}.$$

R^2_{SIG} is the adjusted goodness of fit of a regression with FED and TSLO. R^2_{SIG} is the adjusted goodness of fit of a regression with FED, TSLO, and sig_{INV} or sig_{SPE}. R^2_{GZ} is the adjusted goodness of fit of a regression with FED, TSLO, and GZ.

volatility greatly influences the quantity of credit risk priced by the representative investor. Furthermore, the informational content of these financial indicators for predicting real economic activity can be captured by fluctuations in firms' aggregate asset volatility. All together, my results suggest that fluctuations in aggregate firms' asset volatility are key for the transmission channel that links the fundamental drivers of financial indicators to the real economy. Further research should aim at understanding the links between fundamental shocks, financial indicators, and macroeconomic aggregates in a general equilibrium framework.

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Appendices

A The Marginal Investor Stochastic Discount Factor

To obtain the stochastic discount factor, I first solve for the value function of the representative household as

$$J(C_t, s_t) = \mathbb{E}_t \left[\int_0^\infty f(C_{t+s}, J_{t+s}) ds \right],$$

where the function $f(c, v)$ is a normalized aggregator of consumption and continuation value in each period defined as

$$f(C, J) = \frac{\rho}{1 - 1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)J)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)J)^{\frac{1-1/\psi}{1-\gamma}-1}}.$$

The consumption process of the marginal investor follows:

$$\frac{dC_t}{C_t} = \mu_C dt + \sigma_C(s) dZ_t^A.$$

The Hamilton-Jacoby-Bellman equation in state s (for $s = 1, \ldots, S$) is

$$0 = f(C, J(C, s)) + J_C(C, s) C \mu_C + \frac{1}{2} J_{CC}(C, s) C^2 \sigma_C^2(s) + \sum_{s' \neq s} \zeta_{ss'}^{\mathcal{P}} (J(C, s') - J(C, s)).$$

Conjecture that the solution for J is

$$J(C, s) = \frac{(h(s)C)^{1-\gamma}}{1-\gamma}.$$

Note that

$$
\begin{aligned}
J_C(C,s) &= (h(s)C)^{-\gamma}\,,\\
J_{CC}(C,s) &= -\gamma\,(h(s)C)^{-\gamma-1}\,.
\end{aligned}
$$

Substituting J into the differential equations above, we get the following system of nonlinear equations for h:

$$
\begin{aligned}
0 = {}& \frac{\rho}{1-1/\psi}\left(h(s)^{1/\psi-\gamma} - h(s)^{1-\gamma}\right)C^{1-\gamma} + h(s)^{-\gamma}C^{1-\gamma}\mu_C - \frac{1}{2}\gamma\,(h(s)C)^{1-\gamma}\,\sigma_C^2(s)\\
&+ \sum_{s'\neq s}\zeta_{ss'}^{\mathcal{P}}\left(\frac{(h(s')C)^{1-\gamma}}{1-\gamma} - \frac{(h(s)C)^{1-\gamma}}{1-\gamma}\right).
\end{aligned}
$$

This can be simplified as

$$
\begin{aligned}
0 = {}& \rho\frac{1-\gamma}{1-1/\psi}h(s)^{1/\psi-\gamma} + \left[(1-\gamma)\mu_C - \frac{1}{2}\gamma(1-\gamma)\sigma_C^2(s) - \rho\frac{1-\gamma}{1-1/\psi}\right]h(s)^{1-\gamma}\\
&+ \sum_{s'\neq s}\zeta_{ss'}^{\mathcal{P}}\left(h(s')^{1-\gamma} - h(s)^{1-\gamma}\right).
\end{aligned}
$$

While no algebraic solution exists for this system of nonlinear equations, it is trivial to solve numerically for $h(s)$ $\forall s \in \{1,\ldots,S\}$. Duffie and Skiadas (1994) show that the stochastic discount factor is equal to

$$
\Lambda_t = \exp\left(\int_0^t f_J(C_u, J_u)du\right) f_C(C_t, J_t).
$$

Thus we have

$$
\Lambda_t = \exp\left(\int_0^t \frac{\rho(1-\gamma)}{1-1/\psi}\left(\left(\frac{1/\psi-\gamma}{1-\gamma}\right)h(s_u)^{1/\psi-1} - 1\right)du\right)\rho H(s)^{1/\psi-\gamma}C^{-\gamma}.
$$

Applying Ito's formula with jumps, we get

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s)dt - \eta(s)dZ_t^M + \sum_{s' \neq s} \left(e^{\kappa(s,s')} - 1 \right) dM_t^{s,s'},$$

where

$$\begin{aligned} r(s) &= -\frac{\rho(1-\gamma)}{1-1/\psi} \left[\frac{1/\psi - \gamma}{1-\gamma} h(s)^{1/\psi - 1} - 1 \right] + \gamma \mu_C(s) - \frac{1}{2}\gamma(1+\gamma)\sigma_C^2(s) - \sum_{s' \neq s} (e^{\kappa(s,s')} - 1) \\ \eta(s) &= \gamma \sigma_C(s) \\ \kappa(s, s') &= (1/\psi - \gamma) \log \left(\frac{h(j)}{h(i)} \right). \end{aligned}$$

B Solution to Bond and Equity Valuation

The solution method presented in this appendix is based on Chen, Cui, He, and Milbradt (2016).

Notation for Matrix Formulation I follow the Markov-modulated dynamics approach of Jobert and Rogers (2006). Define y to be the log of the level of cash flows given by:

$$dy = \mu_s dt + \sigma_s dZ_t$$

where Z_t is a Brownian motion. The firm go bankrupt if y becomes lower than given bankruptcy boundaries in each state. There are multiple possible bankruptcy boundaries, yb_s , for each aggregate state s. Order states s such that $s > s'$ implies that $yb_s > yb_{s'}$ and denote the intervals $I_s = [yb_s, yb_{s+1}]$ where $yb_{S+1} = \infty$, so that $I_s \cap I_{s+1} = yb_{s+1}$. Let $\mathbf{yb} = [yb_1, ..., yb_s]^\intercal$ be the vector of bankruptcy boundaries. Let's use the following notation for the solution of debt functions within each interval: $D^j_{s,i} \equiv D^j_s(y)$, $y \in I_i$, that is $D^j_{s,i}$ is the restriction of $D^j_s(y)$ to the interval I_i. Thus, $D^j_{s,i} =$ recovery value for any $i < s$, as it would imply that the company immediately defaults in interval I_i for state s. Let's stack the alive functions along states s but still restricted to interval i so that $\mathbf{D}_i = [D^H_{1,i}, D^L_{1,i}, D^H_{2,i}, D^L_{2,i}, \ldots, D^H_{i,i}, D^L_{i,i}]^\intercal$. Let $\mathbf{I}_i$ be the i-dimensional identity matrix, and let be $\mathbf{1}_i$ a column vector of ones of length i. Let's define matrices of parameters:

$$\mathbf{R}_i = \begin{bmatrix} r_1 + m + \varphi + \zeta_1 & 0 & \cdots & -\zeta_{1,i} & 0 \\ 0 & r_1 + m + \varphi + \zeta_1 & \cdots & 0 & -\zeta_{1,i} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\zeta_{i,1} & 0 & \cdots & r_i + m + \varphi + \zeta_i & 0 \\ 0 & -\zeta_{i,1} & \cdots & 0 & r_i + m + \varphi + \zeta_i \end{bmatrix},$$

where $\zeta_s = \sum_{s' \neq s} \zeta_{s,s'}$

$$\mathbf{Q}_i = \begin{bmatrix} \xi_1^H & -\xi_1^H & \cdots & 0 & 0 \\ -\xi_1^L - \chi_1(1+\beta)/2 & \xi_1^L - \chi_1(1-\beta)/2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \xi_i^H & -\xi_i^H \\ 0 & 0 & \cdots & -\xi_i^L - \chi_i w^H & \xi_i^L - \chi_i w^L \end{bmatrix},$$

$$\mathbf{G}_i = \begin{bmatrix} \zeta_{1,i+1} & 0 & \cdots & \zeta_{1,S} & 0 \\ 0 & \zeta_{1,i+1} & \cdots & 0 & \zeta_{1,S} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \zeta_{i,i+1} & 0 & \cdots & \zeta_{i,S} & 0 \\ 0 & \zeta_{i,i+1} & \cdots & 0 & \zeta_{i,S} \end{bmatrix}, \qquad \mathbf{b}_i = \begin{bmatrix} \alpha_{i+1}^H v_{i+1} \\ \alpha_{i+1}^H v_{i+1} \\ \vdots \\ \alpha_S^H v_S \\ \alpha_S^L v_S \end{bmatrix},$$

$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_S \end{bmatrix} = \begin{bmatrix} r_1 - \mu_1 + \zeta_1 & -\zeta_{1,2} & \cdots & -\zeta_{1,i} \\ -\zeta_{2,1} & r_2 - \mu_2 + \zeta_2 & \cdots & -\zeta_{2,i} \\ \vdots & \vdots & \ddots & \vdots \\ -\zeta_{i,1} & -\zeta_{i,2} & \cdots & r_i - \mu_i + \zeta_i \end{bmatrix}^{-1} \mathbf{1}_S,$$

$\mathbf{N}_i = [0, N, \cdots, 0, N]^{\mathsf{T}}$, $\boldsymbol{\chi}_i = \operatorname{diag}([\chi_1, \chi_1, \cdots, \chi_i, \chi_i])$, $\boldsymbol{\Sigma}_i = \operatorname{diag}([\sigma_1^2, \sigma_1^2, \cdots, \sigma_i^2, \sigma_i^2])$, and $\boldsymbol{\mu}_i = \operatorname{diag}([\mu_1, \mu_1, \cdots, \mu_i, \mu_i])$.

B.1 Debt Value

Debt valuation follows the following differential equation on interval I_i:

$$(\mathbf{R}_i + \mathbf{Q}_i)\mathbf{D}_i(y) = \boldsymbol{\mu}_i \mathbf{D}_i'(y) + 0.5\boldsymbol{\Sigma}_i \mathbf{D}_i''(y) + (c + mp + \varphi p)\mathbf{1}_{2i} - \boldsymbol{\chi}_i \mathbf{N}_i + 1\{i < S\}\mathbf{G}_i \mathbf{b}_i \exp(y)$$

where $\mathbf{G}_i \mathbf{V}_i \exp(y)$ represents the intensity of jumping into default times the recovery in the default state. We can rewrite the problem as a linear system of differential equations as

$$\mathbf{x}'(y) = \begin{bmatrix} \mathbf{D}_i''(y) \\ \mathbf{D}_i'(y) \end{bmatrix} = \begin{bmatrix} -2\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i & 2\boldsymbol{\Sigma}_i^{-1}(\mathbf{R}_i + \mathbf{Q}_i) \\ \mathbf{I}_{2i} & \mathbf{0}_{2i} \end{bmatrix} \begin{bmatrix} \mathbf{D}_i'(y) \\ \mathbf{D}_i(y) \end{bmatrix} + \mathbf{f}_i(y) = \mathbf{A}_i\mathbf{x}_i(y) + \mathbf{f}_i(y)$$

where

$$\mathbf{f}_i(y) = \begin{bmatrix} -(c + mp + \varphi p)2\boldsymbol{\Sigma}_i^{-1}\mathbf{1}_{2i} + 2\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\chi}_i\mathbf{N}_i - 1\{i < S\}2\boldsymbol{\Sigma}_i^{-1}\mathbf{G}_i\mathbf{b}_i\exp(y) \\ \mathbf{0}_{2i} \end{bmatrix}.$$

We know that the general solution $\mathbf{x}_i(y)$ is given by the expression

$$\mathbf{x}_i(y) = \mathbf{x}_i^h(y) + \mathbf{x}_i^p(y).$$

The term $\mathbf{x}_i^h(y)$ is the general solution of the homogeneous equation $\mathbf{x}_i'(y) = \mathbf{A}_i\mathbf{x}_i(y)$ in which are to be found $4i$ arbitrary constants $c_1, \ldots, c_{4i}$. The term $\mathbf{x}_i^p(y)$ is a particular solution of $\mathbf{x}_i'(y) = \mathbf{A}\mathbf{x}_i(y) + \mathbf{f}_i(y)$. The general solution of the linear system $\mathbf{x}_i'(y) = \mathbf{A}\mathbf{x}_i(y)$ is given by

$$\mathbf{x}_i(y) = c_i^1\mathbf{h}_i^1 e^{\lambda_i^1 y} + \ldots + c_i^{4i}\mathbf{h}_i^{4i} e^{\lambda_i^{4i} y} = \mathbf{H}_i\boldsymbol{\Lambda}_i(y)\mathbf{c}_i.$$

where $(\lambda_i^1, \mathbf{h}_i^1)$, ..., $(\lambda_i^{4i}, \mathbf{h}_i^{4i})$ are the eigenpairs of $\mathbf{A}_i$ with $4i$ independent eigenvectors, and

$$\begin{aligned} \mathbf{H}_i &= [\mathbf{h}_i^1, \ldots, \mathbf{h}_i^{4i}], \\ \mathbf{c}_i &= [c_i^1, \ldots, c_i^{4i}]^\top, \\ \boldsymbol{\Lambda}_i(y) &= \operatorname{diag}\left(\left[e^{\lambda_i^1 y}, \ldots, e^{\lambda_i^{4i} y}\right]\right). \end{aligned}$$

We can guess a solution of the kind $\mathbf{k}_i^0 + \mathbf{k}_i^1\exp(y)$ for the particular solution $\mathbf{x}_{i[1:2i]}^p(y)$ where $\mathbf{x}_{[1:2i]}$ selects the first $2i$ rows of vector $\mathbf{x}$. Therefore, we can solve for the coefficients by substituting in:

$$
\begin{aligned}
(\mathbf{R}_i + \mathbf{Q}_i)\left(\mathbf{k}_i^0 + \mathbf{k}_i^1 \exp(y)\right) &= \boldsymbol{\mu}_i \mathbf{k}_i^1 \exp(y) + 0.5\boldsymbol{\Sigma}_i \mathbf{k}_i^1 \exp(y) \\
&\quad + (c + mp + \varphi p)\mathbf{1}_{2i} - \boldsymbol{\chi}_i \mathbf{N}_i + 1\{i < S\}\mathbf{G}_i \mathbf{b}_i \exp(y),
\end{aligned}
$$

and solve for any value of y. This gives us

$$
\begin{aligned}
\mathbf{k}_i^0 &= (\mathbf{R}_i + \mathbf{Q}_i)^{-1}\left((c + mp + \varphi p)\mathbf{1}_{2i} - \boldsymbol{\chi}_i \mathbf{N}_i\right), \\
\mathbf{k}_i^1 &= (\mathbf{R}_i + \mathbf{Q}_i - \boldsymbol{\mu}_i - 0.5\boldsymbol{\Sigma}_i)^{-1} 1\{i < S\}\mathbf{G}_i \mathbf{b}_i.
\end{aligned}
$$

Thus we have

$$
\mathbf{D}_i(y) = \widetilde{\mathbf{H}}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i + \mathbf{k}_i^0 + \mathbf{k}_i^1 \exp(y)
$$

where $\widetilde{\mathbf{H}}_i = \mathbf{H}_{i[1:2i]}$ and $\mathbf{X}_{[1:2i]}$ selects the first $2i$ rows of matrix $\mathbf{X}$, and

$$
\mathbf{D}_i'(y) = \widetilde{\mathbf{H}}_i \boldsymbol{\lambda}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i + \mathbf{k}_i^1 \exp(y).
$$

where

$$
\boldsymbol{\lambda}_i = \operatorname{diag}\left(\left[\lambda_i^1, \ldots, \lambda_i^{4i}\right]\right).
$$

Boundary Conditions The different value functions $\mathbf{D}_i$ for $i \in \{1, \ldots, S\}$ are linked at the boundaries of their domains I_i. Note that $I_i \cap I_{i+1} = yb_{i+1}$ for $i < S$.

For $i = S$, we can immediately rule out all positive solutions to λ_i as debt has to be finite and bounded as $y \to \infty$, so that the entries of $\mathbf{c}_i$ corresponding to positive eigenvalues will be zero:

$$
\lim_{y \to \infty} |\mathbf{D}_S(y)| < \infty.
$$

For $i < S$, we must have value matching of the value functions that are alive across the boundary, and we must have value matching of the value functions that die across the boundary:

$$\mathbf{D}_{i+1}(yb_{i+1}) = \begin{bmatrix} \mathbf{D}_i(yb_{i+1}) \\ \mathbf{d}_{i+1} \exp(yb_{i+1}) \end{bmatrix},$$

where

$$\mathbf{d}_{i+1} = \begin{bmatrix} \alpha_{i+1}^H v_{i+1} \\ \alpha_{i+1}^L v_{i+1} \end{bmatrix}.$$

For $i < S$, we must have smooth pasting of the value functions that are alive across the boundary:

$$\mathbf{D}'_{i+1}(yb_{i+1})_{[1:2i]} = \mathbf{D}'_i(yb_{i+1})$$

where $\mathbf{x}_{[1:2i]}$ selects the first $2i$ rows of vector $\mathbf{x}$. Lastly, for $i = 1$, we must have

$$\mathbf{D}_1(yb_1) = \mathbf{d}_1 \exp(yb_1).$$

Therefore, given $\mathbf{yb}$ the system of $2S^2 + 2S$ equations to solve for $\mathbf{c}_i$ $\forall i \in 1, \ldots, S$ can be given by

$$\widetilde{\mathbf{H}}_{i[2i-1:2i]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \mathbf{k}^0_{i[2i-1:2i]} + \mathbf{k}^1_{i[2i-1:2i]} \exp(yb_i) = \mathbf{d}_i \exp(yb_i) \qquad \text{for } i = 1, \ldots, S$$

$$\begin{aligned} \widetilde{\mathbf{H}}_{i[1:2i-2]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \mathbf{k}^0_{i[1:2i-2]} + \mathbf{k}^1_{i+1[1:2i-2]} \exp(yb_i) &= \widetilde{\mathbf{H}}_{i-1} \mathbf{\Lambda}_{i-1}(yb_i)\mathbf{c}_{i-1} + \mathbf{k}^0_{i-1} + \mathbf{k}^1_{i-1} \exp(yb_i) \\ \widetilde{\mathbf{H}}_{i[1:2i-2]} \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \mathbf{k}^1_{i[1:2i-2]} \exp(yb_i) &= \widetilde{\mathbf{H}}_{i-1} \boldsymbol{\lambda}_{i-1} \mathbf{\Lambda}_{i-1}(yb_i)\mathbf{c}_{i-1} + \mathbf{k}^1_{i-1} \exp(yb_i) \end{aligned}$$

for $i = 2, \ldots, S$, and

$$\lim_{y \to \infty} |\mathbf{D}_S(y)| < \infty.$$

We can define squared matrices $\mathbf{M}_i$, $\widetilde{\mathbf{M}}_i$, $\overline{\mathbf{M}}_i$, $\mathbf{K}_i$, $\widetilde{\mathbf{K}}_i$, and $\bar{\mathbf{K}}_i$ as

$$\mathbf{M}_i(y) = \begin{bmatrix} \widetilde{\mathbf{H}}_i \mathbf{\Lambda}_i(y) \\ \widetilde{\mathbf{H}}_i \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(y) \end{bmatrix}, \qquad \mathbf{K}_i(y) = \begin{bmatrix} \mathbf{k}_i^0 + \mathbf{k}_i^1 \exp(y) \\ \mathbf{k}_i^1 \exp(y) \end{bmatrix},$$

$$\widetilde{\mathbf{M}}_i(y) = \begin{bmatrix} \widetilde{\mathbf{H}}_{i[1:2i-2]} \mathbf{\Lambda}_i(y) \\ \widetilde{\mathbf{H}}_{i[1:2i-2]} \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(y) \end{bmatrix}, \quad \widetilde{\mathbf{K}}_i(y) = \begin{bmatrix} \mathbf{k}_{i[1:2i-2]}^0 + \mathbf{k}_{i[1:2i-2]}^1 \exp(y) \\ \mathbf{k}_{i[1:2i-2]}^1 \exp(y) \end{bmatrix},$$

$$\overline{\mathbf{M}}_i(y) = \widetilde{\mathbf{H}}_{i[2i-1:2i]} \mathbf{\Lambda}_i(y), \qquad \bar{\mathbf{K}}_i(y) = \mathbf{k}_{i[2i-1:2i]}^0 + \mathbf{k}_{i[1:2i-2]}^1 \exp(y).$$

With this notation, we can express $\mathbf{c}_i$ as a linear system of $\mathbf{c}_{i+1}$ with the smooth pasting conditions:

$$\mathbf{c}_i = \mathbf{M}_i(yb_{i+1})^{-1} \widetilde{\mathbf{M}}_{i+1}(yb_{i+1}) \mathbf{c}_{i+1} + \mathbf{M}_i(yb_{i+1})^{-1} \left(\widetilde{\mathbf{K}}_{i+1}(yb_{i+1}) - \mathbf{K}_i(yb_{i+1}) \right).$$

Going forward, we can express $\mathbf{c}_i$ as a linear system of $\mathbf{c}_S$:

$$\mathbf{c}_i = \mathbf{MM}_i \mathbf{c}_S + \mathbf{KK}_i,$$

where

$$\mathbf{MM}_i = \left(\prod_{j=i}^{S-1} \mathbf{M}_j(yb_{j+1})^{-1} \widetilde{\mathbf{M}}_{j+1}(yb_{j+1}) \right),$$

$$\mathbf{KK}_i = \sum_{k=i}^{S-1} \left(\prod_{j=i}^{k-1} \mathbf{M}_j(yb_{j+1})^{-1} \widetilde{\mathbf{M}}_{j+1}(yb_{j+1}) \right) \mathbf{M}_k(yb_{k+1})^{-1} \left(\widetilde{\mathbf{K}}_{k+1}(yb_{k+1}) - \mathbf{K}_k(yb_{k+1}) \right).$$

The default boundary conditions can then be expressed as a linear system of $\mathbf{c}_S$:

$$\overline{\mathbf{M}}_i(yb_i) \mathbf{MM}_i \mathbf{c}_S = \mathbf{d}_i \exp(yb_i) - \overline{\mathbf{M}}_i(yb_i) \mathbf{KK}_i - \bar{\mathbf{K}}_i(yb_i).$$

Stacking all the default boundary conditions, we obtain a squared matrix which can be inversed:

$$\mathbf{c}_S = \begin{bmatrix} \overline{\mathbf{M}}_1(yb_1)\mathbf{MM}_1 \\ \vdots \\ \overline{\mathbf{M}}_{S-1}(yb_{S-1})\mathbf{MM}_{S-1} \\ \overline{\mathbf{M}}_S(yb_S) \\ \mathbf{L}_S \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{d}_1 \exp(yb_1) - \overline{\mathbf{M}}_1(yb_1)\mathbf{KK}_1 - \bar{\mathbf{K}}_1(yb_1) \\ \vdots \\ \mathbf{d}_{S-1} \exp(yb_{S-1}) - \overline{\mathbf{M}}_{S-1}(yb_{S-1})\mathbf{KK}_{S-1} - \bar{\mathbf{K}}_{S-1}(yb_{S-1}) \\ \mathbf{d}_S \exp(yb_S) - \bar{\mathbf{K}}_S(yb_S) \\ \mathbf{0}_{2S\times 1} \end{bmatrix},$$

where $\mathbf{L}_S$ is a $2S \times 4S$ matrix such that $\mathbf{Lc_S}$ is the $2S \times 1$ row vector of c_S^k such that $\lambda_S^k > 0$. Therefore, given $\mathbf{yb}$, solving for bond valuations resorts to solve for $2S^2 - 2S$ eigenpairs and inverse a $4S \times 4S$ matrix.

Example with 2 States, No Search/Liquidity Frictions, and No Restructuration The first second order differential to be satisfied is given by:

$$(r_1 + m + \zeta_1)D_1(y) = \mu_1 D_1'(y) + 0.5\sigma_1^2 D_1''(y) + (c + mp) + \zeta_1\alpha_2 v_2 \exp(y).$$

The solution will be of the form:

$$D_1(y) = c_1^1 e^{\lambda_1^1 y} + c_1^2 e^{\lambda_1^2 y} + k_1^0 + k_1^1 \exp(y).$$

We can solve for the eigenvalues as

$$\mathbf{A}_1 = \begin{bmatrix} -2\sigma_1^{-2}\mu_1 & 2\sigma_1^{-2}(r_1 + m + \zeta_1) \\ 1 & 0 \end{bmatrix},$$

$$\det\left(\begin{bmatrix} -2\sigma_1^{-2}\mu_1 - \lambda & 2\sigma_1^{-2}(r_1 + m + \zeta_1) \\ 1 & -\lambda \end{bmatrix}\right) = \lambda^2 + 2\sigma_1^{-2}\mu_1\lambda - 2\sigma_1^{-2}(r_1 + m + \zeta_1),$$

$$\lambda_1^{1,2} = \frac{-\mu_1 \pm \sqrt{\mu_1^2 + 2\sigma_1^2(r_1 + m + \zeta_1)}}{\sigma_1^2}.$$

The constant terms k_1^0 and k_1^1 need to satisfy

$$(r_1 + m + \zeta_1)\left(k_1^0 + k_1^1 \exp(y)\right) = \mu_1 k_1^1 \exp(y) + 0.5\sigma_1^2 k_1^1 \exp(y) + (c + mp) + \zeta_1 \alpha_2 v_2 \exp(y).$$

Therefore,

$$\begin{aligned} k_1^0 =& \frac{c + mp}{r_1 + m + \zeta_1}, \\ k_1^1 =& \frac{\zeta_1 \alpha_2 v_2}{r_1 + m + \zeta_1 - \mu_1 - 0.5\sigma_1^2}. \end{aligned}$$

The second differential to be satisfied is given by:

$$\begin{bmatrix} r_1 + m + \zeta_1 & -\zeta_1 \\ -\zeta_2 & r_2 + m + \zeta_2 \end{bmatrix} \mathbf{D}_2(y) = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \mathbf{D}_2'(y) + 0.5 \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \mathbf{D}_2''(y) + \begin{bmatrix} c + mp \\ c + mp \end{bmatrix}.$$

The solution will be of the form:

$$\mathbf{D}_2(y) = c_2^1 \mathbf{h}_{2[1:2]} e^{\lambda_2^1 y} + c_2^2 \mathbf{h}_{2[1:2]} e^{\lambda_2^2 y} + \mathbf{k}_2^0,$$

where we already discarded the general solutions with positive eigenvalues. We have to solve for the eigenvalues of

$$\mathbf{A}_2 = \begin{bmatrix} -2\sigma_1^{-2}\mu_1 & 0 & 2\sigma_1^{-2}(r_1 + m + \zeta_1) & -\sigma_1^{-2}\zeta_1 \\ 0 & -2\sigma_2^{-2}\mu_2 & -\sigma_2^{-2}\zeta_2 & 2\sigma_2^{-2}(r_2 + m + \zeta_2) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The presence of ζ_i's make the analytical solution to this problem not feasible to write in this document. The constant terms k_1^0 and k_1^1 need to satisfy

$$\mathbf{k}_2^0 = \begin{bmatrix} r_1 + m + \zeta_1 & -\zeta_1 \\ -\zeta_2 & r_2 + m + \zeta_2 \end{bmatrix}^{-1} \begin{bmatrix} c + mp \\ c + mp \end{bmatrix}.$$

B.2 Equity Value

With regime switching, debt rollover, and restructuration, the equity valuation $E_s(\cdot)$ in state $s \in 1, \ldots, S$ and log cash flows y must satisfy

$$
\begin{aligned}
r_s E_s(y) = &\mu_s E_s'(y) + 0.5\sigma_s E_s''(y) + \exp(y) - (1-\tau)c + m(D_s(y) - p) \\
&+ \sum_{i \neq s} \zeta_{s,i}(E_i(y) - E_s(y)) + \varphi((1-w)F_s(y) - p - E_s(y)),
\end{aligned}
$$

where c is the coupon, m the debt rollover rate, τ the tax benefits of debt, $\zeta_{s,i}$ the transition intensity from state s to state i, φ the arrival intensity of a restructuration event, and w the issuance cost. $F_s(\cdot)$ is the value of the new firm after debt restructuration. In matrix form, and using the same interval notation as for the bond valuation, we have:

$$
\begin{aligned}
\widehat{\mathbf{R}}_i \mathbf{E}_i(y) = &\widehat{\boldsymbol{\mu}}_i \mathbf{E}_i'(y) + 0.5\widehat{\boldsymbol{\Sigma}}_i \mathbf{E}_i''(y) + (\exp(y) - (1-\tau)c)\,\mathbf{1}_i \\
&+ m\,(\mathbf{S}_i \mathbf{D}_i(y) - p\mathbf{1}_i) + \varphi((1-w)\mathbf{F}_i(y) - p\mathbf{1}_i).
\end{aligned}
$$

Matrices specific to solving the equity valuations will be written with a hat. We define the following matrices of parameters:

$$
\widehat{\mathbf{R}}_i = \begin{bmatrix} r_1 + \zeta_1 + \varphi & \cdots & -\zeta_{1,i} \\ \vdots & \ddots & \vdots \\ -\zeta_{i,1} & \cdots & r_i + \zeta_i + \varphi \end{bmatrix},
$$

$$
\mathbf{S}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},
$$

and $\widehat{\boldsymbol{\Sigma}}_i = \mathrm{diag}\left([\sigma_1^2, \cdots, \sigma_i^2]\right)$, $\widehat{\boldsymbol{\mu}}_i = \mathrm{diag}\left([\mu_1, \cdots, \mu_i]\right)$. $\mathbf{S}_i$ is a $i \times 2i$ matrix that selects which debt values the firm is able to issue. We can rewrite the problem as a linear system of differential equations as

$$\widehat{\mathbf{x}}'(y) = \begin{bmatrix} \mathbf{E}_i''(y) \\ \mathbf{E}_i'(y) \end{bmatrix} = \begin{bmatrix} -2\widehat{\boldsymbol{\Sigma}}_i^{-1}\widehat{\mu}_i & 2\widehat{\boldsymbol{\Sigma}}_i^{-1}\widehat{\mathbf{R}}_i \\ \mathbf{I}_i & \mathbf{0}_i \end{bmatrix} \begin{bmatrix} \mathbf{E}_i'(y) \\ \mathbf{E}_i(y) \end{bmatrix} + \widehat{\mathbf{f}}_i(y) = \widehat{\mathbf{A}}_i \widehat{\mathbf{x}}_i(y) + \widehat{\mathbf{f}}_i(y)$$

where

$$\widehat{\mathbf{f}}_i(y) = \begin{bmatrix} -\left(\exp(y) - (1-\tau)c\right)\mathbf{1}_i - m\left(\mathbf{S}_i\mathbf{D}_i(y) - p\mathbf{1}_i\right) - \varphi((1-w)\mathbf{F}_i(y) - p\mathbf{1}_i) \\ \mathbf{0}_i \end{bmatrix}.$$

We know that the general solution $\widehat{\mathbf{x}}_i(y)$ is given by the expression

$$\widehat{\mathbf{x}}_i(y) = \widehat{\mathbf{x}}_i^h(y) + \widehat{\mathbf{x}}_i^p(y).$$

The term $\widehat{\mathbf{x}}_i^h(y)$ is the general solution of the homogeneous equation $\widehat{\mathbf{x}}_i'(y) = \widehat{\mathbf{A}}_i\widehat{\mathbf{x}}_i(y)$ and $\widehat{\mathbf{x}}_i^p(y)$ is a particular solution. The general solution of the linear system $\widehat{\mathbf{x}}_i'(y) = \widehat{\mathbf{A}}\widehat{\mathbf{x}}_i(y)$ is given by

$$\widehat{\mathbf{x}}_i(y) = \widehat{c}_i^1 \widehat{\mathbf{h}}_i^1 e^{\widehat{\lambda}_i^1 y} + \ldots + \widehat{c}_i^{2i} \widehat{\mathbf{h}}_i^{2i} e^{\widehat{\lambda}_i^{2i} y} = \widehat{\mathbf{H}}_i \widehat{\boldsymbol{\Lambda}}_i(y)\widehat{\mathbf{c}}_i.$$

where $(\widehat{\lambda}_i^1, \widehat{\mathbf{h}}_i^1)$, ..., $(\widehat{\lambda}_i^{2i}, \widehat{\mathbf{h}}_i^{2i})$ are the eigenpairs of $\widehat{\mathbf{A}}_i$ with $2i$ independent eigenvectors, and

$$\begin{aligned} \widehat{\mathbf{H}}_i &= [\widehat{\mathbf{h}}_i^1, \ldots, \widehat{\mathbf{h}}_i^{2i}], \\ \widehat{\mathbf{c}}_i &= [\widehat{c}_i^1, \ldots, \widehat{c}_i^{2i}]^\intercal, \\ \widehat{\boldsymbol{\Lambda}}_i(y) &= \mathrm{diag}\left(\left[e^{\widehat{\lambda}_i^1 y_1}, \ldots, e_1^{\widehat{\lambda}_i^{2i} y}\right]\right). \end{aligned}$$

I can guess a solution of the kind $\widehat{\mathbf{k}}_i^0 + \widehat{\mathbf{k}}_i^1 \exp(y)$ for the particular solution $\widehat{\mathbf{x}}_{i[1:i]}^p(y)$ where $\widehat{\mathbf{x}}_{[1:i]}$ selects the first i rows of vector $\widehat{\mathbf{x}}$ generated by the constants and the term in $\exp(y)$. Therefore, we can solve for the coefficients by substituting in and using the particular solution of $\mathbf{D}_i(y)$ and $\mathbf{F}_i(y)$:

$$\begin{aligned}\widehat{\mathbf{R}}_i\left(\widehat{\mathbf{k}}_i^0 + \widehat{\mathbf{k}}_i^1 \exp(y)\right) &= \widehat{\boldsymbol{\mu}}_i \widehat{\mathbf{k}}_i^1 \exp(y) + 0.5\widehat{\boldsymbol{\Sigma}}_i \widehat{\mathbf{k}}_i^1 \exp(y) + \left(\exp(y) - (1-\tau)c\right) \mathbf{1}_i \\ &+ m\left(\mathbf{S}_i \mathbf{k}_i^0 + \mathbf{S}_i \mathbf{k}_i^1 \exp(y) - p\mathbf{1}_i\right) + \varphi((1-w)\mathbf{f}_i \exp(y) - p\mathbf{1}_i),\end{aligned}$$

and solve for any value of y. Note that I assumed that the value of the firm after restructuration has the form $\mathbf{f}_i \exp(y)$. I will prove this later on. This gives us

$$\begin{aligned}\widehat{\mathbf{k}}_i^0 &= \widehat{\mathbf{R}}_i^{-1}\left(-(1-\tau)c\mathbf{1}_i + m\mathbf{S}_i \mathbf{k}_i^0 - mp\mathbf{1}_i\right), \\ \widehat{\mathbf{k}}_i^1 &= \left(\widehat{\mathbf{R}}_i - \widehat{\boldsymbol{\mu}}_i - 0.5\widehat{\boldsymbol{\Sigma}}_i\right)^{-1}\left(\mathbf{1}_i + m\mathbf{S}_i \mathbf{k}_i^1 + \varphi(1-w)\mathbf{f}_i\right).\end{aligned}$$

For the particular part stemming from the general solution of $\mathbf{D}_i(y) = \widetilde{\mathbf{H}}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i$, we can conjecture of non-constant part of the form $\widehat{\mathbf{G}}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i$ which has to satisfy

$$\widehat{\mathbf{R}}_i \widehat{\mathbf{G}}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i = \left(\widehat{\boldsymbol{\mu}}_i \widehat{\mathbf{G}}_i \boldsymbol{\lambda}_i + 0.5\widehat{\boldsymbol{\Sigma}}_i \widehat{\mathbf{G}}_i \boldsymbol{\lambda}_i^2 + m\mathbf{S}_i \widetilde{\mathbf{H}}_i\right)\boldsymbol{\Lambda}_i(y)\mathbf{c}_i$$

for any value of y. Therefore, the following systems have to hold

$$\widehat{\mathbf{R}}_i \widehat{\mathbf{G}}_{i[:,j]} = \widehat{\boldsymbol{\mu}}_i \widehat{\mathbf{G}}_{i[:,j]} \lambda_i^j + 0.5\widehat{\boldsymbol{\Sigma}}_i \widehat{\mathbf{G}}_{i[:,j]} (\lambda_i^j)^2 + m\mathbf{S}_i \widetilde{\mathbf{H}}_{i[:,j]}$$

for all j, where $\mathbf{X}_{i[:,j]}$ selects the jth column of matrix $\mathbf{X}_i$ and $\mathbf{X}_{i[k,j]}$ selects the scalar in the kth row jth column of matrix $\mathbf{X}_i$. Solving for $\widehat{\mathbf{G}}_{i[:,j]}$, we have

$$\widehat{\mathbf{G}}_{i[:,j]} = \left(\widehat{\mathbf{R}}_i - \widehat{\boldsymbol{\mu}}_i \lambda_i^j - 0.5\widehat{\boldsymbol{\Sigma}}_i (\lambda_i^j)^2\right)^{-1} m\mathbf{S}_i \widetilde{\mathbf{H}}_{i[:,j]}.$$

Finally, the general solution is given by

$$\mathbf{E}_i(y) = \widehat{\widehat{\mathbf{H}}}_i \widehat{\boldsymbol{\Lambda}}_i(y)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_i \boldsymbol{\Lambda}_i(y)\mathbf{c}_i + \widehat{\mathbf{k}}_i^0 + \widehat{\mathbf{k}}_i^1 \exp(y)$$

where $\widehat{\widehat{\mathbf{H}}}_i = \widehat{\mathbf{H}}_{i[1:i]}$ and $\mathbf{X}_{[1:i]}$ selects the first $2i$ rows of matrix $\mathbf{X}$, and

$$\mathbf{E}'_i(y) = \widehat{\widehat{\mathbf{H}}}_i \widehat{\boldsymbol{\lambda}}_i \widehat{\boldsymbol{\Lambda}}_i(y) \widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_i \boldsymbol{\lambda}_i \boldsymbol{\Lambda}_i(y) \mathbf{c}_i + \widehat{\mathbf{k}}_i^1 \exp(y)$$

where

$$\widehat{\boldsymbol{\lambda}}_i = \operatorname{diag}\left(\left[\widehat{\lambda}_i^1, \ldots, \widehat{\lambda}_i^{2i}\right]\right).$$

Boundary Conditions The different value functions $\mathbf{E}_i$ for $i \in \{1, \ldots, S\}$ are linked at the boundaries of their domains I_i. Note that $I_i \cap I_{i+1} = yb_{i+1}$ for $i < S$.

For $i = S$, we can immediately rule out all solutions to $\widehat{\lambda}_i$ bigger than 1 as equity over cash flow has to be finite and bounded as $y \to \infty$, so that the entries of $\mathbf{c}_i$ corresponding to eigenvalues larger than 1 will be zero:

$$\lim_{y \to \infty} |\mathbf{E}_S(y) \exp(-y)| < \infty.$$

For $i < S$, we must have value matching of the value functions that are alive across the boundary, and we must have value matching of the value functions that die across the boundary:

$$\mathbf{E}_{i+1}(yb_{i+1}) = \begin{bmatrix} \mathbf{E}_i(yb_{i+1}) \\ 0 \end{bmatrix}.$$

For $i < S$, we must have smooth pasting of the value functions that are alive across the boundary:

$$\mathbf{E}'_{i+1}(yb_{i+1})_{[1:i]} = \mathbf{E}'_i(yb_{i+1})$$

where $\mathbf{x}_{[1:i]}$ selects the first i rows of vector $\mathbf{x}$. Lastly, for $i = 1$, we must have

$$\mathbf{E}_1(yb_1) = 0.$$

Therefore, given $\mathbf{yb}$ the system of $2S(2S+1)/2$ equations to solve for $\widehat{\mathbf{c}}_i$ $\forall i \in$

$1, \ldots, S$ can be given by

$$\widehat{\widetilde{\overline{\mathbf{H}}}}_{i[i]} \widehat{\mathbf{\Lambda}}_i(yb_i)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_{i[i]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^0_{i[i]} + \widehat{\mathbf{k}}^1_{i[i]} \exp(yb_i) = 0 \qquad \text{for } i = 1, \ldots, S$$

$$\begin{aligned} &\widehat{\widetilde{\overline{\mathbf{H}}}}_{i[1:i-1]} \widehat{\mathbf{\Lambda}}_i(yb_i)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_{i[1:i-1]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^0_{i[1:i-1]} + \widehat{\mathbf{k}}^1_{i[1:i-1]} \exp(yb_i) \\ &= \widehat{\widetilde{\overline{\mathbf{H}}}}_{i-1} \widehat{\mathbf{\Lambda}}_{i-1}(yb_i)\widehat{\mathbf{c}}_{i-1} + \widehat{\mathbf{G}}_{i-1} \mathbf{\Lambda}_{i-1}(yb_i)\mathbf{c}_{i-1} + \widehat{\mathbf{k}}^0_{i-1} + \widehat{\mathbf{k}}^1_{i-1} \exp(yb_i) \end{aligned}$$

$$\begin{aligned} &\widehat{\widetilde{\overline{\mathbf{H}}}}_{i[1:i-1]} \widehat{\boldsymbol{\lambda}}_i \widehat{\mathbf{\Lambda}}_i(yb_i)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_{i[1:i-1]} \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^1_{i[1:i-1]} \exp(yb_i) \\ &\quad = \widehat{\widetilde{\overline{\mathbf{H}}}}_{i-1} \widehat{\boldsymbol{\lambda}}_i \widehat{\mathbf{\Lambda}}_i(yb_i)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_{i-1} \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^1_{i-1} \exp(yb_i) \end{aligned}$$

for $i = 2, \ldots, S$, and

$$\lim_{y \to \infty} |\mathbf{E}_S(y) \exp(-y)| < \infty.$$

We can define squared matrices $\widehat{\mathbf{M}}_i$, $\widehat{\widetilde{\mathbf{M}}}_i$, $\widehat{\overline{\mathbf{M}}}_i$, $\widehat{\mathbf{K}}_i$, $\widehat{\widetilde{\mathbf{K}}}_i$, and $\widehat{\overline{\mathbf{K}}}_i$ as

$$\widehat{\mathbf{M}}_i(y) = \begin{bmatrix} \widehat{\widetilde{\overline{\mathbf{H}}}}_i \widehat{\mathbf{\Lambda}}_i(y) \\ \widehat{\widetilde{\overline{\mathbf{H}}}}_i \widehat{\boldsymbol{\lambda}}_i \widehat{\mathbf{\Lambda}}_i(y) \end{bmatrix}, \qquad \widehat{\mathbf{K}}_i(y) = \begin{bmatrix} \widehat{\mathbf{G}}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^0_i + \widehat{\mathbf{k}}^1_i \exp(y) \\ \widehat{\mathbf{G}}_i \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^1_i \exp(y) \end{bmatrix},$$

$$\widehat{\widetilde{\mathbf{M}}}_i(y) = \begin{bmatrix} \widehat{\widetilde{\overline{\mathbf{H}}}}_{i[1:i-1]} \widehat{\mathbf{\Lambda}}_i(y) \\ \widehat{\widetilde{\overline{\mathbf{H}}}}_{i[1:i-1]} \widehat{\boldsymbol{\lambda}}_i \widehat{\mathbf{\Lambda}}_i(y) \end{bmatrix}, \quad \widehat{\widetilde{\mathbf{K}}}_i(y) = \begin{bmatrix} \widehat{\mathbf{G}}_{i[1:i-1]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^0_{i[1:i-1]} + \widehat{\mathbf{k}}^1_{i[1:i-1]} \exp(y) \\ \widehat{\mathbf{G}}_{i[1:i-1]} \boldsymbol{\lambda}_i \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^1_{i[1:i-1]} \exp(y) \end{bmatrix},$$

$$\widehat{\overline{\mathbf{M}}}_i(y) = \widehat{\widetilde{\overline{\mathbf{H}}}}_{i[i]} \widehat{\mathbf{\Lambda}}_i(y), \qquad \widehat{\overline{\mathbf{K}}}_i(y) = \widehat{\mathbf{G}}_{i[i]} \mathbf{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^0_{i[i]} + \widehat{\mathbf{k}}^1_{i[i]} \exp(y).$$

With this notation, we can express $\widehat{\mathbf{c}}_i$ as a linear system of $\widehat{\mathbf{c}}_{i+1}$ with the smooth pasting conditions:

$$\widehat{\mathbf{c}}_i = \widehat{\mathbf{M}}_i(yb_{i+1})^{-1}\widehat{\widetilde{\mathbf{M}}}_{i+1}(yb_{i+1})\widehat{\mathbf{c}}_{i+1} + \widehat{\mathbf{M}}_i(yb_{i+1})^{-1}\left(\widehat{\widetilde{\mathbf{K}}}_{i+1}(yb_{i+1}) - \widehat{\mathbf{K}}_i(yb_{i+1})\right).$$

Going forward, we can express $\widehat{\mathbf{c}}_i$ as a linear system of $\widehat{\mathbf{c}}_S$:

$$\widehat{\mathbf{c}}_i = \widehat{\mathbf{MM}}_i\widehat{\mathbf{c}}_S + \widehat{\mathbf{KK}}_i,$$

where

$$\widehat{\mathbf{MM}}_i = \left(\prod_{j=i}^{S-1}\widehat{\mathbf{M}}_j(yb_{j+1})^{-1}\widehat{\widetilde{\mathbf{M}}}_{j+1}(yb_{j+1})\right),$$

$$\widehat{\mathbf{KK}}_i = \sum_{k=i}^{S-1}\left(\prod_{j=i}^{k-1}\widehat{\mathbf{M}}_j(yb_{j+1})^{-1}\widehat{\widetilde{\mathbf{M}}}_{j+1}(yb_{j+1})\right)\widehat{\mathbf{M}}_k(yb_{k+1})^{-1}\left(\widehat{\widetilde{\mathbf{K}}}_{k+1}(yb_{k+1}) - \widehat{\mathbf{K}}_t(yb_{k+1})\right).$$

The default boundary conditions can then be expressed as a linear system of $\widehat{\mathbf{c}}_S$:

$$\widehat{\widehat{\mathbf{M}}}_i(yb_i)\widehat{\mathbf{MM}}_i\widehat{\mathbf{c}}_S + \widehat{\widehat{\mathbf{M}}}_i\widehat{\mathbf{KK}}_i + \widehat{\mathbf{K}}_i = 0.$$

Stacking all the default boundary conditions, we obtain a squared matrix which can be inversed:

$$\widehat{\mathbf{c}}_S = \begin{bmatrix} \widehat{\widehat{\mathbf{M}}}_1(yb_1)\widehat{\mathbf{MM}}_1 \\ \vdots \\ \widehat{\widehat{\mathbf{M}}}_{S-1}(yb_{S-1})\widehat{\mathbf{MM}}_{S-1} \\ \widehat{\widehat{\mathbf{M}}}_S(yb_S) \\ \widehat{\mathbf{L}}_S \end{bmatrix}^{-1} \begin{bmatrix} -\widehat{\widehat{\mathbf{M}}}_1\widehat{\mathbf{KK}}_1 - \widehat{\mathbf{K}}_1 \\ \vdots \\ -\widehat{\widehat{\mathbf{M}}}_{S-1}\widehat{\mathbf{KK}}_{S-1} - \widehat{\mathbf{K}}_{S-1} \\ -\widehat{\mathbf{K}}_S \\ \mathbf{0}_S \end{bmatrix},$$

where $\widehat{\mathbf{L}}_S$ is a $S \times 2S$ matrix such that $\widehat{\mathbf{L}}\widehat{\mathbf{c}}_{\mathbf{S}}$ is the $S \times 1$ row vector of $\widehat{c}_S^k$ such that $\widehat{\lambda}_S^k > 1$. Therefore, given $\mathbf{yb}$, solving for equity valuations resorts to solve for

$2S(2S+1)/2$ eigenpairs and inverse a $2S \times 2S$ matrix.

Bankruptcy Boundaries The optimality conditions to solve for $\mathbf{yb}$ is given by

$$\mathbf{E}'_{i[i]}(yb_i) = \widehat{\widehat{\mathbf{H}}}_{i[i]}\widehat{\boldsymbol{\lambda}}_i\widehat{\boldsymbol{\Lambda}}_i(yb_i)\widehat{\mathbf{c}}_i + \widehat{\mathbf{G}}_{i[i]}\boldsymbol{\lambda}_i\boldsymbol{\Lambda}_i(yb_i)\mathbf{c}_i + \widehat{\mathbf{k}}^1_{i[i]}\exp(yb_i) = 0,$$

i.e., a smooth pasting condition at the boundaries at which default is declared.

C Baum-Welch Algorithm for hidden Markov model

Let X_t be a discrete hidden random variable with N possible values. We assume that $P(X_t|X_{t-1})$ is independent of time t, which leads to the definition of the time independent stochastic transition matrix $\mathbf{A} = \{a_{ij}\}$ where $a_{ij} = P(X_t = j|X_{t-1} = i)$. The initial state distribution is given by $\boldsymbol{\mu}_0$. The observation variables Y_t can take one of K possible values. The probability of a certain observation at time t for state j is given by

$$\ell_j(y_t) = P(Y_t = y_t|X_t = j).$$

We will represent the observation density as follows: for every $y \in F$, we define the diagonal matrix $\mathbf{L}(y)$ with nonzero elements $\{\mathbf{L}(y)\}_{ii} = \ell_i(y)$. An observation sequence is given by $Y = (Y_0 = y_0, Y_2 = y_2, ..., Y_T = y_T)$. Thus we can describe a hidden Markov chain by $\boldsymbol{\theta} = (\mathbf{A}, \mathbf{L}, \boldsymbol{\mu}_0)$. The Baum-Welch algorithm finds a local maximum for $\boldsymbol{\theta}^* = \arg\max_\theta P(Y|\theta)$.

Initiation Set $\boldsymbol{\theta} = (\mathbf{A}, \mathbf{L}, \boldsymbol{\mu}_0)$ with random initial conditions.

Forward procedure Let $\pi_{i,t} = P(Y_0 = y_0, ..., Y_t = y_t, X_t = i|\boldsymbol{\theta})$ be the probability of seeing the $y_0, y_1, ..., y_t$ and being in state i at time t. First, we get

$$\begin{aligned} c_0 &= \mathbf{1}'\mathbf{L}(y_0)\boldsymbol{\mu}_0, \\ \boldsymbol{\pi}_0 &= \mathbf{L}(y_0)\boldsymbol{\mu}_0/c_0. \end{aligned}$$

Then for $k = 1, \ldots, N$

$$\begin{aligned} \widetilde{\boldsymbol{\pi}}_k &= \mathbf{L}(y_k)\mathbf{A}'\boldsymbol{\pi}_{k-1}, \\ c_k &= \mathbf{1}'\widetilde{\boldsymbol{\pi}}_k \\ \boldsymbol{\pi}_k &= \widetilde{\boldsymbol{\pi}}_k/c_k \end{aligned}$$

Backward procedure Let $\beta_{i,t} = P(Y_{t+1} = y_{t+1}, ..., Y_T = y_T|X_t = i, \boldsymbol{\theta})$ that is the probability of the ending partial sequence $y_{t+1}, ..., y_T$ given starting state i at time t.

We calculate $\beta_{i,t}$ as

$$\boldsymbol{\beta}_{N|N} = \mathbf{1}/c_N$$

then or $k = 1, \ldots, N$

$$\begin{aligned}
\boldsymbol{\beta}_{N-k|N} &= \mathbf{AL}(y_{N-k+1})\boldsymbol{\beta}_{N-k+1|N}/c_{N-k}, \\
\boldsymbol{\pi}_{N-k,N-k+1|N} &= \mathrm{diag}(\boldsymbol{\pi}_{N-k})\mathbf{AL}(y_{N-k+1})\mathrm{diag}(\boldsymbol{\beta}_{N-k+1|N}), \\
\boldsymbol{\pi}_{N-k|N} &= \boldsymbol{\pi}_{N-k,N-k+1|N}\mathbf{1},
\end{aligned}$$

where $\pi_{i,N-k|N} = P(X_t = i|Y, \boldsymbol{\theta})$ is the probability of being in state i at time t given the observed sequence Y and the parameters $\boldsymbol{\theta}$. We can now update $\boldsymbol{\theta}$ as

$$\begin{aligned}
\boldsymbol{P}_{ij} &= \frac{\sum_{k=1}^{n}(\boldsymbol{\pi}_{k-1,k|N})_{ij}}{\sum_{k=1}^{N}(\boldsymbol{\pi}_{k-1|N})_{i}}, \\
\boldsymbol{\mu}_0 &= \boldsymbol{\pi}_{0|N},
\end{aligned}$$

and the parameters of $\mathbf{L}(\cdot)$ as their empirical counterparts.

The algorithm applied to the General Electric Company Stock Price is shown in Figure 15. The algorithm detects three states: (1) high growth (0.16) high volatility (0.27), (2) low growth (0.11) low volatility (0.15), and (3) negative growth (-0.36) very high volatility (0.62).

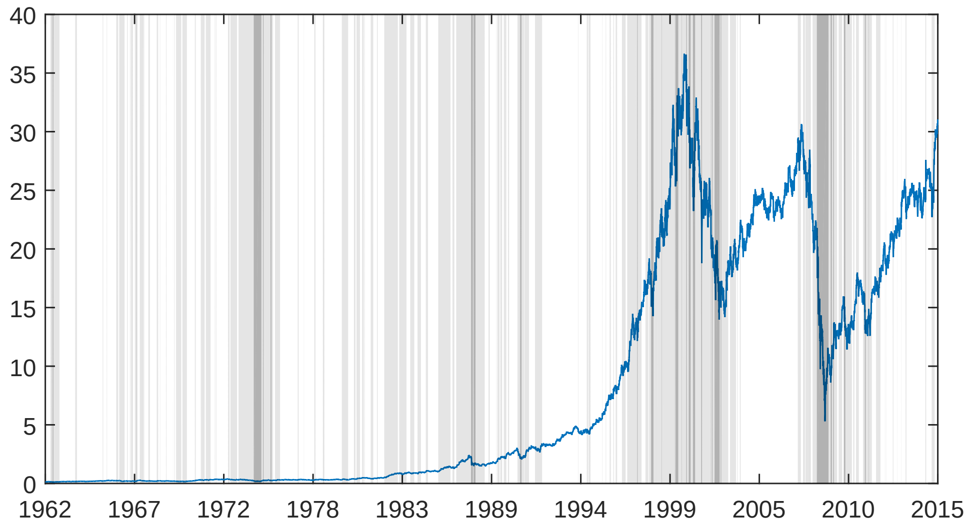

Figure 15: General Electric Company Stock Price

D Continuous- to Discrete-Time Poisson Process

Define the probability of being in state i at time t as $P_i(t)$. With the transition from state i to j following an exponential distribution with parameter ζ_{ij}, we can consider the first-order system equations for the n-state system expressed in matrix form as

$$\boldsymbol{\zeta}^\mathsf{T}\mathbf{P} = \begin{bmatrix} -\zeta_1 & \zeta_{12} & \cdots & \zeta_{1n} \\ \zeta_{21} & -\zeta_2 & \cdots & \zeta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{n1} & \zeta_{n2} & \cdots & -\zeta_n \end{bmatrix}^\mathsf{T} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} = \begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \vdots \\ \dot{P}_N \end{bmatrix} = \dot{\mathbf{P}}$$

where

$$\zeta_i = \sum_{j \neq i} \zeta_{ij}.$$

The general solution to this homogeneous linear system with constant coefficients is

$$\mathbf{P}(t) = \mathbf{h}_1 e^{\lambda_1 t} c_1 + \ldots + \mathbf{h}_n e^{\lambda_n t} c_n = \mathbf{H}\boldsymbol{\Lambda}(t)\mathbf{c}$$

where $(\lambda_1, \mathbf{h}_1)$, ..., $(\lambda_n, \mathbf{h}_n)$ are the eigenpairs of $\boldsymbol{\zeta}^\mathsf{T}$ with n independent eigenvectors, and

$$\begin{aligned} \mathbf{H} &= [\mathbf{h}_1, \ldots, \mathbf{h}_n], \\ \mathbf{c} &= [c_1, \ldots, c_n]^\mathsf{T}, \\ \boldsymbol{\Lambda}(t) &= \operatorname{diag}\left(\left[e^{\lambda_1 t}, \ldots, e^{\lambda_n t}\right]\right). \end{aligned}$$

We can find $\mathbf{c}$ with the initial condition $\mathbf{P}(0)$ solving

$$\mathbf{c} = \mathbf{H}^{-1}\mathbf{P}(0).$$

Thus we can get the discrete-time transition matrix $\mathbf{\Pi}$ according to

$$\mathbf{\Pi} = (\mathbf{H}^{-1})^{\mathsf{T}} \mathbf{\Lambda}(1/12) \mathbf{H}^{\mathsf{T}}.$$

E The Credit Spread Common Factor and the Excess Bond Premium

In the main text, I show how a single common factor is driving credit spreads, leverage, and equity return volatility. In this appendix, I will uncover the credit spreads common factor and show how it relates to the excess bond premium of Gilchrist and Zakrajšek (2012), firms' leverage, and firms' equity return volatility using more convoluted arguments than a simple principal component analysis of these series.

Following Gilchrist and Zakrajšek (2012), I want to decompose credit spreads into (1) a component that captures the systematic movements in default risk of individual firms and (2) a residual component – sometimes called the excess bond premium – that represents the time variation in the average price of bearing exposure to US corporate credit risk, above and beyond the compensation for expected defaults. However, if some variables that proxies for credit risk are highly correlated with the excess bond premium, such an estimation cannot be done consistently by taking the average of the error term of a panel regression of credits spreads on credit risk variables. Indeed, in that case, the exogeneity condition of the error term will be violated. As I showed in Section 3, credit spreads are highly correlated with aggregate firm leverage and aggregate equity return volatility, themselves important indicators of credit risk. Therefore, I estimate the common factor in corporate bond credit spreads – the common time varying fluctuations in credit spreads not explained by cross-sectional differences in credit risk variables – by using a time fixed effect. This time fixed effect will absorb any effect coming from aggregate time varying changes, whether it comes from changes in aggregate firms' credit risk or from a bond risk premium. In section 6, I resort to a structural approach to decompose aggregate credit spreads into these two components.

The log of the option adjusted credit spread issued by firm i at time t is assumed to be related linearly to a vector of firm- and bond-specific characteristic $\mathbf{x}_{it}$ that

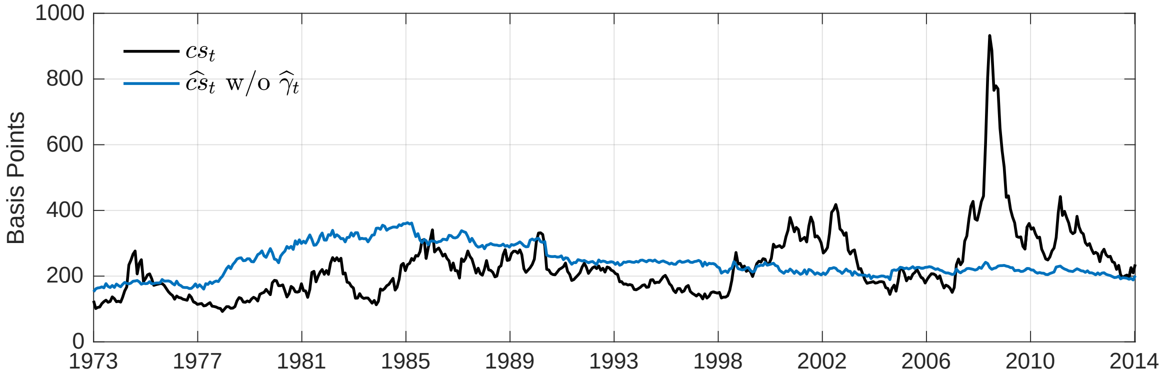

Figure 16: Predicted Monthly Average Credit Spread The average option adjusted credit spread oas_t is constructed as $\frac{1}{N}\sum_{i=1}^{N} oas_{it}$. The predicted monthly average credit spread $\widehat{oas}_t$ is given by $\frac{1}{N}\sum_{i=1}^{N}\exp(\widehat{c}_i+\widehat{\gamma}_t+\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}+\widehat{\sigma}^2/2)$, while the predicted monthly average credit spread $\widehat{oas}_t$ without the time trend γ_t is given by $\frac{1}{N}\sum_{i=1}^{N}\exp(\widehat{c}_i+\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}+\widehat{\sigma}^2/2)$. Grey area represents U.S. recession as defined by National Bureau of Economic Research.

proxies for default risk according to

$$\log(oas_{it}) = c_i + \gamma_t + \mathbf{x}_{it}\boldsymbol{\beta} + \varepsilon_{it}.$$

The time fixed-effect γ_t will capture time variations in the average credit spread. The necessary assumption required for consistent estimation is strict exogeneity of the error with the explanatory variables of all past, current and future time periods of all individuals, i.e.

$$\mathbf{E}\left[\varepsilon_{it}\middle|\,\mathbf{X},\mathbf{y}\right] = 0.$$

I can estimate the model by double-demeaning the data:

$$\ddot{y}_{it} = y_{it} - \bar{y}_i - \bar{y}_t + \bar{\bar{y}},$$
$$\ddot{\mathbf{x}}_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i - \bar{\mathbf{x}}_t + \bar{\bar{\mathbf{x}}},$$

where $y_{it} = \log(oas_{it})$ and

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \qquad \bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_i, \qquad \bar{\bar{y}} = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N} y_i.$$

Then I regress $\ddot{y}_{it}$ on $\ddot{\mathbf{x}}_{it}$ to get $\widehat{\boldsymbol{\beta}}$ as I now have

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}\boldsymbol{\beta} + \ddot{\varepsilon}_{it}.$$

I allow for the error term to be correlated over time for each individual and across individuals for each time period by clustering by firm and month. The firm-specific explanatory variables include: Merton distance to default dd, monthly estimate of firm's equity returns volatility σ_E, quasi-market leverage ratio[13] lev, monthly average equity returns ret, income to total asset ratio[14] inc, distance to maturity in months mat, bond's duration dur, bond's coupon rate $coup$, the outstanding amount par, and the age of the issue in months age. I also include dummy variables for firm's industry code and $S\&P$ firm's rating.[15] All bond specific variables are interacted with a dummy variables of the bond's callability. Table 11 shows the results of different specifications. The time fixed effect can then be retrieved as

$$\widehat{\gamma}_t = \bar{y}_t - \bar{\bar{y}} - (\bar{\mathbf{x}}_t - \bar{\bar{\mathbf{x}}})^{\mathsf{T}}\widehat{\boldsymbol{\beta}}.$$

---

[13]The quasi-market leverage ratio is defined as:

$$lev_{it} = \frac{DLTT_{it} + DLC_{it}}{DLTT_{it} + DLC_{it} + CSHO_{it} \times PRCC_{it}},$$

where $DLTT$ and DLC are the Compustat long-term debt and debt in current liabilities, $CSHO$ is the number of shares outstanding and $PRCC$ the stock price from CRSP.

[14]The income to total asset ratio is defined as:

$$inc_{it} = \frac{OIBDP_{it}}{AT_{it}}$$

where $OIBDP$ and AT are the Compustat earnings before interests, taxes, and depreciation and book assets, respectively.

[15]Replaced by Merrill Lynch or Lehman/Warga bond rating when not available.

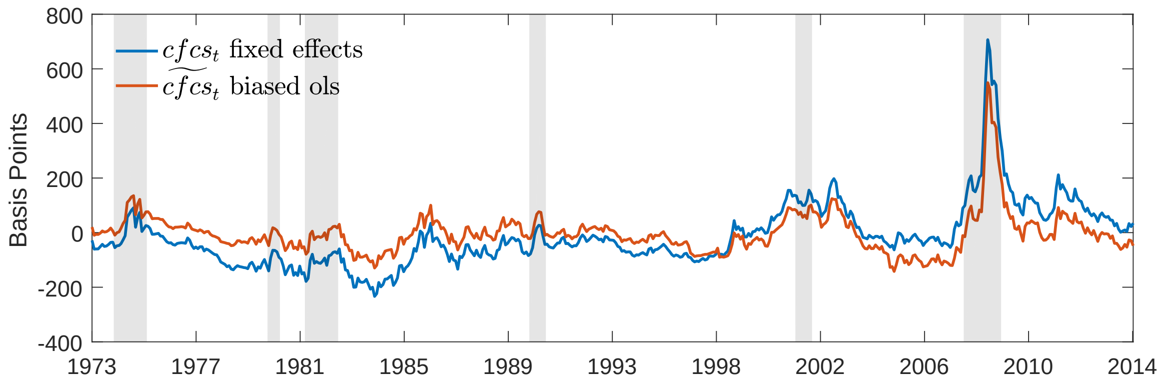

Figure 17: Biased and Non-Biased Credit Spreads Common Factor See text for the definition of the non-biased common factor in credit spreads $CSCF$ and the biased common factor in credit spread $\widetilde{CSCF}$. Grey area represents U.S. recession as defined by National Bureau of Economic Research.

Assuming normally distributed disturbances, the predicted level of the spread for firm i at time t is given by

$$\widehat{oas}_{it} = \exp\left(\widehat{c}_i + \widehat{\gamma}_t + \mathbf{x}_{it}\widehat{\boldsymbol{\beta}} + \widehat{\sigma}_x^2/2\right)$$

where $\widehat{\sigma}_x^2$ is the estimated variance of the disturbance term ε_{it}. The CSCF in period t is then defined by the following linear decomposition[16]:

$$CSCF_t = \frac{1}{N}\sum_{i=1}^{N} oas_{it} - \frac{1}{N}\sum_{i=1}^{N}\widehat{oas}_{it}.$$

Figure 16 shows the actual and predicted credit spreads while Figure 17 shows the estimated CSCF. In the same figure, I also show the excess bond premium estimated by Gilchrist and Zakrajšek (2012) and a time time fixed effect that would result from the biased OLS estimation of

$$\log(oas_{it}) = \mathbf{x}_{it}\widetilde{\boldsymbol{\beta}} + \widetilde{\varepsilon}_{it}.$$

[16]We could also estimate the CSCF with $\exp(\widehat{\gamma}_t)$. The results are essentially the same with $corr(CSCF_t, \exp(\widehat{\gamma}_t)) = 0.98$.

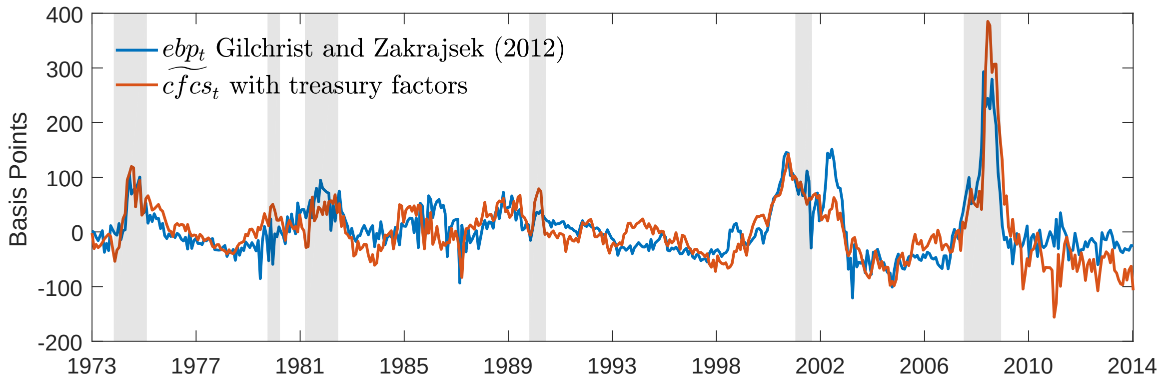

Figure 18: Excess Bond Premium and Biased Credit Spreads Common Factor with Treasury Factors See text for the definition of the biased credit spread time trend $\widetilde{CSCF}$ with treasury factors. The excess bond premium is taken directly from the website of Gilchrist. Grey area represents U.S. recession as defined by National Bureau of Economic Research.

This specification suffer from an omitted variable bias with no time trend variable included in the regression. It is easy to verify that the residuals from this regression are highly cross-correlated, and principal components analysis implies that they are mostly driven by a single common factor. The predicted level of the spread for firm i at time t is then given by

$$\widehat{\widetilde{oas}}_{it} = \exp\left(\mathbf{x}_{it}\widehat{\widetilde{\boldsymbol{\beta}}} + \widehat{\widetilde{\sigma}}_x^2/2\right)$$

where $\widehat{\widetilde{\sigma}}_x^2$ is the estimated variance of the disturbance term $\widetilde{\varepsilon}_{it}$. The biased CSCF can then be constructed as

$$\widetilde{CSCF}_t = \frac{1}{N}\sum_{i=1}^{N} oas_{it} - \frac{1}{N}\sum_{i=1}^{N}\widehat{\widetilde{oas}}_{it}.$$

By introducing factors controlling for the monetary stance[17] in this biased OLS regression, I obtain a time series very close to the excess bond premium estimated

[17] Gilchrist and Zakrasjek (2012) use thee factors: the level, slope, and curvature of the Treasury yield curve.

by Gilchrist and Zakrajšek (2012). The comparison can be see in Figure 18.

Table 10: Panel Fixed Effect Regressions

	log($oas_{i,t}$)					
	(1)	(2)	(3)	(4)	(5)	(6)
$i.rating_{it}$	X	X	X	X	X	X
$i.sic_i$				X	X	X
dd_{it}		-0.019			-0.005	-0.014
		(0.001)			(0.001)	(0.002)
$\log(\sigma_{it})$			0.124		0.082	0.134
			(0.008)		(0.008)	(0.017)
$\log(lev_{it})$				0.238	0.182	0.162
				(0.018)	(0.017)	(0.02)
$\log(ret_{it})$					0.562	-0.679
					(0.352)	(0.812)
$\log(inc_{it})$					-0.063	-0.065
					(0.007)	(0.006)
$i.call_i$						X
$\log(mat_{it})$						0.83
						(0.067)
$\log(dur_{it})$						-0.963
						(0.09)
$\log(coup_{it})$						0.383
						(0.05)
$\log(par_{it})$						-0.008
						(0.003)
$\log(age_{it})$						0.016
						(0.006)
Adjusted R^2	0.791	0.794	0.794	0.796	0.801	0.837
Within R^2	0.227	0.236	0.235	0.245	0.261	0.386

Table 11: N = 203,109

The CSCF can be decomposed into three components: (1) the first principal component of median log equity return volatility by rating classes, (2) the first principal

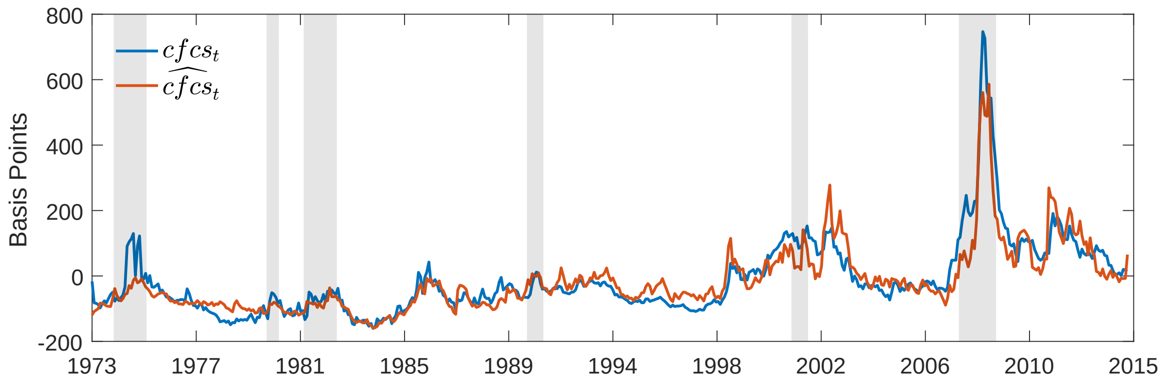

Figure 19: Predicted Credit Spreads Common Factor The predicted common factor in credit spreads $\widehat{\gamma}_t$ is constructed by regressing $CSCF$ on (1) the first component of median log idiosyncratic volatility by rating classes, (2) the first component of median log quasi-market leverage by rating classes, and (3) Treasury yield curve level and slope. Grey area represents U.S. recession as defined by National Bureau of Economic Research.

component of median log quasi-market leverage by rating classes, and (3) Treasury yield curve level and slope. The result is shown in Figure 19. More precisely I regress the estimated CSCF $\widehat{\gamma}_t$ on these three components according to

$$\begin{aligned}\widehat{\gamma}_t &= \alpha_0 + \alpha_1 FPC\,(lev)_t + \alpha_2 FPC\,(\sigma)_t + \alpha_3 TLEV_t + \alpha_4 TSLO_t \\ &\quad + \alpha_5 \mathbf{1}\,\{t \leq \text{NOV 1982}\}\,TLEV_t + \alpha_6 \mathbf{1}\,\{t \leq \text{NOV 1982}\}\,TSLO_t + \nu_{it}, \qquad (16)\end{aligned}$$

where $FPC[lev]_t$ is the first principal component of leverage by rating classes, $FPC[\sigma]_t$ is the first principal component of monthly equity return volatility by rating classes, and $TLEV_t$ and $TSLO_t$ are the level and the slope of the Treasury yield curve. The level and slope factors correspond, respectively, to the first two principal components of nominal Treasury yields at 3-month, 6-month, and 1-, 2-, 3-, 5-, 7-, 10-, 15-, and 30-year maturities. $\mathbf{1}\,\{t \leq \text{NOV 1982}\}$ is a dummy variable equal to one if the date occurs before November 1982. The date of the structural break for Treasury yields curve factors is chosen in order to maximize the Akaike information criterion. This regression has an adjusted-R^2 of 81%. The results are detailed in Table 13. The

Table 12: Components of the Common Factor in Credit Spreads

	$cstt_t$			
	(1)	(2)	(3)	(4)
Treasury Factors	X	X	X	X
$PCA(lev)_t$		0.330		0.168
		(0.036)		(0.013)
$PCA(\sigma)_t$			0.086	0.045
			(0.006)	(0.005)
Adjusted R^2	0.582	0.779	0.727	0.81

Table 13: 502 monthly observations from January 1973 to October 2014. $PCA[lev]_t$ is the first principal component of leverage by rating classes, $PCA[\sigma]_t$ is the first principal component of monthly equity return idiosyncratic volatility by rating classes, and Treasury Factors include the level and the slope of the Treasury yield curve. The level and slope factors correspond, respectively, to the first two principal components of nominal Treasury yields at 3-month, 6-month, and 1-, 2-, 3-, 5-, 7-, 10-, 15-, and 30-year maturities. I also included a dummy variables equal to one if the date occurs before November 1982 that interact with the Treasury Factors only.

predicted CSCF is then given by:

$$\widehat{CSCF}_t = \exp\left(\mathbf{z}_t\widehat{\boldsymbol{\alpha}} + \widehat{\sigma}_z^2/2\right),$$

where $\mathbf{z}_t$ is the vector of explanatory variables from (16) and $\widehat{\sigma}_z^2$ is the estimated variance of the disturbance term ν_{it}. Figure 19 shows the results of this decomposition, while Figure 20 illustrates the contribution of each component.

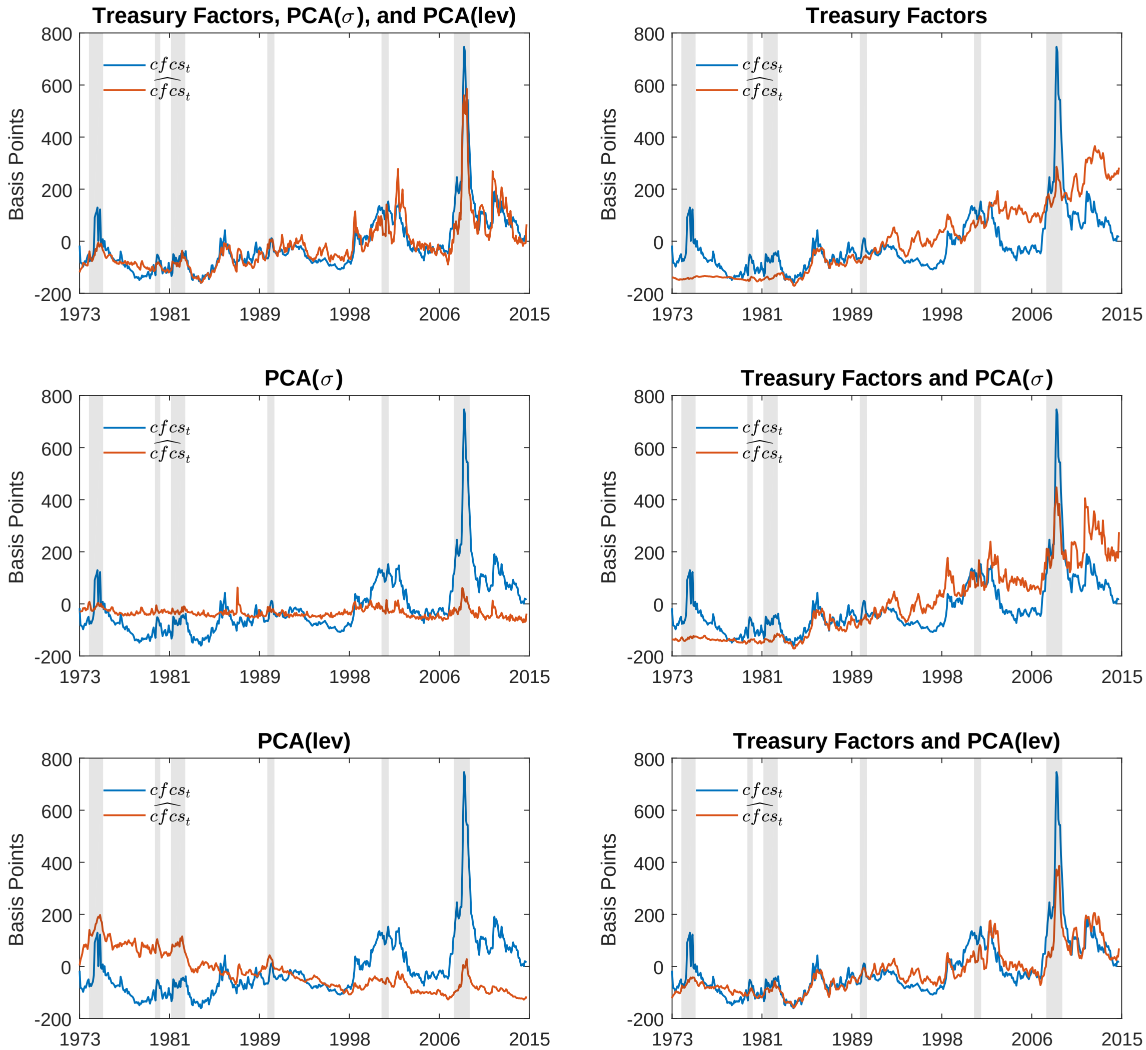

Figure 20: Credit Spreads Common Factor Components Each graph shows the contribution of different components combination. For example, the graphs "Treasury Factors" shows the CSCF predicted with treasury factors only while holding the first principal components of leverage and equity volatility constants at their means but using the coefficient from the full regression. Grey area represents U.S. recession as defined by National Bureau of Economic Research.

F Approximation to Aggregate Asset Volatility

This appendix details the approximation used in Section 7. The environment is consistent with Modigliani and Miller's (1958) theorem. Define y_t to be the level of cash flows generated by firm's assets:

$$\frac{dy_t}{y_t} = \mu_{Y,F} dt + \sigma_{Y,A} dZ_t^A$$

where Z_t^A is a Brownian motion representing aggregate shocks. Assume that corporate debt is a perpetual bond with coupon rate c and that there is no option to default. Investors assume that aggregate asset volatility and the risk-free rate are constant over time. The valuation of equity by a risk-neutral investor is then given by

$$E = \frac{y_t}{r - \mu} - D$$

where the perpetual bond value is given by

$$D = \frac{c}{r}.$$

Aggregate equity volatility then satisfy

$$\sigma_{E,A} = \frac{E + D}{E} \sigma_{Y,A}.$$

component	log(cs)	lev	$\log(\sigma_E)$	$\log(\sigma_{E,A})$	$\log(\sigma_{E,I})$
1	89.04	82.05	94.66	95.77	94.64
2	6.62	11.26	3.16	3.35	2.98
3	2.77	4.99	1.17	0.60	1.26
4	1.02	1.10	0.73	0.19	0.81
5	0.55	0.60	0.28	0.08	0.32

Table 14: Percentage of the Total Variance Explained by each Principal Component with 5 Rating Classes This table displays the percentage of the total variance explained by the principal components of each variable averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC, where log(cs) is log credit spread, lev is market leverage, $\log(\sigma_E)$ is log equity return total volatility, $\log(\sigma_{E,A})$ is log equity return aggregate volatility, and $\log(\sigma_{E,I})$ is log equity return idiosyncratic volatility.

G Additional Graphs and Tables

Figure 21 complements Figure 1 and displays firms' leverage, firms' aggregate equity return volatility, and firms' idiosyncratic equity return volatility averaged by rating class from 1973 to 2014. As one can observe from the fact that the times series move parallel to each others, the first principal component can account for a lot of the total variation. Table 14 shows the percentage of the total variance explained by the principal components. Very similar results arise when using more granular rating groups as shown in Table 15. Overall, almost all the variation in the times series can be accounted for by the first two principal components. From the principal component coefficients, it is easy to conclude that the first component is an average of each series while the second component measure the dispersion of the series (see the coefficient for credit spreads in Table 16).

Since all these series are driven by a single factor, it is natural to ask how they relate to each others. Thus I measure the correlation between the first and second principal components of each variables in Table 1 and 17. Note that to measure the correlation between these objects, I first remove the linear trend from the components to have stationary series.

component	log(cs)	lev	$\log(\sigma_E)$	$\log(\sigma_{E,A})$	$\log(\sigma_{E,I})$
1	89.04	82.05	94.66	95.77	94.64
2	6.62	11.26	3.16	3.35	2.98
3	2.77	4.99	1.17	0.60	1.26
4	1.02	1.10	0.73	0.19	0.81
5	0.55	0.60	0.28	0.08	0.32

Table 15: Percentage of the Total Variance Explained by each Principal Component with 5 Rating Classes This table displays the percentage of the total variance explained by the principal components of each variable averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC, where log(cs) is log credit spread, lev is market leverage, $\log(\sigma_E)$ is log equity return total volatility, $\log(\sigma_{E,A})$ is log equity return aggregate volatility, and $\log(\sigma_{E,I})$ is log equity return idiosyncratic volatility.

	1	2
AAA/AA	0.45	-0.58
BBB	0.45	-0.43
BB	0.46	0.09
B	0.44	0.33
CCC	0.43	0.61

Table 16: First Two Principal Component Coefficients This table displays the first two principal component coefficients of log credit spreads averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC.

	log(cs)	lev	$\log(\sigma_E)$	$\log(\sigma_{E,A})$	$\log(\sigma_{E,I})$
log(cs)	1.0000	0.2946	0.6178	0.4647	0.5903
lev	0.2946	1.0000	0.0823	-0.1613	0.2095
$\log(\sigma_E)$	0.6178	0.0823	1.0000	0.7074	0.8741
$\log(\sigma_{E,A})$	0.4647	-0.1613	0.7074	1.0000	0.4316
$\log(\sigma_{E,I})$	0.5903	0.2095	0.8741	0.4316	1.0000

Table 17: Second Principal Components Correlation Matrix This table displays the correlation between the second principal components of each variable averaged within 5 rating classes: AAA/AA, BBB, BB, B, and CCC where log(cs) is log credit spread, lev is market leverage, $\log(\sigma_E)$ is log equity return total volatility, $\log(\sigma_{E,A})$ is log equity return aggregate volatility, and $\log(\sigma_{E,I})$ is log equity return idiosyncratic volatility.

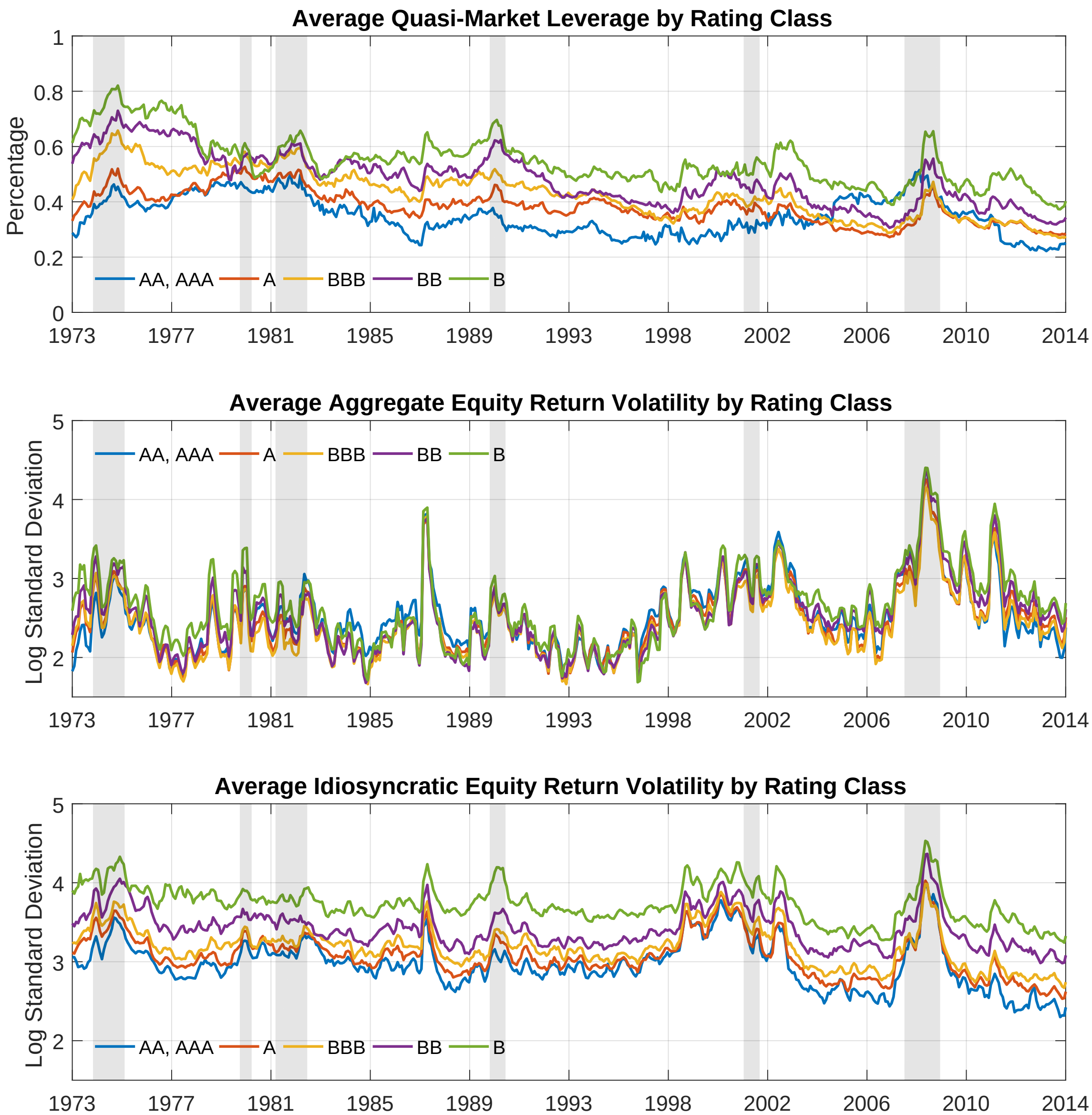

Figure 21: Average Time Series by Rating Class

		FINANCIAL INDICATORS								
		$\sigma_{E,A}$		$\sigma_{E,I}$		credit		bid-ask		default
		inv	spe	inv	spe	inv	spe	inv	spe	all
SHOCKS	no liquidity	3	1	6	3	42	38	**100**	**100**	4
	$\alpha(s) = \overline{\alpha}$	1	1	2	3	3	4	1	1	6
	$\sigma_C(s) = \overline{\sigma}_C$	11	4	22	11	45	53	10	18	9
	$\sigma_A(s) = \overline{\sigma}_A$	**101**	**102**	28	17	**49**	**53**	11	18	43
	$\sigma_I(s) = \overline{\sigma}_I$	6	6	**97**	**110**	10	11	3	3	**56**
	$y = \overline{y}$	**68**	**54**	**139**	**201**	**64**	**76**	**18**	**22**	**140**

Table 18: Shock Decomposition of Financial Indicators This table shows ratios of root mean squared errors (RMSE) of different versions of the model multiplied by 100. The numerator corresponds to the RMSE between the full model and the model without one of the fundamental shocks. The denominator corresponds to the RMSE between the full model and the model without any fundamental shocks. The sample period comprises the whole period from January 1973 to October 2014.